14.381 Problem Set 8
Statistics Fall, 2004

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Due Monday November 15, 6:00pm (in E52-204).

1. Find the maximum likelihood estimator of the following parameters:
   (a) λ for the case of an exponential distribution: $f(x|\theta) = \frac{1}{\lambda}\exp(-x/\lambda), \qquad 0 \leq x < \infty, \lambda > 0;$
   (b) μ and σ for $N(\mu, \sigma^2)$;
   (c) θ if the pdf is $f(x|\theta) = \theta x^{-2}, \qquad 0 < \theta \leq x < \infty.$
2. C&B 7.7
3. C&B 7.12
4. Suppose that the random variables $y_1, ..., y_n$ satisfy

$$y_i = \beta x_i + \varepsilon_i, \qquad i = 1, ..., n,$$

   where $x_1, ..., x_n$ are fixed constants, and $\varepsilon_1, ..., \varepsilon_n$ are $iid\ N(0, \sigma^2)$, σ^2 unknown.
   (a) Find the MLE of β, and show that it is an unbiased estimator of β;
   (b) Find the distribution of the MLE of β. (Hint: note that $\widehat{\beta}_{ML}$ can be written as a linear combination of the y_i, in particular $\widehat{\beta}_{ML} = \sum_{i=1}^{n} a_i y_i$).