14.41 Problem Set 5 Solutions
Fall 2005

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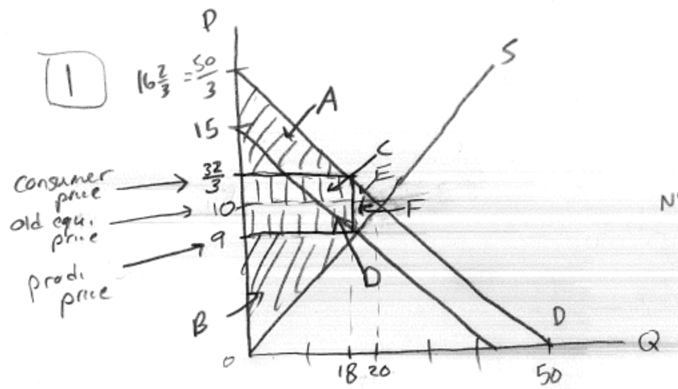


OLD D: $Q = 50 - 3P$

$P^o = \frac{50}{3} - \frac{Q}{3}$

NEW D (including $\tau = 5/3$)

$P^n = P^o - \tau = \frac{45}{3} - \frac{Q}{3}$

$P^n = 15 - \frac{Q}{3}$

new market equ:

$\frac{Q}{2} = 15 - \frac{Q}{3} \Rightarrow Q = 18$

Prod. price $= \frac{Q}{2} = 9$

Cons. price $= 9 + \frac{5}{3} = \frac{32}{3}$

So $\left(\frac{32}{3} - 10\right)$ = consumer burden (i.e. change in consumer price) $= \frac{2}{3}$

$10 - 9 = 1$ = change in producer burden

producers bear $1/_{1+2/3} = 1/_{5/3} = 3/5$ of total burden

Tax revenue $= C + D = \frac{5}{3} \times 18 = 30$

$DWL = E + F = \frac{5}{3} \times 2 \times \frac{1}{2} = \frac{5}{3}$

$CS^{OLD} = \left(\frac{50}{3} - 10\right) \times 20 \times \frac{1}{2} = \frac{200}{3}$ $\quad CS^{NEW} = \left(\frac{50}{3} - \frac{32}{3}\right) \times 18 \times \frac{1}{2} = 54$

$\Delta CS = \frac{200}{3} - \frac{162}{3} = \frac{38}{3}$

$P_S^{OLD} = 10 \times 20 \times \frac{1}{2} = 100$ $\quad P_S^{NEW} = 9 \times 18 \times \frac{1}{2} = 81$

$\Delta PS = 19$

1aiii)

$$\eta_S = \frac{dQ}{dP}\frac{P}{Q} = 2\frac{P}{2P} = 1$$

$$\eta_D = \frac{dQ}{dP}\frac{P}{Q} = -3\frac{P}{50-3P} = \frac{-3}{2}$$

note: we want to use *pre-tax* prices and quantities because we want to know the elasticities at the pre-tax equilibrium, because these elasticities will dictate how supply and demand will change in response to a market distortion.

consumer burden=change in consumer price:

$$\frac{\eta_s}{\eta_s - \eta_d}\tau = \frac{1}{1+1.5}\frac{5}{3} = \frac{2}{3}$$

producer burden=change in producer price:

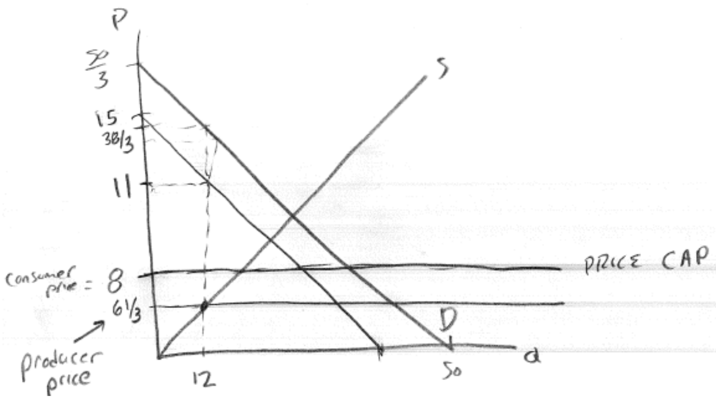
$$\frac{\eta_d}{\eta_s - \eta_d}\tau = \frac{1.5}{1+1.5}\frac{5}{3} = 1$$

iv)

$$DWL = -.5\frac{\eta_s \eta_d}{\eta_s - \eta_d}\tau^2\frac{Q}{P} = -.5\left(\frac{1.5}{2.5}\right)\left(\frac{5}{3}\right)^2\frac{20}{10} = \frac{5}{3}$$

v) In this example, deadweight loss represents inefficiency from efficient trades that should have occurred, but were prevented due to the tax. Inefficiency is measured by the difference between social marginal benefit and social marginal cost curves at the new market outcome – so in this case, since the marginal benefit is greater than the marginal cost at the new outcome, efficient trades are prevented from occurring. Deadweight loss could also occur if *too many* trades are induced due to the tax – i.e. if the social marginal cost curve is above the social marginal benefit curve at the new market outcome. This could occur if one good is subsidized rather than taxed, or if there are positive production/consumption externalities yet the market is taxed.

b) consider first the case where the price cap means that consumers can pay no more than \$8. Then the maximum amount that producers can receive is \$6.33. This means that producers will only supply 12. If 12 are supplied, consumers are willing to pay suppliers \$11 per unit (or are willing to pay \$12.67 in total, including the tax). Since consumers are willing to pay more than producers require, the price cap will be binding and consumers will pay \$8, and producers will receive \$6.33. Hence, producers bear the full burden of the tax (i.e. the per-unit amount they take home for each unit sold falls by the exact amount of the tax). The following graph illustrates this:



If the price cap means that the most producers can take home at the end of the day is \$8, then they can charge consumers \$8 per-unit, and consumers would pay \$9.67 per-unit (including the tax). Under this price, producers will supply 16, and consumers will be willing to pay \$9.67 for each one. Now, consumers bear the full burden.

ci) think about the *total per-unit* tax caused by this scheme: for each unit sold, consumers pay a tax of \$1.67, but producers receive a subsidy of \$.83. Hence, the total amount that is paid to the government for each unit bought and sold is \$.83 (\$1.67-\$.83). Since there is a smaller tax in this market, intuitively we should expect deadweight loss to be less.

In fact, the subsidy shifts the supply curve to the left – now that they're getting a subsidy, they require less from the consumer for each unit sold, and are willing to supply more for any price. Hence, the market quantity is closer to the equilibrium level under this tax and subsidy scheme – subsidizing production encourages more production, which partially corrects the distortion caused by the consumption tax. So this additional distortion has actually *helped* efficiency in this market, since it moves the market outcome closer to the efficient level. When thinking about the efficiency effects of a tax, don't automatically assume that a tax or subsidy distorts the market inefficiently! It depends on other factors, such as pre-existing taxes, or externalities.

ii) Because all that's changed in the market is the effective per-unit tax on goods, the *share* of the burden borne by each party should not be different from (a). We can see this quite easily:
total burden= τ . (the total burden is the sum of the producer and consumer burdens, which represents the sum of producer and consumer price changes – which is the amount

of the tax!) Producer burden=$\dfrac{\eta_d}{\eta_s - \eta_d}\tau$. Share of burden=$\dfrac{\eta_d}{\eta_s - \eta_d}$, which is independent from the level of the tax – and it depends only on elasticities!

So a subsidy for producers will effectively lower the amount of the tax, which lowers the *level* of the burden borne by each party – but not the share.

You could also demonstrate this numerically. The old equilibrium had P=10. With a tax of 5/3 on consumers, the price that they'll pay producers for a given Q is P=15-(Q/3). With a 5/6 subsidy on producers, the price that they require from consumers is P=(Q/2)-(5/6). The equilibrium Q from equating those expressions is 19 (now, higher than before by 1 – since the equilibrium Q is closer to the pre-tax Q, deadweight loss must have fallen). The price that producers receive from consumers is 26/3. The price that producers receive in total for each unit sold is (26/3)+(5/6)=57/6. The price that consumers pay in total for each unit sold is (26/3)+(5/3)=31/3. The consumer burden is therefore (31/3)-10=(1/3), and the producer burden is 10-(57/6)=(1/2). The share of the burden that producers bear is (1/2)/(5/6), since 5/6 is the total burden (i.e. the tax minus the subsidy) which is 3/5. This is the same thing that we got in (a).

iii) From the previous discussion, a tax of 5/6 would have the same effect. It could be placed on either consumers or producers. It would have the same "market outcome" in that the quantity sold would be the same as if consumers were taxed 5/3 and producers were subsidized by 5/6.

2a) Although we often write aggregate demands in terms of price as a function of quantity, this would be very complicated for this example because A's demand for gasoline is perfectly inelastic (i.e. A is willing to pay anything to get 30 units of gasoline). Instead, write aggregate quantity demanded in terms of price. This will be a convenient format for below, where we want to calculate elasticities.

$$D_g = 30 + \frac{40}{P_g}$$

$$D_d = \frac{100 - 30P_g}{P_d} + \frac{120}{P_d} = \frac{220 - 30P_g}{P_d}$$

$$D_y = \frac{40}{P_y}$$

b) From section, we know that the formula for the elasticity of demand/supply at a point is: $\eta_i^D = \frac{dQ}{dP}\frac{P}{Q}$. So:

$$\eta_g^D = -\frac{40}{P_g^2}\frac{P_g}{\left(30 + \frac{40}{P_g}\right)} = -\frac{40}{P_g}\frac{P_g}{30P_g + 40} = -\frac{40}{30P_g + 40}$$

$$\eta_d^D = -\frac{220 - 30P_g}{P_d^2}\frac{P_g}{\left(\frac{220 - 30P_g}{P_d}\right)} = -1$$

$$\eta_y^D = -\frac{40}{P_y^2}\frac{P_y}{\left(\frac{40}{P_y}\right)} = -1$$

c) From an efficiency standpoint, we'd like to tax the most inelastic goods the most – because behavioral changes resulting from the taxation will be smallest, and hence there will the smallest amount of deadweight loss. Plugging in prices, the elasticity of demand for gasoline is -.4, so demand for gasoline is less elastic than donuts and yachts – so we would want to tax donuts and yachts more than gasoline.

Using the Ramsey Rule, we can calculate the exact ratio of taxes. However, the Ramsey Rule in the book assumes that prices are all 1, which isn't the case here, so we'll need to derive a new Ramsey Rule. The intuition behind the Ramsey Rule is that the ratio of the marginal cost to taxing a good (i.e. the marginal deadweight loss) relative to the marginal benefit to taxing a good (i.e. the marginal revenue raised) should be equivalent for all goods. The deadweight loss from taxing good *i* (assuming infinite elasticity of supply) is: $-\frac{1}{2}\eta_i^D \frac{Q_i}{P_i}\tau_i^2$, so the marginal deadweight loss from taxation is $\eta_i^D \frac{Q_i}{P_i}\tau_i$. The revenue raised from taxing a good is simply $Q_i\tau_i$, so the marginal revenue raised is Q_i. Hence the ratio of the two is $\frac{\eta_i^D \tau_i}{P_i}$. This should be equivalent across all goods, so for goods *i* and j, $\frac{\eta_i^D \tau_i}{P_i} = \frac{\eta_j^D \tau_j}{P_j} \Rightarrow \frac{\tau_i}{\tau_j} = \frac{\eta_j}{\eta_i}\frac{P_i}{P_j}$. So $\frac{\tau_g}{\tau_d} = \left(\frac{-1}{-.4}\right)\left(\frac{2}{3}\right) = \frac{3}{2}$, $\frac{\tau_g}{\tau_y} = \left(\frac{-1}{-.4}\right)\left(\frac{2}{3}\right) = \frac{3}{2}$, $\frac{\tau_y}{\tau_d} = \left(\frac{-1}{-1}\right)\left(\frac{3}{3}\right) = 1$.

This could be inequitable because it's focused only on efficiency, rather than equity. In particular, this taxes gasoline more heavily than donuts or yachts, and in this example the poorest person (A) has his most heavily consumed goods (gasoline) taxed the most.

d) So the government can only tax donuts and yachts. From above, we know that $\tau_y = \tau_d$. So the government must set a tax rate τ on donuts and yachts to raise 50 in revenue. Using our aggregate demand functions, this means:

$$D_d\tau + D_y\tau = 50 \Rightarrow \left(\frac{160}{3+\tau}\right)\tau + \left(\frac{40}{3+\tau}\right)\tau = 50 \Rightarrow \frac{200\tau}{3+\tau} = 50$$
$$150 + 50\tau = 200\tau \Rightarrow \tau = 1$$

e)
Consider first taxing only person A. Person A doesn't consume yachts, so you'd want to tax yachts as much as possible. If B really liked yachts, you could probably get the total amount of necessary tax revenue from taxing only yachts. If you had to tax another good as well, you'd probably want to tax donuts more than gasoline, since B really likes to consume donuts.

More specifically, you could note that there is no way for the government to raise 50 by taxing yachts. This is because the more it taxes yachts, fewer yachts are consumed, and hence fewer yachts are subject to the tax. You can solve for the maximum amount of tax revenue that can be received from taxing yachts, and find that the revenue maximizing tax is an infinite tax on yachts; given B's demand, he will always consume some yachts (just a very, very small amount). A very high (infinite) tax on yachts will raise 40 in revenue, leaving 10 more to be raised by taxing gasoline or donuts. Since A will always consume 30 gasoline, and use whatever's left for donuts, the government should choose a tax on gasoline and donuts to maximize A's donut consumption. A consumes fewer donuts than gasoline, and B consumes more donuts than gasoline, so we'd probably raise more revenue and hurt A less by taxing donuts more heavily than gasoline.

To find the actual solution, one would want to maximize A's donut consumption with respect to the revenue constraint:

$$\max \frac{40 - 30\tau_g}{3+\tau_g}$$
$$s.t. \left(30 + \frac{40}{3+\tau_g}\right)\tau_g + \left(\frac{160 - 30\tau_g}{3+\tau_d}\right)\tau_d$$

It's a bit less clear if we only care about person B. Given that B cares equally about yacht and gasoline consumption, and A only consumes gasoline (and inelastically, at that), the government would definitely want to tax gasoline more than yachts. It's not immediately obvious how they would want to tax donuts – A consumes donuts, but B really likes donuts. The clearest conclusion one can make is that the government should tax gasoline more than yachts.

To find the exact tax rates, one would need to maximize B's utility function with respect to the revenue constraint. Although we aren't given B's utility function, it can be figured out pretty easily if you're familiar with the workings of the Cobb-Douglass (i.e. log) utility functions that we've used so many times for this course. When utility functions are of the Cobb-Douglass form (i.e. $U = a\log(C) + b\log(D) + c\log(E)$, the individual will spend a fraction $\frac{a}{a+b+c}$ on good C, $\frac{b}{a+b+c}$ on good D, and $\frac{c}{a+b+c}$ on good E. Since person B has 200 in income, his utility function is $U = \log(gas) + 3\log(donuts) + \log(yachts)$. The following maximization problem would yield the optimal tax rates:

$$\max\ \log(gas) + 3\log(donuts) + \log(yachts)$$
$$s.t. \left(30 + \frac{40}{3+\tau_g}\right)\tau_g + \left(\frac{160 - 30\tau_g}{3+\tau_d}\right)\tau_d + \left(\frac{40}{3+\tau_y}\right)\tau_y$$

An alternative way to think about it would be minimizing the sum of deadweight losses to B – that is, maximizing B's utility is the same as minimizing B's deadweight loss (i.e. the deadweight loss from focusing only on B's utility). You could use a Ramsey-type rule to simplify the maximization (recognizing that the quantity in the marginal deadweight loss expression is B's consumption, while the quantity in the marginal revenue expression is both A and B's consumption).

3.) (a) Jim is more impatient, because his value of consumption in the first period relative to the second is less than Chris's. Formally, you could show that the intertemporal marginal rate of substitution for Jim is less than that of Chris, *i.e.,* Jim is less willing to give up a unit of consumption today for a unit of consumption tomorrow.

$$MRS_{Jim} = \frac{1}{2}\left(\frac{C_2}{C_1}\right)^{\frac{1}{2}} < MRS_{Chris} = \left(\frac{C_2}{C_1}\right)^{\frac{1}{2}}$$

(b) For each agent, the intertemporal budget constraint equals:

$$1.1(100 - C_1) = C_2$$

Chris's maximization is thus

$$\max_{C_1} C_1^{\frac{1}{2}} + (110 - 1.1C_1)^{\frac{1}{2}}$$

With a the first-order condition

$$\frac{1}{2C_1^{\frac{1}{2}}} - 1.1\frac{1}{2(110 - 1.1C_1)^{\frac{1}{2}}} = 0$$

$$1.1C_1^{\frac{1}{2}} - (110 - 1.1C_1)^{\frac{1}{2}} = 0$$

$$1.21C_1 = 110 - 1.1C_1$$

$$2.31C_1 = 110$$

$$C_1 = 47.62$$

$$C_2 = 1.1(100 - 47.62) = 57.62$$

Jim's maximization is

$$\max_{C_1} C_1^{\frac{1}{2}} + \frac{1}{2}(110 - 1.1C_1)^{\frac{1}{2}}$$

With a the first-order condition

$$\frac{1}{2C_1^{\frac{1}{2}}} - 1.1\frac{1}{4(110 - 1.1C_1)^{\frac{1}{2}}} = 0$$

$$0.55C_1^{\frac{1}{2}} - (110 - 1.1C_1)^{\frac{1}{2}} = 0$$

$$0.30C_1 = 110 - 1.1C_1$$

$$1.40C_1 = 110$$

$$C_1 = 78.43$$

$$C_2 = 1.1(100 - 78.43) = 23.73$$

(GRAPHS TO COME…)

c) The budget constraint, with the subsidy, is now

$$1.1(1+s)(100-C_1)=C_2$$

To keep the math simple, define S=1+s and perform the maximization:

$$\max_{C_1} C_1^{\frac{1}{2}}+\frac{1}{2}(110S-1.1S*C_1)^{\frac{1}{2}}$$

FOC:

$$\frac{1}{2C_1^{\frac{1}{2}}}-1.1S\frac{1}{4(110S-1.1S*C_1)^{\frac{1}{2}}}=0$$

$$0.55S*C_1^{\frac{1}{2}}-(110S-1.1S*C_1)^{\frac{1}{2}}=0$$

$$0.30S^2*C_1=110S-1.1S*C_1$$

$$(0.30S+1.1)C_1=110$$

$$C_1=\frac{110}{0.3S+1.1}$$

Because $C_1=C_2$, we can substitute into the budget constraint

$$1.1(S)(100-\frac{110}{0.3S+1.1})=\frac{110}{0.3S+1.1}$$

$$(0.3S+1.1)*110S-121S=110$$

$$33S^2-110=0$$

$$S=1.83$$

$$s=0.83$$

And $C_1=C_2=66.7$

d) With the subsidy, Jim saves 100-66.7=33.3. This is the lowest amount of savings we could subsidize for Jim to reach $C_1=C_2$. Jim receives a subsidy of \$33.30

Will Chris change his consumption if the first \$33.30 of savings were subsidized? The answer is no. One way to see this is to do the optimization for Chris using the \$33.30 subsidy to wealth (an outward shift of the budget set). If the new optimum is feasible (that is, Chris saves enough to get the entire subsidy), then Chris will prefer that point to anywhere on the budget set. The wealth-subsidized budget constraint is

$$1.1(133.30-C_1)=C_2$$

$$\max_{C_1} C_1^{\frac{1}{2}}+(146.63-1.1C_1)^{\frac{1}{2}}$$

$$\frac{1}{2C_1^{\frac{1}{2}}} - 1.1\frac{1}{2(146.63-1.1C_1)^{\frac{1}{2}}} = 0$$

$$1.1C_1^{\frac{1}{2}} - (146.63-1.1C_1)^{\frac{1}{2}} = 0$$

$$1.21C_1 = 146.63 - 1.1C_1$$

$$2.31C_1 = 146.63$$

$$C_1 = 63.47$$

$$C_2 = (133.30-63.47)*1.1 = 69.82*1.1 = \$76.81$$

Because Chris saves \$69.82, this solution lies on his (kinked) budget constraint, as shown in the graph.

In this case, Chris's relative consumption is $\frac{C_1}{C_2} = 1.21$, the same as in part (b).

e) This part is a little tricky…We do the same maximization all over again (for the last time in this problem!), with the new utility function:

$$\max_{C_1} 25C_1 + \ln(110S - 1.1S*C_1)$$

$$25 - \frac{1.1S}{(110S-1.1S*C_1)} = 0$$

$$100 - 1.1C_1 = \frac{1.1}{25}$$

$$C_1 = 99.96$$

Now Jim is extremely impatient! In addition, consumption does not depend on *S*. There is no way we could subsidize consumption so that Jim would save more.

f) The income and substitution effects cancel! Changing the return to savings (*i.e.,* subsidizing it) does not change consumption in the first period or the dollar amount of savings. In most cases (and in this case) the income effect of the change on period 1 consumption is positive. Jim consumes more in period 1 as he becomes richer. Because the effects cancel, the substitution effect must be negative.

g) This is only the case when period 2 consumption is a Giffen good. For a Giffen good, the amount consumed *falls* when the price *rises*. A subsidy to savings is effectively a decrease in the price of period 2 consumption. This situation is highly unlikely, if not impossible, though. It is hard enough to find Giffen goods in static consumption models, let alone in an intertemporal model!

4)

Cutting the estate tax:

Horizontal equity:

- the estate tax is a form of double-taxation, because an individual who builds an estate through labor income will be taxed first on the labor earnings, and then again when the estate is passed on to heirs. A person who saves builds an estate will pay more in taxes than one who chooses to spend on consumption, or give his wealth in gifts, rather than save it as an estate. This is horizontally inequitable, because taxation is based on what people chose to do with their wealth, rather than the amount that was accumulated in the first place. Since the estate tax is horizontally inequitable, a cut in the tax rate would improve horizontal equity.

Vertical equity:

- because the estate tax applies only to the very rich (in 2004, it applied to estates in excess of $1.5 million), it is a very progressive form of taxation (i.e. the rich pay it, the poor don't). Since it is highly vertically equitable to begin with, a cut in the rate worsens the vertical equity of the tax.

Efficiency:
There are at least four concerns:

- the estate tax may cause rich elderly individuals to run down their estate by increasing purchases substantially before their expected age of death (i.e. take the extended family on a cruise, purchase the grandson a Corvette, etc.). These purchases are likely inefficient (i.e. the marginal benefit to the purchaser is less than the true social marginal cost), but the marginal cost to the individual may be much less because the money is heavily taxed if not spent. Because the estate tax could lead to this inefficient behavior, cutting the tax would improve efficiency on these grounds (there would be less inefficient spending before death, because more of the estate would be passed on to heirs).
- If individuals save in order to build an estate, then an estate tax may distort savings decisions – which could be inefficient. However, it's not clear whether estate taxes increase or decrease savings, as this depends on the magnitude of the income and substitution effects: increasing the estate tax lowers the value of the estate, so individuals may want to save more so that after tax, their heirs have more; on the other hand, increasing the estate tax reduces the returns to the next dollar saved for an estate, which could reduce saving for an estate. Hence, the actual effects on savings are uncertain, and therefore so are the efficiency effects.
- Similarly, changing the estate tax could distort decisions related to earning money for an estate. i.e. if elderly rich people continue to work in order to build a large estate to pass on to their heirs, then altering the estate tax effectively changes the value of work to them – and therefore could distort their labor supply decisions. However, as with savings, it's unclear which direction this effect goes, since there are opposing income and substitution effects.

- Additionally, if large estates are passed on to heirs, then this huge income shock could result in inefficient labor supply decisions by the heirs. Consider really bright relatives of a rich elderly person. Without an inheritance, these people may go on to build a corporate empire, become president, find a cure for AIDS, solve world poverty, etc. – but a large inheritance may reduce their desire to do this. So, the social marginal benefit from these sort of people's labor supply might be very high – so we want them to work – but the large inheritance discourages them from doing so. In this way, estate taxes could be efficient – so cutting the tax would have negative efficiency consequences.

Cutting the capital gains tax:

Horizontal equity:

- if you think of capital gains as returns from selling a business after building it, then the capital gains tax is horizontally inequitable because it is generally lower than the income tax. That is, an individual could choose to work for a firm his entire life and pay annual taxes on his income – or, he could choose to build his own firm, and the largest returns to this would probably be the returns he gets from selling it at a later date. However, the returns he gets from selling this firm would be subject to the capital gains tax, not the income tax, and so he would be taxed at a lower rate. Hence, because the capital gains tax is lower than the income tax, two otherwise identical individuals who exert the exact same amount of work effort (and, perhaps, are exactly as "productive") would be taxed at different rates – which is horizontally inequitable. Lowering the capital gains tax further would reduce horizontal equity to an even greater extent.

Vertical equity:

- although the capital gains tax applies to everyone who sells assets, since most capital gains realizations accrue to the richest of the population, the tax turns out to effectively be quite progressive (i.e. the tax revenue from the richest is much greater than from the poor). Hence, a cut in the capital gains tax worsens vertical equity.

Efficiency:

- "lock in" effect: because capital gains are taxed on realization instead of accrual (i.e. you pay the tax once you sell the assets, rather than on the appreciation in the value of the assets every year), the PDV of tax payments is lower the longer one holds on to an asset, which encourages people to hold assets longer than what might be optimal. (easy way to think of it: consider a world with no inflation, for simplicity, and imagine that there is a return on investment only in the first period. After the first period, there is no change in the value of the asset. Then, regardless of when the asset is sold, taxes of X(1+r)t must be paid (where X is the investment amount, r is the return to investment). However, taxes paid in the future cost less than taxes paid today, because their PDV is lower)). Since the capital gains tax applies only on realization, it encourages inefficient lock in. Reducing the capital gains tax would reduce this inefficiency.

- Changes to the capital gains tax affects returns to entrepreneurial behavior, since the returns from entrepreneurship are generally received in capital gains (i.e. selling a business). Hence, lowering the capital gains tax would increase returns to entrepreneurship; if we think that not enough entrepreneurship (i.e. a less than efficient amount) is undertaken in the economy, a reduction in the capital gains tax could encourage more.

There is no right answer regarding a policy recommendation; it depends entirely on how you weight the horizontal and vertical equity effects with efficiency effects. In reality, I suspect that the negative vertical equity concerns from cutting the estate tax would receive extremely high weight among most politicians.