

## 14.41 Problem Set #2

Due: 10/7/05

1) Suppose a passenger aircraft crashes into a remote Pacific island, leaving only three survivors. After days of waiting, it becomes apparent that help may never arrive without some effort by the survivors. Fortunately, this small island is covered with coconut trees. One survivor (Michael) suggests building a raft with the coconut trees to sail off the island and search for help. Another survivor (Shannon) doubts that help would be found with a raft, and prefers making a shelter with the wood, to make island life more comfortable. The third survivor (Jack) cares equally for building a raft and building a shelter.

The castaways must determine how many trees to devote to the raft, and how many to shelter. The more trees used for the raft, the more likely it is to be successful. The more trees used on shelter, the more comfortable the shelter will be. Suppose there exists 100 trees for use on this island.

Specifically, the three castaways have the following utility:

$$\text{Jack: } U_J = \frac{1}{2} \ln R + \frac{1}{2} \ln S$$

$$\text{Michael: } U_M = \frac{3}{4} \ln R + \frac{1}{4} \ln S$$

$$\text{Shannon: } U_S = \frac{1}{4} \ln R + \frac{3}{4} \ln S$$

Where  $R$  is the number of trees used in the building of the raft, and  $S$  is the number of trees used to build the shelter.

- How many trees would each person choose to devote to the raft, and how many would each person devote to the shelter, if the decision were solely theirs to make?
- On a single graph, sketch a plot of each person's utility as a function of the number of trees used for the raft. (Hint: plot the person's optimal number of trees for the raft, and think about what the function would look like if more or less trees were used than that optimal amount).
- Suppose that the decision on how many trees to devote to a raft is made by majority vote. Based on your results from b, would you expect majority voting to yield a consistent outcome? Explain.
- What final outcome would you expect under majority voting? Explain intuitively why a majority vote would settle at this outcome.
- Suppose four more survivors (from the tail section of the plane) are discovered.

Three of these survivors have utility functions  $U_1 = \frac{4}{5} \ln R + \frac{1}{5} \ln S$ , and the other

has utility function  $U_2 = \frac{1}{5} \ln R + \frac{4}{5} \ln S$  (so three really want to be rescued, and

- one really likes shelter). If these new survivors were also given votes, how would you expect the decision under majority voting to change from that in (c)? Why? Without carrying out additional calculations, can you determine the exact decision that would result from majority voting?
- f) Suppose instead of those four survivors, only two additional survivors are found. They each have utility functions  $U_3 = \max(R, T)$  - so they don't care whether trees are used for a raft or a shelter, as long as resources are only devoted to one use. Add this utility function to your sketch from (b). Will a consistent outcome emerge from majority voting? If so, what is the new outcome. If not, explain or demonstrate why.

2) After years of pleading, MIT has finally convinced the city of Cambridge to consider converting Memorial Drive to park land, such that all future generations of students and faculty would finally be able to enjoy the peaceful and scenic stretch of land along the Charles. It has been proposed to turn Memorial Drive into an underground tunnel, and then tear up the road and convert it into a park (similar to the Esplanade) that anyone could use. Due to the nature of the tunnel, the speed limit would need to be reduced from its current level of 30 mph to 25 mph.

The city of Cambridge is trying to decide whether this project is worthwhile in the long run.

- a) Besides the physical costs associated with building the tunnel, what other costs associated with this project would Cambridge want to consider? Give some benefits that would result from this project. Are these long-run (many year) benefits, or short-run? How could the town go about estimating these benefits?
- b) Suppose Cambridge decides to estimate these benefits by polling residents of the city and asking them how much a park along the river would be worth. What are the pluses and minuses of this approach?
- c) The city is considering other benefits that the park would bring besides just the value of the new park land to people who will use it. One possible benefit is that because the new park makes life as an MIT student happier, MIT is able to raise tuition more than it would have. Should this extra revenue to MIT count as an additional benefit? Why or why not.
- d) Suppose that, instead of taking the new Memorial Drive tunnel, many motorists decide to drive on Broadway. As a result, shops and restaurants along Broadway receive an increase in business. Should this extra revenue to businesses be counted as additional benefits? Why or why not.
- e) Now, let's try to determine the costs of this project. Suppose the only three costs of the project are the initial construction expenses, the annual maintenance costs, and the cost to drivers from the reduction in the speed limit. The initial expense in digging the tunnel and replacing the road with park is \$200 million. Yearly maintenance costs to the tunnel are \$5 million per year *above* what costs would have been if Memorial Drive was left as it is. 200,000 cars use Memorial Drive annually, and for simplicity, assume that the average length that someone drives on Memorial Drive is five miles. Assume that the cost of a person's time is \$.30 per minute. For simplicity, assume that the project initial expenses must be paid in the current year, but the maintenance and slow-down in driving costs don't begin accruing until the following year.
  - i) Assuming a social rate of discount of 10%, what are the aggregate costs (PDV) of this project?
  - ii) In calculating the costs, the city assumes no change in behavior for drivers. Why might this assumption be incorrect? Would this possible error imply that the city's cost estimate is too high or too low?
  - iii) Suppose that the annual benefits to the park are constant and begin in the following year. What is the minimum yearly benefit that the park must provide for the project to be undertaken (still assuming a social discount rate of 10%)?

3) The only public good that residents of Cambridge care about is parks. Suppose (for simplicity) that everyone in town has identical preferences and income, and is taxed equally for public good provision. Suppose their aggregate preferences for parks and private goods spending can be represented with a utility function of the following form:  $U = \frac{1}{10} \ln P + \frac{9}{10} \ln X$ , where P represents park land in acres, and X represents private good spending. Suppose parks cost \$10,000 per acre.

a) Assuming that total resources in the community are \$5,000,000, how many acres of parkland will be demanded? Demonstrate the optimal choice graphically (using a budget constraint and indifference curve).

A report is released stating that Massachusetts residents are more obese this year than in previous years. The state would like to reverse these trends by encouraging exercise. Towards this goal, it decides to give communities grants to improve local parks and encourage outdoor activities.

b) Suppose the state gives Cambridge a block grant of \$300,000 – no strings attached (although they say it should be used for parks, Cambridge is entirely at its own discretion).

- i) How would you expect provision of park land to change? Should this be an income effect, a substitution effect, or a mixture of both? Show this change graphically with budget constraints and indifference curves. Determine mathematically the new amount of park land provided.
- ii) It turns out that, in actuality, the full amount of \$300,000 is used for park land (so park land increases by 30 over your answer in (a) ). What might be some reasons for this?
- iii) If the full amount of \$300,000 is used for park land, but city residents object to this use of the grant (they view it as inefficient), how does the Tiebout model suggest this will be corrected?

c) Now suppose that the state gives a conditional block grant instead: the state will give Cambridge \$300,000, but it *must* be used for park land. How do you expect park provision to change from your answers in (a) and (b)? Show this change graphically with budget constraints and indifference curves, and determine mathematically the new amount of park land provided.

d) Suppose instead that Cambridge receives a matching grant, such that each \$1 of spending for a park is matched by \$.40 from the state. Now, how do you expect park provision to change? Should this be an income effect, a substitution effect, or a mixture of both? Show this change graphically with budget constraints and indifference curves, and determine mathematically the new amount of park land provided.

e) From the perspective of the state, which of these options (block grant, conditional block grant, or matching grant) should be preferred?

f) Give one instance when a state should give a matching grant rather than a conditional block grant, and one instance when a state should give a conditional block rather than a matching grant.

4) For several years, the US experienced budget surpluses. Some members of Congress proposed that we spend part of the surplus on education, but they couldn't agree on how the money should be spent. Identify the key economic aspects of the following questions in your answer.

a) The federal Hope Scholarship program, which began in 1998, gives families who pay income taxes (those with incomes above about \$20,000/year) a credit of up to \$1500 against their federal income taxes for money spent on the first two years of college. Congressman A, whose district is in the suburbs, says that we should increase the amount of the Hope Scholarship program because people who go to college earn more and are better able to support their families. Is Congressman A correct? Evaluate his point as a justification for this program.

b) Congressman B, whose district is in the inner city, says that we should spend more to improve our public elementary schools instead, because everyone goes to elementary school but only middle-class and upper-class people go to college. Evaluate this rationale for government spending on elementary education

c) Congressman C, a former public economics student, overhears their conversation and says that the government shouldn't spend any more money on elementary education because each family has already chosen its preferred level of elementary education spending. Explain what her argument is and why you do or do not think it applies.

d) Congressman D, a Republican, says that we should give each community a block grant that must be spent on local education. Congressman E, a Democrat, argues that we should instead have a program where we provide matching grants to localities to match their spending on hiring teachers. Which proposal is likely to lead to the largest increase in education spending, and why (use graphs in your answer)?

e) The state of Massachusetts is also considering increasing its spending on education due to a state budget surplus. One proposal is to lower tuition at state universities by \$1000. A second proposal is to give state residents a \$1000 grant they can use to attend any university in the state. Discuss how these two proposals will affect college attendance and private spending on education. Explain which proposal you would support and why.