14.41 Problem Set #3 Solutions

Problem 1:

a) Each superhero's utility exhibits diminishing marginal returns (you can see this by simply plotting the level of utility against consumption, or by finding that the second derivative is negative). This is equivalent to saying that the individual is risk averse. Since this is true, and since insurance is fair, the price that each type pays for a unit of insurance will be equal to the probability that they get caught – and further, we know from class and section that risk-adverse individuals (i.e. utility with diminishing marginal returns) will fully insure if insurance is fair. No one got penalized for not mentioning risk aversion, but keep in mind that a 100% complete answer would have included it.

You could also get this result by solving for the optimal level of insurance that each type will choose.

So, for clumsy superheros, the problem would be:

$$Max\ pU_{accident} + (1-p)U_{NoAccident} \Rightarrow Max\ .9(100-50+b-.9b)^{.7} + .1(100-.9b)^{.7}$$

$$\Rightarrow \frac{(.9)(.5)(.1)}{(50+b-.9b)^{.7}} - \frac{(.1)(.5)(.9)}{(100-.9b)^{.7}} = 0 \Rightarrow (.9)(.1)(100-.9b)^{.7} = (.1)(.9)(50+b-.9b)^{.7}$$

$$\Rightarrow 100-.9b = 50+.1b \Rightarrow b = 50$$

where *b* is the amount of insurance the clumsy superhero buys (which is equal to the amount he gets from the insurance company if he is caught). The procedure is similar for skilled superheros.

bi) the maximum amount that each would be willing to pay for insurance is equal the amount that makes him exactly as well off with insurance as without.

Without insurance, the expected utility of each type is:

$$EU_{clumsy} = .9(100-50)^{.7} + .1(100)^{.7} \approx 16.428$$

$$EU_{skillful} = .3(100-50)^{.5} + .7(100)^{.5} \approx 9.121$$

Full insurance would mean that the superhero receives full compensation for injury if injured, resulting in complete consumption smoothing across the two possible states (caught or not caught). Hence, his income in each state is simply his full income less insurance costs. So to solve for the maximum each type would be willing to pay for full insurance, solve the following:

$$EU_{clumsy,no\ insurance} = EU_{clumsy,\ full\ insurance} \Rightarrow 16.428 = (100-X)^{.5} \Rightarrow X = 45.48$$

$$EU_{skillful,no\ insurance} = EU_{skillful,\ full\ insurance} \Rightarrow 1.553 = (100-X)^{.5} \Rightarrow X = 16.80$$

So the clumsy type is willing to spend 45.48 for full insurance (for a per-unit price of .910) and the skillful type is willing to spend 16.80 (for a per-unit price of .336). Note that each is willing to pay more than the actuarially fair price – this is because their utility function exhibits diminishing marginal returns to consumption (risk aversion).

ii) If ACME is looking for a single market price such that everyone will fully insure, it will have to offer the maximum price that the skillful are willing to pay: 16.80. However, when it does this it will collect 12*16.80=201.6 in revenue for every 11*.3*50+.9*50=210 it expects to pay out – since expected revenue is less than expected pay out, ACME won't stay in business if it offers this plan. Hence, there's a market failure: we know that each type is willing to pay *more* than the actuarially fair price for full insurance, and ACME is certainly willing to offer insurance at a greater-than-fair price. The problem is that types are unobservable (that is, asymmetric information exists in this insurance market), so this arrangement cannot occur. Because there are transactions that would occur under full information that would make everyone better off, a market failure exists.

c) Now, let's consider the expected utility that each type would receive under each plan:

$$EU_{clumsy,\, plan\ A} = .9(100-50+20-7)^{.7} + .1(100-7)^{.7} \approx 18.747$$
$$EU_{skillful,\, plan\ A} = .3(100-50+20-7)^{.5} + .7(100-7)^{.5} \approx 9.132$$

$$EU_{clumsy,\, plan\ B} = .9(100-50+50-34)^{.7} + .1(100-34)^{.7} \approx 18.779$$
$$EU_{skillful,\, plan\ B} = .3(100-50+50-34)^{.5} + .7(100-34)^{.5} \approx 8.124$$

Comparing these expected utilities to those without insurance, as calculated in (bi), we see that clumsy types prefer plan B to plan A, and are better off by purchasing insurance than not purchasing insurance. The skillful types prefer plan A to plan B, and are also better off by purchasing insurance. So if these packages were offered, all the clumsy would purchase type B, and all the skillful would purchase plan A. ACME revenues are 11*7+1*43=120. ACME expected payout is 11*.3*20+.9*50=111. ACME would make profit from this package, so it's willing to offer them – and now everyone has some insurance!

However, there is still a market failure for the same reason as before – the skillful types still wish to purchase full insurance at actuarially fair prices, and the insurance company would offer insurance for those prices – but due to asymmetric information, this is impossible. The outcome may be more favorable than before, since now everyone gets some insurance, but a market failure still exists.

di) There might be an incentive here for the skillful type to purchase the test. Consider what happens if the skillful pay for the test and reveal their type: given that there is perfect competition in the insurance market, the skillful will be offered actuarially fair insurance, so they would fully insure. The clumsy type would never purchase the test, because if they do their type will be revealed, and they will receive actuarially fair insurance – which is more expensive than the insurance plan they receive from c! To determine whether the skillful types will purchase the test, consider their expected utility if they do:

$$EU_{clumsy} = .9(100-50)^{.7} + .1(100)^{.7} \approx 16.428$$

$$EU_{skillful,test} = (100-1.5-.3*50)^{.5} \approx 9.138$$

Since the skillful are slightly better off by paying $1.50 for the test, receiving fair insurance and fully insuring, they will be willing to pay for Dr. Brain's test.

Now, ACME will offer two insurance packages. The first will be for those revealed to be skillful, who will be able to purchase insurance for a per-unit of coverage cost of .3. The second will be available only to those who haven't revealed themselves with a test (i.e. the clumsy), who will be able to purchase insurance for a per-unit of coverage cost of.9. Both types will fully insure!

dii) The skillful are better off than in (c), because they are now fully insured. The clumsy are worse off, because although they are still fully insured, they have to pay more in order to be fully insured.

diii) Now, there is no longer a market failure! The presence of the market for blood tests has eliminated the asymmetric information problem, and everyone is fully insured at actuarially fair prices (although the skillful are technically paying more than actuarially fair prices to receive full insurance, since they first have to purchase the test. Some people cited this as a source of market failure. This would be correct if the $1.50 for the test represented the true cost of the test and not a transfer to Dr. Brain).

Problem 2:

a) Don't get confused because we're thinking about consumption in different periods – this is no different from the sort of utility maximization that we're used to! Just treat the problem as deciding how much to spend on two goods: consumption today and consumption tomorrow. The easiest way to solve intertemporal utility maximization is to consider how much the person can consume in the second period, given how much he chose to consume in the first period. So in this example, if the person lives into the second period, his consumption is: $C_2 = (1+r)*(100 - C_1) = 1.1*(100 - C_1)$. Now just plug this in for $C_2$, and maximize the expected utility function with respect to $C_1$.

$$\max U = \ln C_1 + \frac{1}{3} p \ln(C_2) \Rightarrow \max U = \ln C_1 + \frac{1}{3} p \ln(1.1(100 - C_1))$$

$$\Rightarrow \frac{1}{C_1} - \frac{p}{3}\left(\frac{1}{100 - C_1}\right) = 0 \Rightarrow 300 - 3C_1 = pC_1 \Rightarrow C_1 = \frac{300}{3+p}, \quad C_2 = 1.1(100 - C_1) = 1.1\left(\frac{100p}{3+p}\right)$$

So p=.75 for type As, so $C_1^A = 80, saving = 20, C_2^A = 22$. p=.2 for type Bs, so $C_1^B = 93.75, saving = 6.25, C_2^B = 6.875$. Type As are saving more, and hence consuming less in period 1 than Bs do but more in period 2, because the probability that they will live to see the second period is greater.

b) Yes, people would be better off, because annuities provide a higher return on investment. Remember, the price of fair insurance is the probability of payout (i.e. the probability of having an accident). Here, the probability of payout is the probability of living into the second period. Type Bs only have a 20% chance of reaching the second period, so the price of one unit of annuity payout is .2. Provided the type B reaches the second period, the return on his investment is huge- for every dollar he spends on insurance, he gets five dollars if he lives into the second period, for a net return of four dollars. This certainly greater than his returns from saving. Type As must pay $.75 for $1 of coverage, so As receive $1.33 for every $1 invested if they live into the second period. The return on their annuity investment is 33%, which is again higher than returns from saving.

The reason that fair annuities can provide higher returns is because risks are pooled – everyone pays in, but only a fraction receive payment. In reality, annuities are valuable because they assist in consumption smoothing throughout old age – in this simplified two period model, however, consumption smoothing isn't a concern, because this can be accomplished through private savings.

ci) The government receives 30T in revenue (since it taxes all 30 people equally), and invests it at an interest rate of 10%, so that it has 30T*1.1=33T in funds to pay out in period two. It pays out .75*B in expectation for every type A, and pays out .2*B in expectation for every type B – so in sum, it expects to pay out 20*.75*B+10*.2*B=17B.

Since the budget must break even at the end of period two, this implies that 33T=17B, or the benefits the government pays out to anyone who lives into period two is 33T/17.

cii) The government is now trying to decide what T to choose to maximize the sum of utilities. In other words, its maximization problem is:

$$\max \textit{social welfare} = 20*\left(\ln(100-T)+\frac{1}{3}*.75*\ln\left(\frac{33T}{17}\right)\right)+10*\left(\ln(100-T)+\frac{1}{3}*.2*\ln\left(\frac{33T}{17}\right)\right)\Rightarrow$$

$$\frac{-20}{100-T}+\frac{5}{T}-\frac{10}{100-T}+\frac{2}{3T}=0\Rightarrow T=\frac{1700}{107}\approx 15.89,\ B=30.84$$

So each person pays 15.89 in taxes, and receives a benefit in period two of 30.84 (if they survive that long).

$$EU_{no\ SS}^{A}=\ln(80)+\frac{1}{4}\ln(22)\approx 5.15$$

$$EU_{no\ SS}^{B}=\ln(93.75)+\frac{1}{15}\ln(6.875)\approx 4.67$$

$$EU_{SS1}^{A}=\ln(84.11)+\frac{1}{4}\ln(30.84)\approx 5.29$$

$$EU_{SS2}^{A}=\ln(84.11)+\frac{1}{15}\ln(30.84)\approx 4.66$$

A is better off with social security than without – this is because it is highly probable that a type A reaches the second period, and because the return from “saving” with social security is greater than the returns from private investment. Since B=33T/17 $\approx$ 1.94T, one dollar “saved” (taxed) through social security returns 1.94 dollars if the person reaches the second period, for returns of 94% (which is certainly greater than the 10% return from the private market). From A’s perspective, this isn’t the optimal social security system – the optimal system would in fact force more saving through an even higher tax – but because the high return results in high period two consumption, As find this system better than without a system.

Bs, however, are just slightly worse off than before. This is because the tax rate is much higher than what they would’ve chosen (see answers to the next part). So even though the rate of return is much higher than what private savings yields in the absence of social security, Bs are worse off because the tax rate is too high.

One could view this system as redistributive since it taxes Bs more than they wish, and these tax revenues are partially redistributed to As since As are more likely to receive social security benefits.

ciii) now the government chooses T to maximize B’s utility:

$$\max B's\ utility = \ln(100-T) + \frac{1}{3} * .2 * \ln\left(\frac{33T}{17}\right) \Rightarrow \frac{-1}{100-T} + \frac{1}{15T} = 0 \Rightarrow T = \frac{100}{16} = 6.25,\ B = 12.13$$

which implies:

$$EU_{SS2}^{A} = \ln(93.75) + \frac{1}{4}\ln(12.13) \approx 5.16$$
$$EU_{SS2}^{B} = \ln(93.75) + \frac{1}{15}\ln(12.13) \approx 4.71$$

Now B is better off with this system than without, and is better than the first proposed system. A is still better off than without social security (and if you assume that A can supplement social security income with additional private savings, A's utility will be higher than 5.16), but A is worse off than with the first social security system. This is because the forced savings in this system are lower than the other system – since this system doesn't account for A's preferences whatsoever, but the prior system partially weighted A's preferences, A must be better off with the first social security system. However, assuming A can save in addition to social security, A is *not* worse off than without social security – this is because A can always save on top of social security at a 10% interest rate, but the returns on some investment (the 6.25 "invested" with taxes) is greater than the market rate of 10%.

Again, this system redistributes from Bs to As – this is because the ratio of expected payout to payin is much higher for As than Bs. However, Bs are better off with this sort of redistribution than they are if the social security system didn't exist.

d) There are many ways in which the American social security system is redistributive – for instance, since the PIA is calculated as a progressive function of the AIME, the social security system redistributes from the poor to the rich (the replacement rate for social security benefits is higher for the poor than the rich). What is most relevant from this example, however, is that the system also redistributes from the short lived to the long lived – because if two people are identical except for life expectancy, they'll pay the same amount in taxes, but the short lived person will receive less in benefits than the long lived person will. In America, women tend to live longer than men, and whites tend to live longer than blacks, and richer people tend to live longer than poorer people – so due to differences in life expectancy, the system is redistributive from the first group to the second (of course, the progressivity of benefit calculations may make the system actually less redistributive from the first to second groups, or actually redistribute in aggregate from the second group to the first).

3) No, the fact that people who do not receive UI are unemployed for a shorter period of time does not prove that UI causes longer durations of unemployment. There are several differences between those who receive UI and those who don't:

- Only people who are laid off are eligible for UI, not those who are fired or quit;
- Of those who are eligible for UI, only 2/3 decide to take up benefits.

One alternative explanation for the president's table is that people who know they can find another job easily don't bother to apply for UI. The key is that receiving UI is not a random experiment with a treatment and control group, so there could be other differences between the groups that account for the difference in outcomes.

Better evidence about the effect of UI on unemployment durations can be found in the work of economist Bruce Meyer. In one study of state law changes (a natural experiment where some people were affected by the law change and others weren't), he found that a 10% increase in benefits was associated with an 8% increase in durations. In another study where he looked at the probability of going back to work each week conditional on being unemployed for that amount of time (the hazard rate), he found that lots of people found jobs at 26 weeks, when their unemployment benefits were running out. Both these studies suggest that UI benefits do affect the duration of unemployment.

b. No, for at least two reasons:

- Longer durations of unemployment could lead to better job matches, which is a benefit to society. We do not want brain surgeons working at McDonald's just because it takes a little while to find a new brain surgeon position. Relatively generous benefits allow people to take the time they need to make a good job match. However, the Meyer study of hazard rates and evidence showing that people who are unemployed longer do not get higher wages suggests that this is not a very important consideration in reality.
- Perhaps there are two kinds of unemployed people: lazy rich people, who can get a job whenever they want, and poor hardworking people, who have a hard time finding a job. Then, while reducing UI would induce the lazy rich people to look harder for a new job, it would also hurt the consumption of the poor people who are already looking as hard as they can. We would have to weigh these implications to decide whether it was worth it to reduce the program's generosity.

ci) With individual perfect experience rating, firms don't pay any of the cost of layoffs, so we would expect more layoffs than under a policy of firm perfect experience rating, where firms pay the cost. (If firms are able to pass the cost of unemployment benefits through to the worker in the form of lower wages, then individuals may pay under both policies.) With individual perfect experience rating, we would expect unemployment durations to be shorter; since individuals have to pay back all the benefits they receive while unemployed, they stay unemployed only long enough to find a good job match.

cii) Individual perfect experience rating provides individuals with good incentives to find a job quickly, but does not provide them any insurance against becoming unemployed (they bear the full cost of the layoff). Firm perfect experience rating provides individuals with insurance against layoffs, but does not provide any insurance for firms (though it gives the firm the right incentives about whether to lay off a worker). Just because individual perfect experience rating helps workers smooth consumption while unemployed *does not* imply that it is actually insurance! By definition, insurance compensates someone in event of a loss – in doing so, it helps to smooth consumption between states of nature in which there is and there is not a loss. Individual experience rating doesn't compensate unemployed people for becoming unemployed – it merely gives them a loan with they have to pay back.

4a) The rationale behind the first proposal would be that we could give higher benefits for injuries that are harder to fake—then we could make disabled people better off (which, after all, is the point of the program) without encouraging people to fake injuries for the higher benefit (since we would only give the higher benefit for injuries that are hard to fake.)

Think of the "lottery" of applying for DI as a faker—you have some probability p of getting approved even though you're not disabled, and a benefit B if you get approved. So the expected value of the lottery is pB, and the size of the expected value is how fakers decide whether it's "worth it" to quit work for 5 months. For injuries with a lower p, we can increase B without changing the expected value of the lottery.

Many of you pointed out that some disabilities still allow a person to work, so that the *difference* in wages should be paid for these disabilities, as opposed to 100%. This was also correct.

The rationale behind the second proposal is, again, that fakers compare pB to the cost of quitting work for 5 months when they decide whether it's worth it to apply. So increasing the cost of applying, by extending the period they would have to go without work, would decrease the number of fakers.

4b) The drawback to the first program is that those who are truly injured with injuries that are hard to fake would have smaller benefits.

The drawback to the second approach is that the truly disabled people—who are the point of the program—would be hurt by having to wait, if they were credit constrained and had to decrease consumption even though they were going to get reimbursed (also, remember that the truly disabled also have some chance of getting rejected from the program).

4c) You would want to lower benefits for those whose injuries are easy to fake (e.g. back pain) relative to those whose injuries are difficult to fake (blindness, paraplegia).