14.41 Problem Set #1
Due: 9/23/05

Note: In mathematical problems, half of the credit will be given for correctly laying out the problem, and providing proper intuition for your answer (or, if you cannot solve it mathematically, intuition for what you think the answer should be). The other half will be given for deriving the correct mathematical solution.

1.) The newly elected president of an island nation has pledged to reduce air pollution. The nation has no close neighbors – the only sources of air pollution are the two domestic chemical plants run by firm A and firm B. Firm A has been in operation for 50 years and has pollution abatement (i.e. reduction) costs of x^3, where x is a unit of pollution. Firm B, which operates a 6-month old plant, has a pollution abatement cost of x^2. Assume that neither firm is initially engaging in pollution abatement. The per unit benefit to a unit of pollution abatement experienced by the island's citizens is constant at $300.

a) What is the socially optimal overall level of pollution abatement? What is the socially optimal allocation of that pollution across the two firms?

b) The president considers engaging in command-and-control style quantity regulation and declares that each firm must engage in 80 units of pollution abatement. Is this socially optimal? Why or why not?

c) Alternatively, the president considers providing a subsidy of $300 per unit of pollution abatement. What is the per firm and total level of pollution abatement? Is this socially optimal?

The president also considers issuing pollution permits and establishing a market for these permits. For reasons associated with the relative generosity of the firms to his recent election campaign, firm A is given permits such that it must engage in 100 units of pollution abatement if it fails to enter the permit market. Firm B is given permits such that it must engage in 60 units of pollution abatement if it fails to enter the market. Each permit allows the firm holding it to engage in one unit of pollution. Assume the market for pollution permits is perfectly competitive. Also assume that no firm may increase the amount of pollution it is engaged in (i.e. negative abatement is not allowed). This assumption restricts firm A to buying 100 or fewer permits and firm B to buying 60 or fewer permits.[1]

What is the market clearing price for pollution permits? What level of pollution abatement does each firm engage in? Is this socially optimal? How does the level of abatement compare here with part c? Dropping the perfect competition assumption, what problems are likely to arise in practice in the permit market?

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[1] Mathematically, this problem has 2 solutions, one of which is nonsensical in the context of the problem. This assumption rules out the nonsensical solution.

2.) Great Southern is a manufacturer of paper products in a small town. The plant releases chemical by-products of the chorine bleaching process into the river that runs through the town. These chemicals prevent the residents of the town from fishing in the river, or from using the river for recreation. The manufacturer can cut down on chemical emissions, and faces total costs $C(a) = a^2$ of doing so, where *a* is a unit of pollution abatement. A management consultant comes to town and reports that the benefit to the townsfolk of abatement is 10 per unit.

a) Draw a graph depicting the societal marginal costs and benefits of abatement, and label the efficient outcome. What is the exact level of pollution abatement that is socially efficient? If the town institutes a per-unit tax on chemical emissions, what level of tax will reach the socially optimal amount of abatement?

The town manager is considering either taxing the firm's pollution or requiring the firm to reach a certain level of abatement. Abatement technology is always improving, and the firm suspects that its costs of abatement might be $C(a) = a^2 - 6a$ if technology improves. The town manager wants to incorporate this uncertainty into her decision on whether to tax or regulate.

b) Suppose that the town manager institutes the tax you found in (a). If the true costs of abatement end up being $C(a) = a^2 - 6a$, then how much will the firm abate? Answer by first drawing a new graph that illustrates the societal marginal costs (under both the predicted and actual marginal cost schedule) and marginal benefits of abatement. Label the social optimum and actual outcome under the actual marginal cost schedule – and indicate the deadweight loss, if any. Then, calculate the exact level of abatement undertaken by the firm. Is this level socially optimal? Why or why not? Compute the actual amount of deadweight loss associated with mismeasuring the costs of abatement.

c) Suppose that the town manager institutes a mandatory abatement at the socially optimal level you found in (a). If the true costs of abatement end up being $C(a) = a^2 - 6a$, then how much will the firm abate? As in (b), illustrate the problem and solution graphically. What is the exact level of abatement undertaken by the firm? Is this level socially optimal? Why or why not? Compute the deadweight loss associated with mismeasuring the costs of abatement.

d) Given the uncertainty in abatement costs, which strategy makes the most sense for reducing pollution?

e) A new study comes out linking the chemicals released by the paper mill to cancer in humans. The town must now re-think the abatement analysis given this new information. The management consultant returns and declares that given the cancer risk, the marginal benefit to abatement is now 96-4*a*.

i) Is there a chance that your answer to (d) may not hold under the new marginal benefit curve? Why or why not?

ii) Using this new marginal benefit schedule, find the optimal level of pollution abatement and the tax on the paper mill required to reach this level. As in (a), illustrate this graphically, and determine the exact solution as well.

The firm still isn't sure about its abatement costs, and the town manager wants to again decide between tax or regulation, taking this uncertainty into account.

iii) Suppose that the town manager institutes the tax you found in (ii). If the true costs of abatement end up being $C(a) = a^2 - 6a$, then how much will the firm abate? Again, demonstrate the solution graphically as in (b), and also calculate the exact level of abatement. Compute the amount of deadweight loss associated with mismeasuring the costs of abatement.

iv) Suppose that the town manager instead institutes a mandatory abatement at the socially optimal level you found in (ii). If the true costs of abatement end up being $C(a) = a^2 - 6a$, then how much will the firm abate? Demonstrate the solution graphically as in (c). What is the exact level of abatement? Compute the deadweight loss associated with mismeasuring the costs of abatement.

v) Given the uncertainty in abatement costs, which strategy makes the most sense for reducing pollution?

f) Given the uncertainty of abatement costs, does the policy strategy differ when the marginal benefit to abatement is 10 per unit than when the marginal benefit is 96-4*a*? If so, how? Explain the intuition for this result as best you can.

3.) For each of the examples below, answer the following three questions:

i) Is there an externality? If so, describe it, including references to whether it is positive or negative, and whether it is a consumption or production externality. If you are uncertain, describe the conditions under which there may, or may not, be an externality

ii) If there is an externality, does it seem likely that private markets will arise that allow this externality to be internalized?

iii) If there is an externality, contrast the effectiveness of each of the following corrective measures that the government can use to address the problem:

1) direct quantity regulation
2) taxes/subsidies
3) permits with trading

For each of (1)-(3), discuss the advantages and disadvantages, and conclude as to which would be best, or whether government intervention would do more harm than good.

a) Purchasing products that use large amounts of packaging
b) Subsisting on a diet of fast food
c) Talking on a cellphone in the library
d) Getting a high school education

4.) The town of Springfield has been hit by a bizarre crime wave (bands of armed robbers attacking DMV employees). In response to this crime wave – and since the current local law enforcement is extraordinarily incompetent - a new police department will be formed. The city has two residents : Patty and Selma. Each has a utility function over cigarettes (X) and total policemen (M), of the form: $U = 2*\log(x) + \log(M)$. The total provision of policemen hired, M, is the sum of the number hired by each of the two persons: $M = M_P + M_S$. Patty and Selma both have income of 100, and the price of both cigarettes and a policeman is 1. They are limited, for the purposes of this problem, to providing between 0 and 100 policemen.

a) How many policemen are hired if the government does not intervene? How many are paid for by Patty? By Selma?

b) What is the socially optimal number of policemen? If your answer differs from (a), why?

c) Suppose the government is not happy with the private equilibrium, and it decides to provide 10 policemen. It taxes Patty and Selma equally to pay for the new hires – but Patty and Selma are free to hire additional policemen if they would like. (So to be clear, Patty and Selma each begin with an income of 100, but are taxed equally for the public provision of 10 policemen – and may hire more policemen if they prefer). What is the new total number of policemen? How does your answer compare to (a)? Have we achieved the social optimum? Why or why not?

d) Suppose now that the government is still not happy, so it decides to provide a total of 35 policemen. It taxes Patty 10 to pay for them, and it taxes Selma 25. (So as in (c), Patty and Selma each have a pre-tax income of 100, and can hire policemen in addition to the 35 already provided - if they find it optimal to do so). What is the new total number of policemen? How many are provided by Patty, and how many by Selma? How does this compare to the level of provision in (c), and why?