14.41, Fall 2005
PS #1 solutions:

Question 1:

1a) The social optimum is where each firm's marginal cost of abatement is equal to the marginal benefit of abatement:

$$MC_A=dC/dx=3x^2$$

$\rightarrow$at the optimum, where MC_A=SMB,

$$3x^2=300$$
$$x=10$$

$$MC_B=dC/dx=2x$$

$\rightarrow$at the optimum, where MC_B=SMB,

$$2x=300$$
$$x=150$$

The social optimum is 160 units of abatement, 10 by Firm A and 150 by Firm B.

b) At 80, Firm A's marginal cost of abatement is:

$$MC_A=3(80^2)=19200$$

while Firm B's marginal cost is:

$$MC_B=2(80)=160$$

Since Firm B's marginal cost is below Firm A's, Firm A could abate one unit less and Firm B one unit more and society would save 19200-160=$19,040 while achieving the same level of abatement. Thus, this outcome is not socially optimal.

c) Here, Firm A will set marginal cost to $300, as will Firm B, and (as shown in part a) Firm A will abate 10 while Firm B will abate 150, for a total of 160. This is socially optimal, as each firm has internalized the $300 social benefit of abatement because of the Pigouvian tax (subsidy).

d) The easiest way to think about this is to realize that, if there is perfect competition in the permit market, the firms will trade permits until the marginal costs of abating an additional unit equals the price of the permit. Why? Think of it this way: without trading, firm A must abate 100 units. The marginal cost of abating that 100th unit is extremely high (30,000). Firm B must only abate 60 units, and the marginal cost of abating that last unit is only 120. Clearly there is room for trades here, since firm A is willing to pay up to $30,000 not to abate that last unit, but additional abatement only costs firm B $120. Under perfect competition, the price that results will be the one such that the marginal costs of abatement for each firm are equal – because then there are no additional gains to trade to be had.

Denote the amount that Firm A abates by x_A, and the amount that Firm B abates by x_B. Setting marginal costs equal:

$$3x_A^2 = 2x_B \Leftrightarrow x_B = \frac{3}{2}x_A^2$$

Using the constraint that total abatement must be 160 (Firm A begins by having to abate 100, Firm B begins having to abate 60), the abatement constraint is:

$$x_A + x_B = 160$$

So plugging in:

$$x_A + x_B = 160 \Leftrightarrow \frac{3}{2}x_A^2 + x_A - 160 = 0 \Leftrightarrow 3x_A^2 + 2x_A - 320 = 0 \Rightarrow x_A = 10 \ or \ x_A = -\frac{32}{3}$$

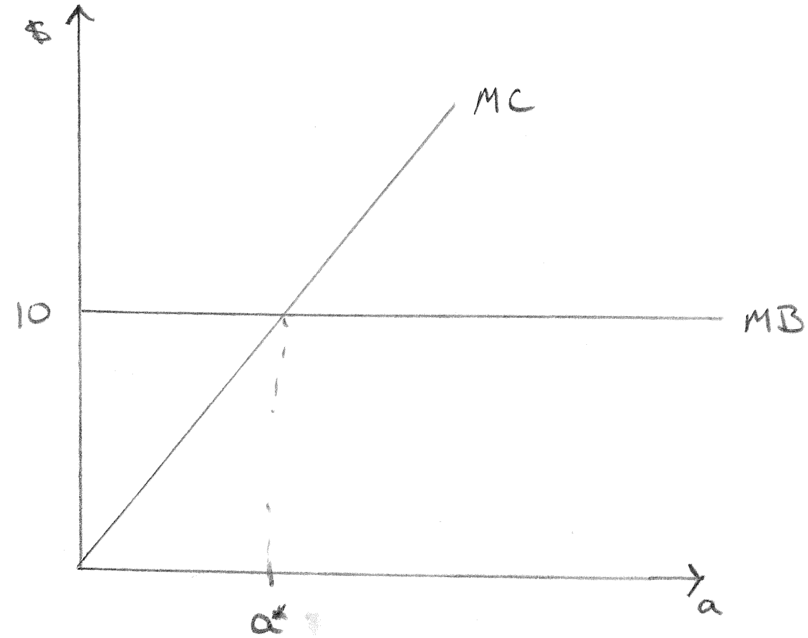
But the problem stated that only positive abatement is allowed, so Firm A abates 10 units, which leaves 150 units to be abated by Firm B. This is the socially optimal solution from (a). The price of an abatement voucher in this market will be the marginal cost of abatement, $300 – because with this price, neither firm is willing to make additional trades.

In practice the market may not be perfectively competitive when there are only two participants. From the initial endowment of abatement levels, Firm A's marginal costs to abatement are much greater than Firm B's - so Firm B has monopoly power in this situation, and may choose to set the price for abatement vouchers above $300. On the other hand, Firm A is the only buyer in the market, so if Firm A has greater bargaining power, it may be able to bargain the price of abatement vouchers below $300. (As an extreme example, suppose Firm B happens to have all the bargaining power. For every unit of abatement that Firm B offers to abate instead of Firm A, Firm B can change Firm A's full marginal benefit from the transaction – so that Firm B gets all the surplus from the trading relationship. In other words, with complete bargaining power, Firm B can act as a price discriminating monopolist.) Either way, this would result in a distribution of abatement between the two firms that is different from the socially optimal level in (a). Another potential problem is political credibility: the market will only function if the participants believe the government will honor the property rights conveyed by the permits. If the firms believe the president will not be re-elected and a new president will not honor the property rights conveyed by the permits (or if firms have different beliefs on the probability that permits will be honored in the future), then the market may not function properly – and the socially optimal distribution of abatement may not be reached.

Problem 2

a) Marginal cost of abatement $= \frac{d}{da}(c(a)) = \underline{2a}$

Marginal benefit $= 10$

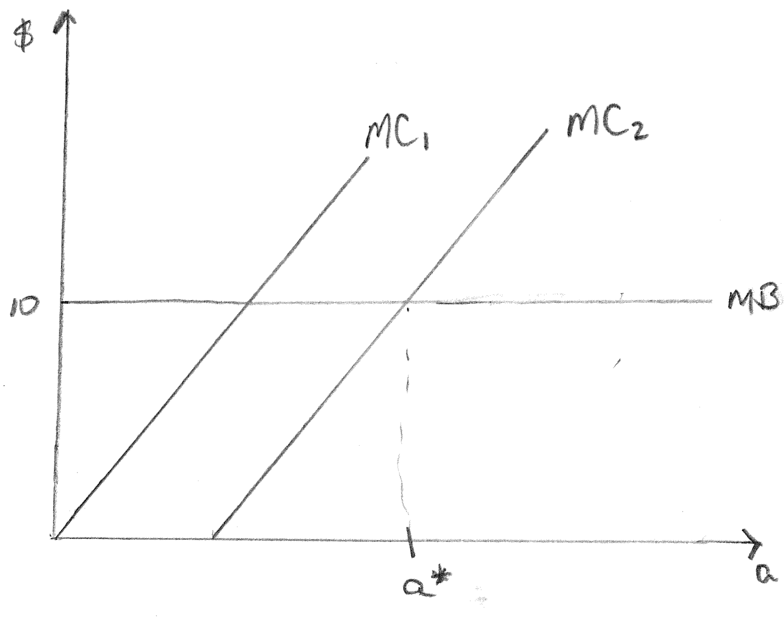


Setting $MC = MB$, $2a = 10$, $\underline{a^* = 5}$

The per-unit tax required to reach a^* is $\tau = 10$

b) MC_1 (predicted) $= 2a$

MC_2 (true) $= 2a - 6$

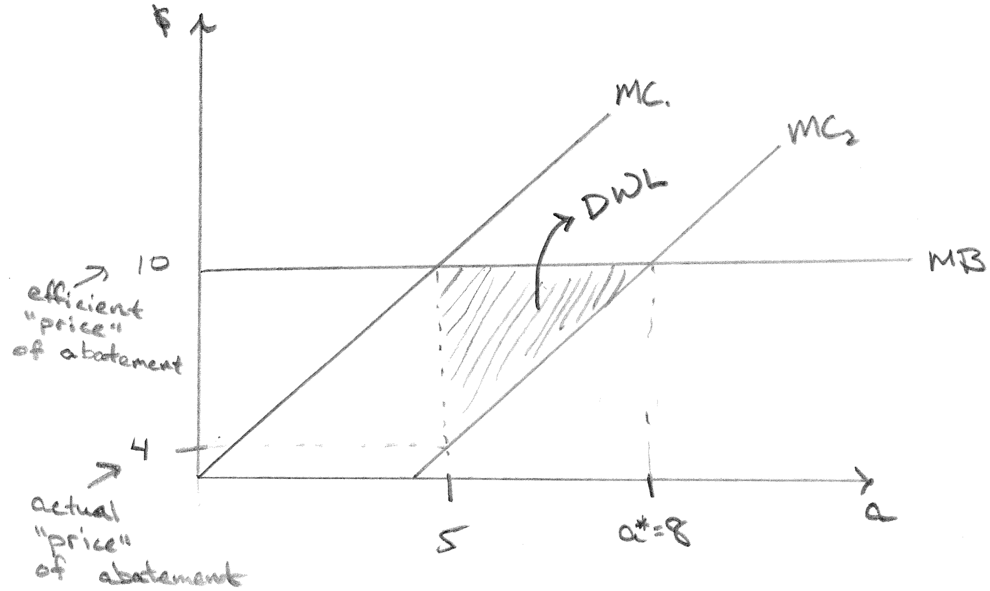


Because the marginal benefit curve is flat, the optimal "price" of abatement will not change if the true MC curve changes. Thus the tax found in part (a) is still efficient and there is no DWL. The new optimum is where

$2a - 6 = 10$, $\underline{a^* = 8}$

Problem 2

c) The firm will abate the required 5 units.



In this case, the firm will abate less than the socially efficient level. The DWL is given by the area of the shaded triangle.

$DWL = \frac{1}{2}(6)(3) = \underline{9}$

d) In this case, a tax will be efficient regardless of the true Marginal cost curve. We therefore prefer the tax.

e) (i) Now that the MB curve is downward sloping, a tax may not always be efficient. We need to compare the DWL associated with mispredicting the cost under a tax & regulation to determine which policy is preferred now.

(ii) Setting MB = MC, $96 - 4a = 2a$

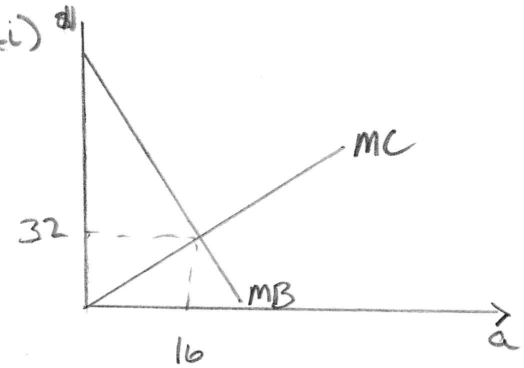
$6a = 96$

$a^* = \underline{16}$

The socially optimal tax is $\tau = 96 - 4(16) = \underline{32}$.

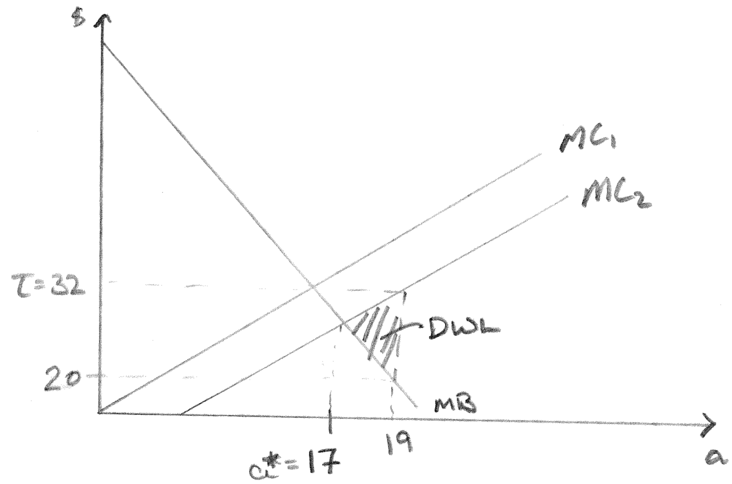
Problem 2

e) (ii)



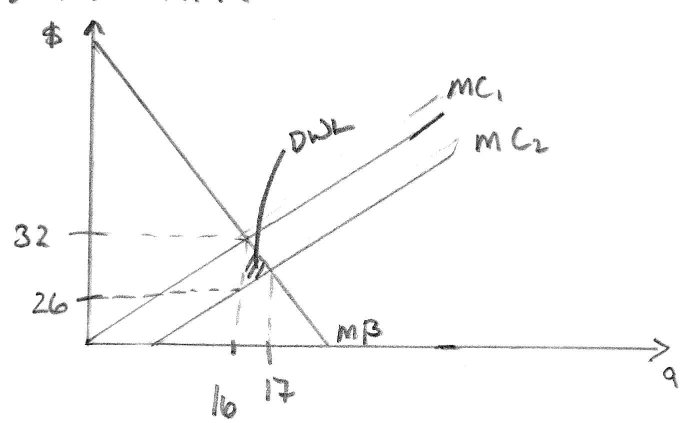
(iii) Under the tax, the firm will abate until $MC_2 = \tau$ or $2a - 6 = 32$

$\Rightarrow$ $\underline{a = 19}$



$DWL = \frac{1}{2}(2)(12) = \underline{12}$

(iv) The firm will abate the required 16 units



$DWL = \frac{1}{2}(1)(6) = \underline{3}$

(v) In this case, the quantity regulation minimizes the DWL associated with mispredicting the costs of abatement.

Problem 2.

f) With the new marginal benefit schedule, the optimal policy is now quantity regulation instead of taxation.

The intuition for this result is tricky, and I was accordingly generous with the grading. Here's the basic idea: If the MB curve is flat, uncertainty in MB implies large uncertainty in quantities but relatively little in prices. Thus, the tax you set won't change much when the MC curve changes. On the other hand, the optimal quantity will change a lot, so if you fix the quantity using regulation you will have large inefficiencies. (Take it as given that missing <u>either</u> tax or quantity by a lot will result in greater inefficiency -- this turns out to be the case.)

If the MB curve is steep, then the optimal quantity won't change much with a surprise shift in marginal costs, but the optimal tax will. Thus, fixing the tax will result in greater efficiency loss than quantity regulation.

Question 3:

A couple repeated mistakes arose throughout question 3. First, *consumption* externalities result when an individual's private consumption of a good causes another to experience some disutility. A *production* externality results when something about the production process of a good causes another to experience some disutility. In the cellphone example, excess noise is produced due to the *consumption* of cellphone minutes; there is no production of a product to sell, and so the externality in this case is consumption rather than production.

Also, there was some confusion as to what a "private market solution" would be. The way a private market would settle externalities is generally by paying the one who creates the externality to reduce their externality-generating action (or otherwise threatening or coercing them to stop). So, for instance, in the education example, the existence of private schools is *not* a private solution to education externalities. Private schools allow parents to purchase higher quality education, if they think that the relative benefits of a private schooling education are greater than the tuition costs. Private schools do nothing to help anyone internalize the externality of education. A "solution" to the externality would be anything that encourages those who don't receive a high school education to do so. So one private market solution would be if everyone banded together and paid dropouts to remain in school.

a) There is very likely a consumption externality here. This is because consumers generally do not face the marginal costs of their excess garbage, unless they live in communities where garbage collection fees are charged on a per-pound, or per-bag basis (in some communities in America, one has to purchase garbage tags from the local government, and only garbage bags with tags will be collected). Even if consumers faced the true marginal costs of their garbage, negative consumption externalities could still exist if consumers threw packaging away in public garbage cans, or if packaging often ends up being improperly disposed of as litter. Under any of these circumstances, the consumer doesn't face the true marginal costs of excess packaging, so there is an externality that private markets wouldn't correct. (Even though extra packaging should cost the consumer more, this still doesn't internalize the externality, since extra packing imposes two costs: the cost of the material used, and the disposal costs. If firms charge consumers more for extra packaging, chances are this is simply to cover the costs of material and not to correct an externality.)

Some argued that a negative production externality exists, if pollution is generated from the production of extra packaging. This is also a possible externality.

This is a situation where the costs of reducing packaging are likely to be highly uncertain (what do firms lose from having to reduce packaging? Products may be more liable to breakage, or demand for certain products may fall if the excessive packaging is necessary for ease of display in a store, or to catch a shopper's eye. In any case, this is probably really difficult to measure). It's also a situation where the marginal benefit curve is probably quite flat, so the proper intervention is a tax rather than quantity regulation.

One way a tax could work is by taxing packaging per-ounce or per-pound. The tax could be levied on either the consumer or the producer.

b) As Professor Gruber mentioned in lecture (and as is written in the book), the biggest negative consumption externality from eating lots of fast food is likely higher health care costs due to obesity. As many pointed out, this isn't a problem if everyone pays the direct costs of their own health care, but if health care is entirely provided for a certain segment of the population (as Medicare and Medicaid are), then the obese who receive this free health care do not face the marginal costs of their own obesity. There are likely other negative consumption externalities, such as increased pollution (if they are more likely to drive than walk due to health problems) – but health care is the main one. There is also a positive consumption externality, because obese individuals are likely to die sooner and hence receive less from social security funds over their lifetime.

Clearly, quantity regulation enforced by preventing people from consuming excess fast food is infeasible, impractical, and likely unconstitutional. Quantity regulation enforced by limiting the number of fast food restaurants might be more feasible, but would be hard to implement (how to define unhealthy food? How to enforce?) and would undoubtedly be unpopular. Taxation might work: as mentioned in the book, a per-person tax on body weight (or "skinny subsidy") are possible, as well as taxation of fast food. Permits would be difficult to operationalize, as it would require giving all Americans a certain quantity of fast food vouchers, which one would need to present when purchasing fast food. If permits existed, they would help internalize the externality, because people who wanted to consume more fast food would have to pay more to purchase additional permits (remember, from an efficiency viewpoint, it's okay if the obese remain obese – as long as they face the true costs of their actions).

c) Clearly this is a negative externality. It's a consumption externality because the consumption of cell phone minutes results in disutility to others in the library (it's not a production externality because the cell phone users aren't using the phone to produce anything of value to others). A private market solution would exist if everyone in the library banded together to pay the cellphone users to stop – or, if one person with an intense dislike for library cellphone users were to pay them on his own. Non-monetary private solutions include the threat of physical violence, or the existence of strong social norms that shun cell phone use. Quantity regulation seems like the right solution, which is the current system in place (quantity is limited to zero), because taxes would be too difficult to implement (you'd have to go to and pay a librarian every time you wanted to use your phone).

d) We intended you to think of this problem from the perspective of an individual who receives education, so that education is a positive consumption externality – but some looked at it from the perspective of the school, so that education is a positive production externality. Either way is an acceptable way to think of the problem, as long as it was clear to us that you put thought into the nature of possible externalities. There are a number of positive externalities that result from education: educated people are more likely to vote, are less likely to commit crimes, know more about proper health and hygiene such that they are healthier, and have more knowledge to transmit to others through informal or mentoring relationships. It might also be the case that smarter people make those around them smarter, or smarter people raise smarter kids, who make their classmates smarter. Smarter cities may be more likely to attract higher paying jobs, benefiting everyone in the area.

Given these positive externalities from education, a private solution will not exist. Why? Because to correct this externality, people would have to pay high school dropouts to go back and get their diploma, which is unlikely to happen. Being paid higher wages because of higher education *does not* correct the externality, because that only compensates the individual for his higher productivity and value due to education – a firm would never pay a well educated employee more because that person has smart kids who go to school and make their classmates smarter. Similarly, the existence of private schools is not a private solution, because they exist so that parents can purchase a higher quality education – they're not encouraging dropouts to get additional education so that their peers benefit.

Depending on your perspective, likely government solutions include quantity regulation (providing public schools and mandating people receive a certain amount of education) or subsidies (subsidizing education by providing free schools). Permits are completely infeasible – they would require people who want to dropout to purchase "dropout permits" from people who want to stay in school (in this manner, dropouts would have to *pay* to dropout, which would help them face the costs of their actions).

Question 4:

a) Because both Patty and Selma have the same utility functions, we say that the problem is *symmetric* – so the solution should be the same for each. Consider the maximization for Patty:

$$\max \; 2\log(x_p) + \log(M_p + M_s)$$
$$\textit{such that: } 100 = x_p + M_p$$

i.e. she's choosing x and M to maximize her utility subject to her budget constraint, given that Selma is providing some level of policemen M_s. $M = M_p + M_s$ will be the total number of policemen that end up being provided. The important thing to note is that Patty decides how many policemen to provide *in addition* to the amount that Selma is already providing.

To solve for the amount of cigarettes consumed and policemen provided by Patty, I find it easiest to:

1) rearrange the budget constraint in terms of x_p: $x_p = 100 - M_p$
2) plug this back into the utility function: $U = 2\log(100 - M_p) + \log(M_p + M_s)$
3) maximize this utility function with respect to M_p, by taking the derivative and setting it equal to zero:

$$\frac{dU}{dM_p} = \frac{-2}{100 - M_p} + \frac{1}{M_p + M_s} = 0$$
$$-2M_p - 2M_s + 100 - M_p = 0$$
$$3M_p = 100 - 2M_s$$
$$M_p = \frac{100}{3} - \frac{2}{3}M_s$$

This function gives the optimal level of policemen that Patty would provide, given Selma provides M_s.

Since the problem is symmetric, Selma's solution takes a similar form[1]:

$$M_s = \frac{100}{3} - \frac{2}{3}M_p$$

[1] If you have any game theory experience, you should recognize these functions as "best response functions" – since they give one person's "best response" (optimal choice) given the other person's actions. You should also recognize that the solution to this problem is a Nash equilibrium, found by calculating the intersection of the two best response functions.

The key here is that Patty knows that Selma will provide policemen in this fashion, and uses this information when making her own decision. Thus, we can simply take Selma's best response function to Patty and plug it in to Patty's function for providing policemen, and solve:

$$M_p = \frac{100}{3} - \frac{2}{3}M_S = \frac{100}{3} - \frac{2}{3}\left(\frac{100}{3} - \frac{2}{3}M_p\right) = \frac{100}{9} + \frac{4}{9}M_p$$
$$\Leftrightarrow \frac{5}{9}M_p = \frac{100}{9} \Leftrightarrow M_p = 20$$

since the problem is symmetric, $M_S = 20$ (you could also get this by plugging $M_p = 20$ into Selma's best response function), and total policemen provided is 40.

b) We want to find the level of M that maximizes social surplus, since such a level would be socially optimal. The easiest way to do this is to set the sum of the marginal rate of substitution between cigarettes and policemen (MRS) for Patty and Selma equal to the marginal rate of transformation between the two goods (MRT)[2]. The intuition behind this is that the MRT measures the relative marginal cost to society (SMC) of purchasing an additional policeman, and at the social optimum, this should be equal to the relative marginal benefit to society of having another policeman (SMB). However, when providing another policeman, the benefits accrue to *both* Patty and Selma. Since the relative marginal benefit from an additional policeman to one person is their MRS, the relative marginal benefit to society is the sum of their MRS.

$$MRT = \frac{P_M}{P_x} = 1$$

$$MRS_P = \frac{MU_M}{MU_x} = \frac{\frac{1}{M_P + M_S}}{\frac{2}{x_p}} = \frac{x_p}{2(M_P + M_S)}$$

$$MRS_S = \frac{x_S}{2(M_P + M_S)}$$

$$MRS_P + MRS_S = \frac{x_p + x_S}{2(M_P + M_S)} = \frac{x_p + x_S}{2M} = MRT = 1$$

Now, the money used to purchase the policemen must come out of their incomes (you can assume that they're taxed equally for the policemen, but it actually doesn't matter), so we know: $x_p + x_S + M = 200 \Leftrightarrow x_p + x_S = 200 - M$

[2] Note: you could also solve this problem by maximizing a *social welfare function*, which is the sum of Patty and Selma's utility, although while doing this you would in fact derive the condition that the sum of MRS must equal MRT.

Plugging this in to the above equation gives us:

$$\frac{200-M}{2M}=1 \Leftrightarrow 3M=200 \Leftrightarrow M=\frac{200}{3}=66\frac{2}{3}$$

Hence, the socially optimal level of policemen is $66\frac{2}{3}$. (What is two-thirds of a policeman? Perhaps a very incompetent one…?) This level is greater than that in (a) because when Patty or Selma considered adding another policeman, they just considered the additional benefit that it would bring to them individually – ignoring the fact that an additional policeman brings additional social benefit, because the other person would benefit from that additional policeman as well. You could think of it almost as an externality: each person only considers the benefit that the public good would bring to them individually, rather than also considering the benefit that accrues to society from providing an additional unit. (The other difference is that, because the solution is a Nash equilibrium, if one person were to increase their provision even further, it would encourage the other person to *reduce* their own provision of the public good due to the free rider problem).

c) This problem is similar to (a), except now Patty and Selma are taxed equally for 10 policemen – so they go through the same decision process as in (a), but this time have an income of 95 and decide how many more policemen to provide given that 10 are already provided.

So Patty's problem becomes:

$$\begin{aligned}&\max\ 2\log(x_p)+\log(M_p+M_S+10)\\&\textit{such that: } 95=x_p+M_p\end{aligned}$$

To solve:

$$\begin{aligned}&\frac{dU}{dM_p}=\frac{-2}{95-M_p}+\frac{1}{M_p+M_S+10}=0\\&-2M_p-2M_S-20+95-M_p=0\\&3M_p=75-2M_S\\&M_p=25-\frac{2}{3}M_S\end{aligned}$$

Selma clearly has the same best response function, so plugging Selma's BRF in to Patty's, and solving through for M_p yields the solution

$$M_p=15,\ M_S=15,\ M=M_p+M_S+10=40$$

So the number of policemen provided is exactly the same as in (a)! Why? The intuition is that since Patty and Selma are taxed equally for the policemen, and because the amount that is provided is *less* than what they would provide without any government, they treat the tax as if they were just paying for five policemen on their own. Since they wanted to provide 20 each in the absence of government intervention, they'll just supplement the five they're already providing by purchasing an additional 15 each – and in sum, the total number of policemen is the same as in (a).

d) It should be clear that the solution will not be symmetric here, because Patty and Selma are being taxed unequally for the provision of policemen. We should also suspect that Selma may not provide any additional policemen in this case, because she's already providing (through taxes) more than she wanted to provide in (a) (but we'll check this).

So Patty's problem is:

$$\max\ 2\log(x_p)+\log(M_p+M_S+35)$$
$$\textit{such that: } 90=x_p+M_p$$

And Selma's is:

$$\max\ 2\log(x_S)+\log(M_p+M_S+35)$$
$$\textit{such that: } 75=x_S+M_S$$

Solving each individually, we get that the best response functions are:

$$M_S=\frac{5}{3}-\frac{2}{3}M_p$$
$$M_P=\frac{20}{3}-\frac{2}{3}M_S$$

Plugging Patty's into Selma's:

$$M_S=\frac{5}{3}-\frac{2}{3}M_p=\frac{5}{3}-\frac{2}{3}\left(\frac{20}{3}-\frac{2}{3}M_S\right)\Leftrightarrow M_S=-5$$

It turns out that given Patty's best response function, Selma thinks there are *too many* policemen provided! That is, she would prefer kidnapping five policemen and selling them to Shelbyville for \$1, and buying five cigarettes with that money. Since we don't allow any usurping of public goods in this fashion, Selma is stuck providing more policemen than she wants to – and certainly won't provide any additional policemen above the 25 that she's being taxed for. Hence, $M_S=0$. Patty's solution is pretty easy now, since we just plug 0 in for M_S. So $M_P=\frac{20}{3}$, $M=0+\frac{20}{3}+35=\frac{125}{3}=41\frac{2}{3}$.
More policemen are now provided than in the no government intervention case (a and c) – this is because Selma is being forced to pay more than she would in the absence of

government intervention. However, the increase in public good provision relative to (a) and (c) is slight (and far from the socially optimal level) because Patty can free-ride on Selma's provision (in fact, Patty only provides 16 2/3 policemen now – which is less than the 20 she provides without government intervention).