14.41 Problem Set #2 solutions
Fall 2005

1a) each person's utility maximization problem is of the following form:

$$\max \quad a\ln R + b\ln S$$
$$such\ that: 100 = R + S$$

Rearranging the budget constraint in terms of S, and plugging in, we get:

$$U = a\ln R + b\ln(100 - R)$$

Maximizing this by taking the derivative of utility with respect to R, setting it equal to 0, and solving for R, we get $R = \frac{100a}{a+b}$, but since $a + b = 1$, $R = 100a$.

So Jack prefers 50 trees be used for the raft and 50 for shelter (because for Jack, a=.5 and b=.5). Michael prefers 75 trees for the raft, and only 25 for the shelter, while Shannon favors 25 trees for the raft and 75 for shelter.

1b) see graph

1c) Yes. From the graph in b, we can see that each person's utility as a function of R has a single peak (the peak for Michael is 75, for Jack is 50, and for Shannon is 25). We know that majority voting yields a consistent outcome (i.e. the majority outcome doesn't cycle and the outcome won't depend on the order in which options are considered) provided preferences are single peaked, and so in this case majority voting will be consistent.

1d) The median voter theorem states that if everyone's preferences over the decision being voted on are single peaked, then the decision from majority voting will be the same as the choice of the median voter (the person who prefers the median amount of the good). Here, it's easy to see graphically using your graph from b. Suppose the three are trying to decide whether to use 25 trees, more than 25 trees, or less than 25 trees on the raft. Jack and Michael would vote that more trees should be used, while Shannon wants to use exactly 25 trees – so Jack and Michael win the vote, and they'd then carry out the same vote over again using a larger number of trees for the raft. Similarly, if they considered 75 trees, Jack and Shannon would vote to use less trees on the raft, while Michael would be perfectly happy using 75. As they consider using different amounts of trees on the raft, eventually they would settle on 50. When voting on using 50 trees, a majority will not exist to either increase or decrease the number of trees used – and hence, 50 will be the final outcome. Intuitively, this is how a majority vote settles on the preferred choice of the median voter – this is how the median voter theorem would work in practice when voting occurs over a continuum of possibilities for the public good.

1e) By now you should be familiar enough with utility maximization to realize that now, three of the survivors prefer more trees be used for the raft than Michael wanted (because their *a* parameter in their utility function is greater than 3/4), and one wants less trees than Shannon did. Hence, majority vote should result in a greater number of trees used

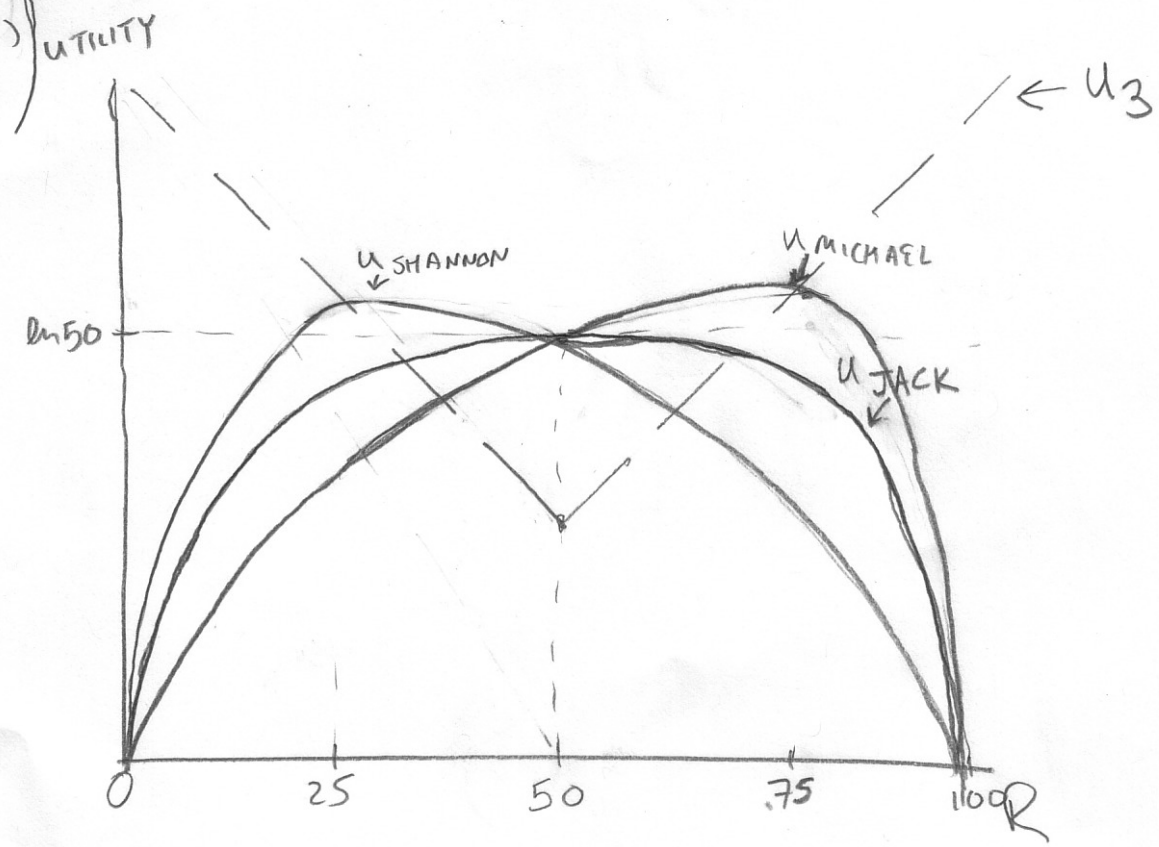
for the raft than before. Since three like using more trees on the raft than Michael does, and three now like using less trees, Michael is the new median voter – so a majority vote will settle on his optimal choice, which is 75 trees used for the raft.

1f) See graph. These new preferences are *not* single-peaked (there is a peak at R=0 and R=100), so the median voter theorem no longer applies. More importantly, majority voting is no longer consistent. The way to see this is to first consider a vote on whether 51 trees should be used on the raft, more than 51, or less than 51. The two new people will vote to use more than 51 trees, as does Michael – while Jack and Shannon want to use less. So next they consider whether to use 52 trees, more or less – and this process continues until the vote settles at 75 trees (because at 75 trees, exactly half want to use more, and half want to use less).

However, what if the initial vote was whether 49 trees, more than 49, or less than 49 should be used? Now, three people vote for less than 49 trees, and the process continues until the majority vote settles on 25 trees (again, half want more than 25 and half want less than 25).

Hence, with these preferences, two outcomes are possible under a majority vote! This is not consistent, because the actual outcome now depends on how the initial vote is structured. This gives lots of power to the *agenda setter* of the vote - for instance, if everyone agreed that Michael could be the moderator for the voting process, he would begin by introducing a vote on using 51 trees for the raft – because by doing so, the process would culminate in an outcome of 75 trees rather than 25 trees.

1B, 1F)



NOTE: u_3 ISNT DRAWN TO SCALE (THE GIVEN UTILITY FCN WOULD PUT IT FAR ABOVE THE OTHER UTILITY CURVES). ALL THAT MATTERS HERE IS THAT IT'S CLEAR THE NEW UTILITY FUNCTION IS <u>NOT</u> SINGLE PEAKED (SINCE THE MAXIMUMS ARE AT R=0 AND R=100). [REMEMBER, WITH UTILITY FUNCTIONS THE <u>LEVEL</u> OF "UTILITY" IS UNIMPORTANT - ALL THAT MATTERS IS THE PREFERENCE RANKING OF GOODS]

2a) Other costs could include: the potential for greater car accidents in the tunnel, or the loss of a pretty river view for drivers on Memorial Drive. Possible benefits: value of park land to Cambridge residents who use the park, value of park land to Cambridge residents who don't use the park but nevertheless value knowing that there is more green space and fewer roads in their city, fewer car-induced pedestrian fatalities on Memorial Drive, less air pollution (if car exhaust is relatively contained within the tunnel, or if the tunnel air is filtered), and less noise pollution (if the cars' noise isn't amplified by the tunnel). These are all long-term benefits which accrue each year, and so their present discounted value should be considered by discounting with a social discount rate.

One way to estimate benefits would be simply to poll residents, but this would have the problems described below. Another solution would be to try to infer this contingent valuation using the *revealed preferences* of Cambridge residents for parks. It's hard to know specifically how this could work, but one possibility is that the city could study how property values changed in the past due to other park projects, and infer valuation for this project using that information – or similarly, the city could study how property values rose in other cities in response to park projects of similar scope. This is because if the project would bring value to people who live along Memorial Drive and the Charles, then this should be reflected in higher housing prices. Although increases in property value from housing near the park won't account for benefits from park visitors who don't live nearby, it does give a lower bound, at least.

b) Pluses: it provides you with a direct answer, and you can target the populations that you're interested in, which you can't necessarily do with contingent valuation (i.e. you can survey both Cambridge and non-Cambridge residents, or anyone else whose benefits you care about). There are (at least) three big problems with this approach. First, there are huge incentives to lie and exaggerate – if you know the purpose of the survey, and you really value the park proposal, you could vastly exaggerate your actual valuation, in hopes of pushing the aggregate benefits high so that the project is approved. There's no cost to you of doing this, because it's only a survey, and you won't be charged your full valuation later on through higher taxes. Second, we know from other studies that the way questions are framed on a survey (for instance, the order in which questions are asked) matters a lot – so it's not clear the responses you get are people's true valuation. Third, it may be the case that people simply don't know their own valuation – it's hard to put an individual price on an improvement of this sort, and so a survey response may be a poor guide for the value one would actually receive from the project.

c) No, this shouldn't be counted as an additional benefit. If the town has correctly counted the direct value to people from using the park (and provided that the town cares about students, so it takes their benefits into account), then including the extra revenue that MIT receives from being able to raise tuition prices is double counting these benefits – it's double counting because the benefits are counted first from the valuation of the students, and again from extra revenue to MIT. The increase in tuition price is simply a transfer from students to the university – the students benefit from the improvement, but transfer this extra value to MIT through higher tuition. In the end, MIT may end up

benefiting from this project, rather than the students – but this is an equity concern, and all Cambridge cares about is correctly accounting for the value of the park to students.

d) If, as a result of this project, people will tend to modify their driving habits to drive down Broadway instead of using the tunnel, and if these drivers would have tended to spend money on businesses along Memorial Drive, then the increase in business on Broadway is simply a shift of revenue from Memorial Drive businesses to Broadway businesses – this would simply be a transfer from one set of private firms to another, and not represent a gain in aggregate business to the town. So if this is the case, the additional revenue earned by Broadway merchants should not be counted.

If, on the other hand, the money spent on Broadway shops is money that would've been spent in a different town, then this represents new revenue to Cambridge and should be counted as benefits to the city (although this revenue is essentially a transfer from other businesses, we're looking at costs and benefits from the perspective of Cambridge – and since this is a new transfer to Cambridge from out of the town firms, it represents an additional benefit to Cambridge from the project).

e i) initial costs=$200 million. Annual expenses are $5,000,000; the costs begin next year and continue forever, so the PDV of these costs is $\frac{\$5{,}000{,}000}{.1} = \$50{,}000{,}000$.
Previously, the average driver spent 10 minutes on Memorial Drive. Now, the average driver will spend 12 minutes on Memorial Drive due to the lower speed limit, so the cost of the project is an extra 2 minutes of driving time. 200,000 drivers per year and time costs of $.30 per minute imply annual costs from the lower speed limit of $120,000 – and with a social discount rate of 10%, the PDV is $1,200,000. Hence, the PDV of total costs of the project is $251,200,000.

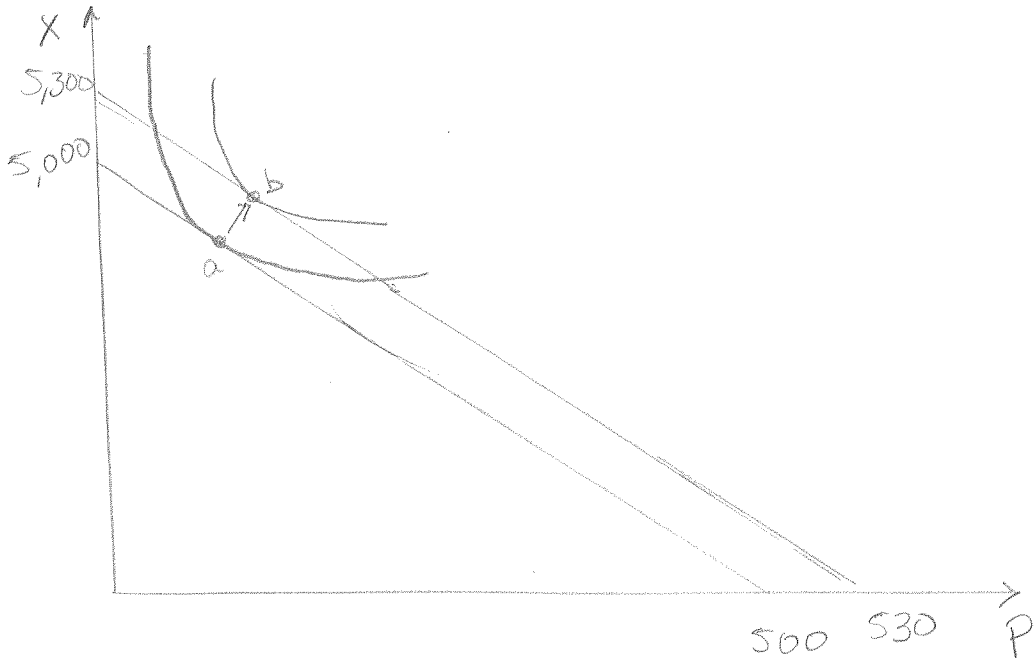
e ii) Since it now takes longer to drive on Memorial Drive (and since it might be less pleasant, since drivers lose the river view), we might expect some drivers to modify their driving habits and choose an alternative route. If drivers choose a different route, then their different route *must* be faster or more pleasant (by revealed preference). Hence, the true time costs are most likely not the costs calculated in (i) – in fact, if drivers switch to a different road because the other road is faster, then the total time lost from the project must be lower than the estimates from (i). This implies that the cost estimate in (i) is an overestimate of the true costs of the project.

e iii) $\frac{benefits}{.1} \geq \$251{,}200{,}000 \Rightarrow benefits \geq \$25{,}120{,}000$

PROBLEM 3.

(a) $U = \frac{1}{10}\ln P + \frac{9}{10}\ln X$ B.C.: $10P + X = 5{,}000$

(units in 1000's)



$$\text{Max } \tfrac{1}{10}\ln P + \tfrac{9}{10}\ln(5000 - 10P)$$

$$\text{FOC: } \frac{1}{10P} + \frac{9}{10}\left(\frac{-10}{5000-10P}\right) = 0$$

$$\Rightarrow 10P = \frac{5000 - 10P}{9}$$

$$\underline{P = 50, \quad X = 4500}$$

(b) This is a pure income effect; it does not change the relative price of parks faced by the city.

The FOC is now $\frac{1}{10P} + \frac{9}{10}\left(\frac{-10}{5300-10P}\right)$

PROBLEM 3, PART (b)

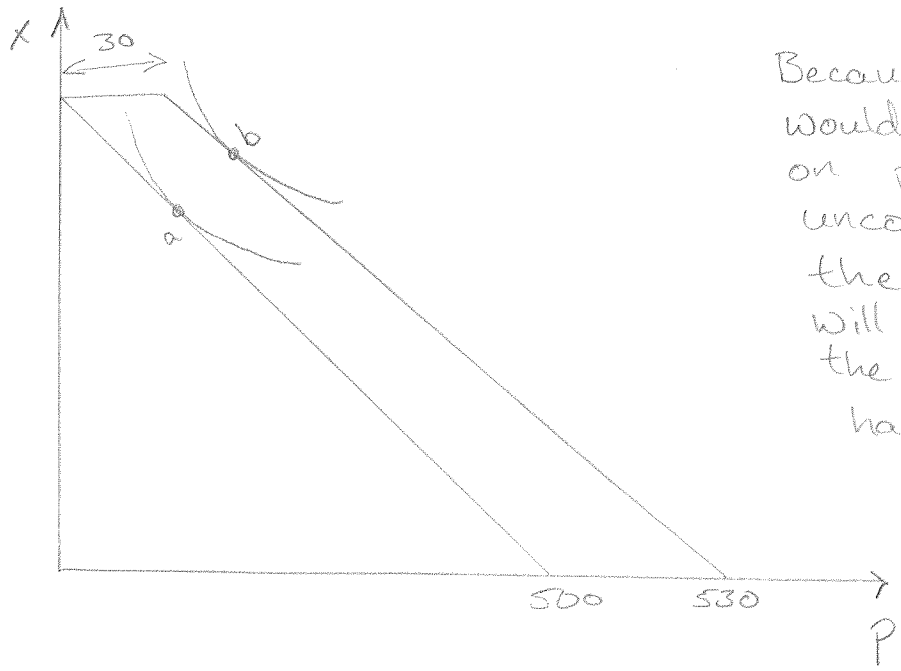
$$\Rightarrow 10P = \frac{5300 - 10P}{9}$$

$$\underline{\underline{P = 53, \quad X = 5247}}$$

(ii) One could attribute this observation to the "flypaper effect." Bureaucracy would like to grow, so in order to please the state and receive more grants it spends all of the block grant on parks. It does this even though it's against the preferences of the town's residents.

(iii) The Tiebout model suggests that competition between towns will force the local bureaucracy to respect the wishes of the residents. If it doesn't, the residents will move to another town that does.

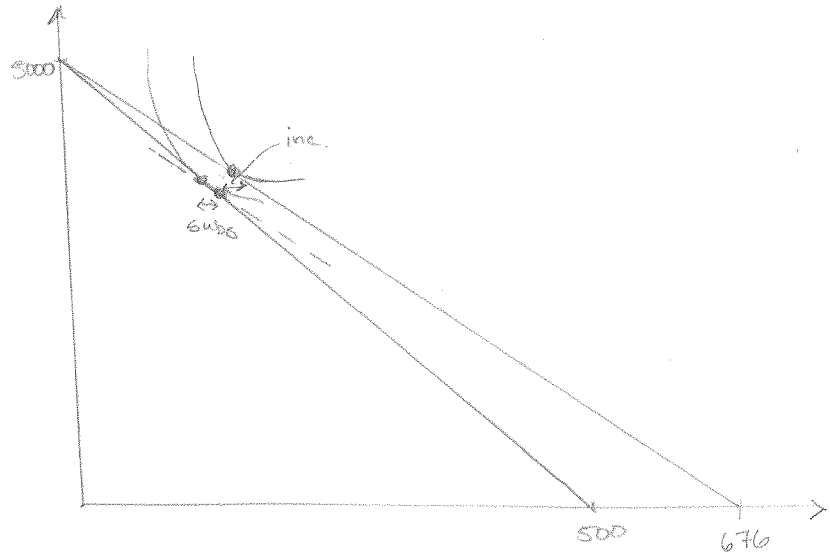
(c)



Because the town would have spent > 30 on parks w/ an unconditional grant, the conditional grant will not further constrain the town. $\Rightarrow$ we still have $P = 53$, $X = 5{,}247$

PROBLEM 3, PART (d)

The price of parks has fallen. Now \$1 of spending buys \$1.40 worth of parks, so the price $= \frac{10}{1.4} = \$7.14$ (still in 1000



The FOC is now $\frac{1}{10P} + \frac{9}{10}\left(\frac{-7.14}{5000-7.14P}\right)$

$$\Rightarrow 642.6P = 50{,}000 - 71.4P$$

$$\underline{P = 70, \quad x = 4500}$$

(e) This depends on the state's goals. If it wants to maximize the welfare of the town, it should go for the block grant. If it wants to encourage spending on parks, it should use the matching grant. The conditional block grant will also encourage spending on education relative to the unconditional grant if parks was already low but this will only happen once previous spending has been crowded out.

PROBLEM 3, PART (f)

If there are large spillovers across towns, and it is important to bring every town up to a minimum level of spending, then a conditional block grant will do the trick. Flu shot subsidization is a good example. On the other hand, if you want to encourage spending on the public good in general (also possibly because of spillovers), then the state should use a matching grant. One could argue that parks provide a good example because the benefits spill over across towns and there is not a minimum level of spending required for people to enjoy these benefits.

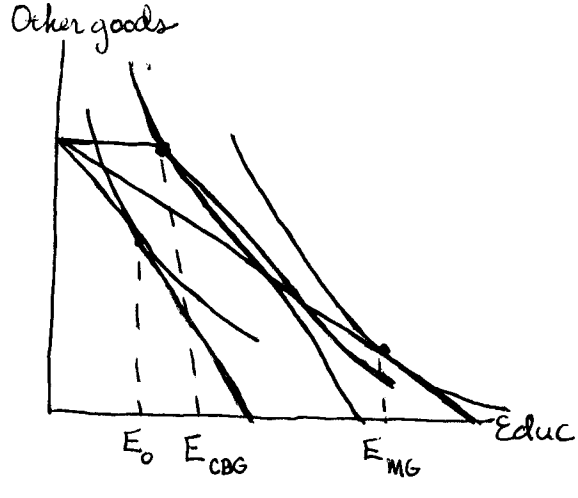
4)

a) Congressman A is incorrect. While there may be good reasons to subsidize college education, this is not one of them. The benefit he is referring to is a private benefit—increased earnings—not a social benefit, and therefore can be handled within the market mechanism (unless he makes another argument that there is market failure, such as credit constraints).

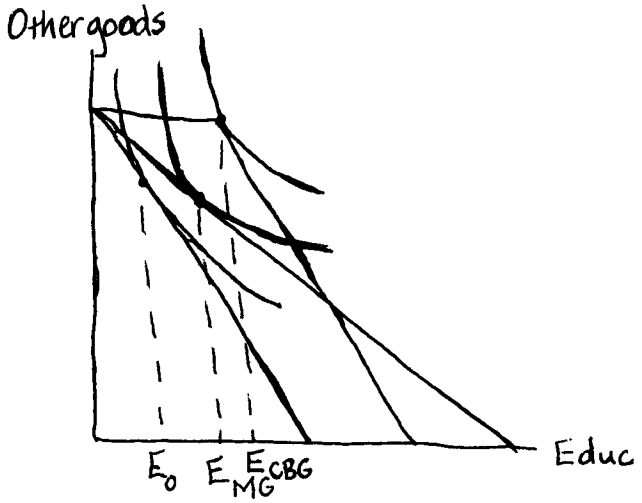
b) Congressman B is incorrect. This is an argument about redistribution, not education. While there may be good reasons for redistribution, he does not give a good reason for using education as a redistributional mechanism. It is probably more efficient, for example, to redistribute through the tax system.

c) Congressman C is making a Tiebout argument—that families sort into school districts that provide the level of elementary education they desire, and therefore that provision is efficient without federal intervention. While there is evidence that families sort, there are several reasons that sorting may not be perfect (and therefore that there may be room for federal intervention). First, there may not be as many towns as there are preferences over education. Second, towns may offer bundles of goods of which education is only one, so that families pick the best available bundle but cannot maximize over education separately. Third, poor people who value education may be credit-constrained, and unable to move into a town with the level of education they prefer.

d) In most cases, matching will lead to a larger increase:



But if spending is very low and a town has strong preferences for other goods rather than education, a conditional block grant may lead to the larger increase:



e)
Both proposals decrease the cost of college relative to other goods, and therefore will increase college attendance as long as one assumes that there are some people currently on the margin between attending and not attending college.

Lowering tuition at state universities makes the higher education system look more like the primary education system: students can get a fixed quantity of education for low cost (or, in the case of the primary system, for free), but to increase the level of education at the margin they must spend much more. Lowering the cost of public education relative to private education will cause people to substitute toward public education. If we assume that state schools cost less than private schools, this will lower spending by people who otherwise would have attended private schools. Plus, the tuition at state schools has gone down, which lowers spending by people who were attending state schools anyway. But note that we also expect to increase college attendance, so the net effect on spending is ambiguous.

Grants for any university in the state are a type of voucher system. The grants will shift some students from attending school out-of-state to attending school (public or private) within state. They will also decrease the spending of people who were already attending schools in-state, since they no longer pay the last $1000 of tuition (although note that in general equilibrium tuition is likely to increase). Again, however, there will be an increase in college attendance—in fact, probably a larger effect, since some people who previously did not attend college may not be interested in attending state schools, but may be on the margin of attending private school. Thus the net effect on spending is ambiguous.

These proposals have the standard financial pros and cons of increased public school spending versus vouchers:

1) Increased public school spending may crowd out private spending, as some students move from higher spending on a private school to the cheaper—but lower-quality—public provision.
2) Vouchers go even to those who are very well-off and unlikely to significantly increase their consumption of education—for them, when the government spends more on education, they simply spend less on education (and more on other goods).

But there is one difference between higher education and primary education, which is that many people do not go to college. So any program that decreases the cost of college, although it decreases spending by those already in the system, will also increase the spending of those who now enter the system.