Problem Set #2

Course 14.451 – Macro I

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1. **Dynamic Programming – Analytic Solution**

Assume the following problem for the social planner:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} U_0 = \sum_{t=0}^{\infty} b^t U(c_t)$$
$$s.t. \quad c_t + k_{t+1} \le f(k_t) \quad \forall t \ge 0$$
$$c_t \ge 0, \; k_{t+1} \ge 0 \quad \forall t \ge 0$$
$$k_0 > 0 \text{ given}$$

where U_0 is the lifetime utility of the representative agent, k_t is physical capital per unit of labor at time t, and c_t is consumption per unit of labor at time t. Assume that the labor supply of the agent is simply fixed at 1, and assume the following functional forms:

$$U(c) = \ln c$$
$$f(k) = A(1-t)k^a$$

where $a > 0$, and A is some constant greater than zero that captures technology in the economy. And finally, you should think of t as some government tax on output. And as most governments do in our world, this one throws the tax revenues in the ocean. ☺

(a) Re-express the above problem in the form of a dynamic programming problem. *(i.e. Write out the Bellman Equation)*

(b) Now, using a guess of $V(k) = E + F \ln k$ for your value function in part (a), solve for the optimal policy rules for consumption and capital.

(c) Plug your policy rules from part (b) into your original dynamic programming problem from part (a) to solve for the constants E and F. ***(Hint:** This will take a bit of math on your part).*

(d) What is the fraction of disposable income that the agent saves each period? How does it depend on b, and what is the intuition for this?

(e) How do higher taxes affect the agent's happiness? What does better technology do for happiness? *(1-2 sentences only… I just want to make sure you check that your value function makes intuitive sense before moving on).*

2. **Dynamic Programming – Numerical Solution**

Write a program in MATLAB to solve the Dynamic Programming problem from part 1A using numerical iteration as I showed you in recitation last week. If you would like your solutions to match up closely to mine, feel free to use the following guidelines:

(i) Use a state vector of 50 possible states.
(ii) Center your state vector around the steady state of the economy using values in a range 10% above and below the steady state.
(iii) Stop the iteration when the absolute difference between all points of your old guess and new guess at the value function is less than .01

Finally, assume the following conditions:

$$A = 1$$
$$a = 0.35$$
$$b = 0.9$$
$$t = 0.3$$

(a) Using your numerical program, plot your value function $V(k)$ and policy functions $c(k)$ and $k'(k)$. Submit these graphs along with your MATLAB code.

(b) Now, again using MATLAB, plot your analytical solutions for the value function and policy functions from Question #1. Do your answers match up?

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