Introducing money.

Olivier Blanchard*

April 2005

* 14.452. Topic 6.

No role for money in the models we have looked at. Implicitly, centralized markets, with an auctioneer:

- Possibly open once, with full set of contingent markets. (Remember, no heterogeneity, no idiosyncratic shocks.) (Arrow Debreu)
- More appealing. Markets open every period.
  Spot markets, based on expectations of the future. For example, market for goods, labor, and one–period bonds. A sequence of **temporary equilibria** (Hicks, Value and Capital, in particular chapters 21 and 22).
  Still no need for money. An auctioneer. Some clearing house.

So need to move to an economy where money plays a useful role.

The ingredients.

- No auctioneer. Geographically decentralized trades.
- Then, problem of double coincidence of wants. Barter is not convenient. Money, accepted on one side of each transaction, is much more so.

Two types of questions:

Foundations

- Why money? What kind of money will emerge?
- Can there be competing monies?
- Fiat versus commodity money?
- Numeraire versus medium of exchange? Should they be the same, or not?

Not just abstract, or history. The rise of barter in Russia in the 1990s. "Natural" dollarization in some Latin American countries. "Units of account", i.e. a numeraire different from the medium of exchange, in Latin America.

But most of the time, we can take it as given that money will be used in transactions, that it will be fiat money, and that the numeraire and the medium of exchange will be the same.

If we take these as given, then we can ask another set of questions:

- How different does a decentralized economy with money look like?
- What determines the demand for money, the equilibrium price level, nominal interest rates?
- How does the presence of money affect the consumption/saving choice?
- Steady state and dynamic effects of changes in the rate of money growth.

Start by looking at a benchmark model. Cash in advance (CIA).

Then, look at variations on the model; money in the utility function.

Then focus on price and inflation dynamics, especially hyperinflation.

1 A cash in advance model

Think in terms of a decentralized economy (although we shall see that there is an optimization problem which replicates the outcome).

1.1 The optimization problem of consumers/workers

Ignore uncertainty. This will make the basic structure of the model much easier to understand.

Consumers/workers maximize:

$$\sum_{i=0}^{i=\infty} \beta^i U(C_{t+i})$$

subject to:

$$P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + M_t + (1+i_t)B_t + (1+r_t)P_t K_t + X_t$$

and

$$P_t C_t \leq M_t$$

Note that I ignore the **labor/leisure choice**. It would be affected. But I leave it out for simplicity. People supply one unit of labor inelastically.

The notation: P_t is the price of goods in terms of the numeraire (the price level). M_t, B_t, K_t are holdings of money, bonds, and capital at the start of period t. W_t and Π_t are the nominal wage and nominal profit received by each consumer respectively. i_t is the nominal interest rate (the interest rate stated in dollars, not goods) paid by the bonds. r_t is the rental rate (in goods) on capital. Money pays no interest.

X_t is a nominal transfer from the government (which has to be there if and when we think of changes in money as being implemented by distribution of new money to consumers).

Now turn to the assumptions underlying the specification:

- Consumers care only about consumption. They do not derive utility from money.

- The first constraint is the budget constraint in nominal terms. It says that nominal consumption plus new asset holdings must be equal to nominal income—wage income (the labor supply is inelastic and equal to one) and profit income—plus initial asset holdings, including interest on the bonds, rental on capital, plus nominal government transfers.
- If the only constraint was the first constraint, then people would hold no money: Bonds pay interest, capital pays a rent, money does not. The second constraint explains why people hold money. It is known as the **cash in advance (CIA)** constraint. People must enter the period with enough nominal money balances to pay for consumption. (Note a bit of arbitrariness here. It would be equally plausible for the constraint to assume that people have both what they carried from last period, and what they receive from the government. So $PC < M + X$. This would, as you will see, change some of the results.)
- One potential story: Households are composed of a worker and a consumer. The worker goes to work. The consumer goes to buy goods, and must do this before the worker has been paid. So he must have sufficient money balances to finance consumption.
- One can think of more sophisticated, smoother, formulations. For example: The cost of buying consumption goods is decreasing in money balances. I shall return to this below.

Let $\lambda_{t+i}\beta^i$ be associated with the budget constraint, $\mu_{t+i}\beta^i$ be associated with the CIA constraint. Set up the Lagrangian and derive the FOC.

$$C_t : \qquad U'(C_t) = (\lambda_t + \mu_t)P_t$$

$$M_{t+1} : \qquad \lambda_t = \beta(\lambda_{t+1} + \mu_{t+1})$$

$$B_{t+1}: \qquad \lambda_t = \beta(1 + i_{t+1})\lambda_{t+1}$$

$$K_{t+1}: \qquad \lambda_t P_t = \beta(1 + r_{t+1})\ \lambda_{t+1} P_{t+1}$$

Interpretation of each.

We can manipulate these conditions to get an intertemporal condition on the marginal utility of consumption:

From the third and the fourth:

$$(1 + i_{t+1}) = (1 + r_{t+1})\frac{P_{t+1}}{P_t} = (1 + r_{t+1})(1 + \pi_{t+1}$$

From the first and second:

$$\lambda_t = \beta\frac{U'(C_{t+1})}{P_{t+1}})$$

$$\lambda_{t+1} = \beta\frac{U'(C_{t+2})}{P_{t+2}}$$

Replacing in the third:

$$\frac{U'(C_{t+1})}{P_{t+1}} = \beta(1 + i_{t+1})\frac{U'(C_{t+2})}{P_{t+2}})$$

Divide both sides by $(1 + i_{t+1})$, and then multiply and divide the second term by $(1 + i_{t+2})$, to get:

$$\frac{U'(C_{t+1})}{1 + i_{t+1}} = \beta(1 + r_{t+2})\frac{U'(C_{t+2})}{1 + i_{t+2}}$$

Interpretation:

Because people have to hold money one period in advance, the effective price of consumption is not 1 but $1 + i$.

Once we adjust for this price effect, then we get the same old relation, between marginal utility this period, marginal utility next period, and the real interest rate.

Note the role of both the **nominal** and the **real** interest rates. Note that if the nominal interest rate is constant, the equation reduces to the standard Euler equation:

$$U'(C_t) = \beta(1 + r_{t+1})U'(C_{t+1})$$

Turn to the characterization of money demand. From the third and the fourth condition:

$$\mu_{t+1} = i_{t+1}\lambda_{t+1}$$

So as long as the nominal interest rate is positive, μ will be positive, and so:

$$\frac{M_t}{P_t} = C_t$$

Pure quantity theory. No interest rate elasticity.

1.2 The General Equilibrium Closure

We close it in the simple way we can (no CIA constraint on firms for example). The assumptions are those we made for the basic Ramsey model, plus those having to do with money.

- Assume firms rent capital and labor, so

$$P_t F_N(K_t, N_t) = W_t \qquad F_K(K_t, N_t) = r_t - \delta$$

- Given constant returns and competitive markets, pure profit Π_t is equal to 0.
- Employment, N_t is equal to 1 (inelastic supply)
- Money is introduced through lump sum transfers to the consumers/workers ("helicopter drops", to use Milton Friedman's expression). (An alternative would be that money is used by the government to buy goods, which are then either given to consumers, or used for defense spending/thrown in the sea. This would however mix fiscal and monetary policy). So

$$M_{t+1} - M_t = X_t$$

- As bonds are issued by agents (not firms, as firms are simply renting capital and labor services) and all agents are identical, in equilibrium, there are no bonds outstanding, so

$$B_{t+1} = B_t = 0$$

Replacing all these conditions in the accumulation equation of consumers-workers gives:

$$P_t C_t + P_t K_{t+1} = P_t F_N(K_t, 1) + (1 + F_K(K_t, 1) - \delta) P_t K_t$$

Or, dividing by P_t, and reorganizing:

$$K_{t+1} = F(K_t, 1) + (1 - \delta) K_t - C_t$$

So, exactly the same condition as in the non monetary economy.

1.3 Dynamics and Steady State

The relevant equations are therefore:

$$\frac{U'(C_{t+1})}{1+i_{t+1}} = \beta(1+r_{t+2})\frac{U'(C_{t+2})}{1+i_{t+2}}$$

$$(1+i_t) = (1+r_t)(1+\pi_t)$$

$$(1+r_t) = 1-\delta+F_K(K_t,1)$$

$$\frac{M_t}{P_t} = C_t$$

Look at the steady state: Suppose that the rate of growth of nominal money is equal to x, so

$$\frac{X_t}{P_t} = x\frac{M_t}{P_t}$$

In steady state, $C_t, K_t, r_t, i_t, \pi_t$ are constant, so:

From the FOC of the consumer, and the demand for capital by firms:

$$(1+r) = 1+F_K(K,1)-\delta = 1/\beta$$

This is the same rule as without money: The modified golden rule.

Using these relations in the budget constraint of the consumer gives:

$$C = F(K,1)-\delta K$$

So, on the real side, the economy looks the same as before. In addition people hold money. And inflation proceeds at the same rate as money growth. The fact that, in steady state, **money growth** has no effect on the real allocation is refered to as the **superneutrality** of money.

In steady state, real money balances must be constant, so:

$$\pi = x$$

Inflation is equal to money growth. And so, $i = \pi + r = x + r$. This one–for–one effect of money growth on the nominal interest rate is known as the **Fisher effect** (from Irving Fisher).

What about dynamic effects? These are harder to characterize, and come on the consumption side from anticipated changes in the effective price of goods, which in turn depends on the nominal interest rate, which in turn depends on inflation, which in turn depends on money growth.

But it is easy to guess the solution to simple paths for money. Check that an unexpected permanent increase in money at time zero leads to an equal proportional increase in the price level, and no change in the real variables.

The same, I believe, holds for an unexpected permanent increase in money growth at time zero, from x to x'. In this case, this leads to no change in the price level today, and inflation at rate x' from then on. Nominal interest rates increase by $x' - x$. Real variables are unaffected.

More complicated paths for expected money can clearly have real effects, as they can lead to anticipated changes in inflation, thus anticipated in nominal interest rates, and thus anticipated changes in the path of consumption. But nothing looks like the effects of money on output we saw when looking at stylized facts in Topic 1.

(To get a sense of dynamics, consider the following short cut. Assume $U(C) = \log(C)$, use the fact that $C = M/P$, and—this is where the short cut is—assume the real rate to be constant and equal to r such that $1 + r\beta^{-1}$. Then, the first order condition for consumption becomes:

$$\frac{P_{t+1}/M_{t+1}}{(1+r)(P_{t+1}/P_t)} = \beta(1+r)\frac{P_{t+2}/M_{t+2}}{(1+r)(P_{t+2}/P_{t+1})}$$

Or

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+2}}{M_{t+1}}$$

The rate of inflation is equal to the rate of money growth next period. (Note that our two guesses above satisfy this condition; and in our two examples the condition on r was indeed satisfied, starting from steady state. In general, consumption will change and so will capital and the marginal product of capital.))

Conclusions:

- We have seen how we can introduce money as a medium of exchange in GE models. The monetary economy does not look very different from the economies we had looked at until now.
- The consumption saving choice is modified , but not transformed. There is now a demand for money is a function of transactions, and the interest rate.
- The real effects of money are limited. In steady state, money is not only neutral, but superneutral
- Changes in money may have dynamic effects on real variables. But these effects appear to be limited.

The CIA model we looked at had an extremely simple cash in advance constraint. There exists others, where for example, different households go to the bank at different times. One of the nicest ones is the GE version of Baumol Tobin, constructed by David Romer (See BF, Chapter 4), in which households choose the timing of their trips to the banks to exchange bonds for money. In these models, changes in money typically have distributional effects (between those who go to the bank and those who do not) and thus effects on real activity for some time. But, again, the effects appear limited (and sometimes exotic).

CIA models can become very unwieldy. Thus, we often use short cuts. The most popular one is to introduce money in the utility function.

2 Money in the utility function

Consider the following optimization problem (known as the Sidrauski model):

Consumers/workers maximize:

$$\sum \beta^i U(C_{t+i}, \frac{M_{t+i}}{P_{t+i}})$$

subject to:

$$P_t C_t + M_{t+1} + B_{t+1} = W_t + \Pi_t + M_t + (1 + i_t)B_t + X_t$$

Think of the utility function as a reduced form of a more complex problem in which by holding more money, households can shop more efficiently, increase leisure time, and so on. Plausibly $U_m > 0$ and $U_{mc} \geq 0$ (why?).

Let us again limit ourselves to the case of certainty here. Let $\lambda_{t+i}\beta^i$ be the lagrange multiplier associated with the constraint. Then the FOC are given by:

$$C_t : \quad U_c(C_t, \frac{M_t}{P_t}) = \lambda_t P_t$$

$$B_{t+1} : \quad \lambda_t = \lambda_{t+1}\beta(1 + i_{t+1})$$

$$M_{t+1} : \quad \lambda_t = \beta[\lambda_{t+1} + \frac{1}{P_{t+1}} U_m(C_{t+1}, \frac{M_{t+1}}{P_{t+1}})]$$

Interpretation. Can rewrite as:

An **intertemporal condition**:

$$U_c(C_t, \frac{M_t}{P_t}) = \beta(1 + r_{t+1})U_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}})$$

An **intratemporal condition**

$$U_m(C_t, \frac{M_t}{P_t})/U_c(C_t, \frac{M_t}{P_t}) = i_t$$

Interpretation. Note that the second says that the ratio of marginal utilities has to be equal to the opportunity cost of holding money, so i, the nominal interest rate.

If for example,

$$U(C, M/P) = \log(C) + a\log(M/P)$$

(not a terribly appealing assumption, as $U_{cm} = 0$, but very convenient)

Then,

$$\frac{1}{C_t} = \beta(1 + r_{t+1})\frac{1}{C_{t+1}}$$

$$\frac{M_t}{P_t} = a\frac{C_t}{i_t}$$

This gives us an *IS* and an *LM* relation.

- The IS: the higher the interest rate, the lower consumption given expected consumption in the future. (Not quite an IS relation, but at least gives us consumption demand—as a function of the real interest rate, and expected consumption in the future)
- The LM: The demand for money is a function of the level of transactions, here measured by consumption, and the opportunity cost of holding money, i.

Under these assumptions, no distortion in the intertemporal consumption decision.

Turn to steady state implications, for the general case. (the firms' side is the same as before).

$$1 + F_K(K, 1) - \delta = 1/\beta$$

$$C = F(K, 1) - \delta K$$

$$U_m(C, \frac{M}{P})/U_c(C, \frac{M}{P}) = (x + r)$$

So, same real allocation again. Superneutrality of money. And a level of real money balances inversely proportional to the rate of inflation, itself equal to the rate of money growth.

What is the optimal rate of money growth? As money is costless to produce, the optimal rate is such as to drive the marginal utility of real money to zero, so to drive $i = x + r$ to zero.

In other words, it is to have $x = -r$, or money growth—or inflation—negative and equal to minus the marginal product of capital. This result is known as the **Optimum Quantity of Money** (the semantics are not great, as this is a result about money growth, not level.)

Dynamic effects of changes in money on real activity? In general, yes, but limited. And nothing which looks like the real effects of money in the real world.

So, bottom line: Money as a medium of exchange, without nominal rigidities gives us a way of thinking about the economy, the price level, the nominal interest rate, but not much in the way of explaining fluctuations.

Very useful however when money growth and inflation become high and variable. Turn to this.

3 Money growth, inflation, seignorage

Inflation and Money Growth during Seven Hyperinflations of the 1920s and 1940s

Country	Beginning	End	P_T/P_0	Av Monthly Inf rate (%)	Av Monthly M Growth (%)
Austria	Oct. 1921	Aug. 1922	70	47	31
Germany	Aug. 1922	Nov. 1923	$1.0\text{x}10^{10}$	322	314
Greece	Nov. 1943	Nov. 1944	$4.7\text{x}10^{6}$	365	220
Hungary 1	Mar. 1923	Feb. 1924	44	46	33
Hungary 2	Aug. 1945	Jul. 1946	$3.8\text{x}10^{27}$	19,800	12,200
Poland	Jan. 1923	Jan. 1924	699	82	72
Russia	Dec. 1921	Jan. 1924	$1.2\text{x}10^{5}$	57	49

P_T/P_0: Price level in the last month of hyperinflation divided by the price level in the first month.

Source: Philip Cagan, "The Monetary Dynamics of Hyperinflation," in Milton Friedman ed., Studies in the Quantity Theory of Money, *(Chicago: University of Chicago Press, 1956), Table 1.*

Start with the money demand in the spirit of that we have just derived:

$$\frac{M_t}{P_t} = C_t \; L(r_t + \pi_t^e)$$

If money growth and inflation are high and variable, M, P and π^e will move a lot relative to C and r. So assume, for simplicity, that $C_t = C$, and $r_t = r$, so:

$$\frac{M_t}{P_t} = C \; L(r + \pi_t^e)$$

This gives a relation between the price level and the expected rate of inflation. The higher expected inflation, the lower real money balances, the higher the price level.

This relation, together with an assumption about money growth, and the formation of expectations, allows us to think about the behavior of inflation. This is what Cagan did. Looking at hyperinflations, he asked;

- Was hyperinflation the result of money growth, and only money growth?
- Why was money growth so high? Did it maximize seignorage. And if not, then why?

Now have a quick look at his model (Read the paper, written in 1956. It is a great read, even today). Also, read BF4-7, and BF10-2. What follows is just a sketch.

Continuous time, more convenient here. Assume a particular form for the demand for money:

$$M/P = \exp(\, -\alpha\pi^e)$$

So, in logs:

$$m - p = -\alpha\; \pi^e$$

Log real money balances are a decreasing function of expected inflation. Or differentiating with respect to time:

$$x - \pi = \alpha\; d\pi^e/dt$$

Assume that people have adaptive expectations about expected inflation. (In an environment such as hyperinflation, this assumption makes a lot of sense. More on rational expectations below).

$$d\pi^e/dt = \beta\; (\pi - \pi^e)$$

Money growth and inflation

Suppose money growth is constant, at x. Will inflation converge to $\pi = x$? To answer, combine the two equations above and eliminate $d\pi^e/dt$ between the two, to get:

$$x - \pi = -\alpha\beta(\pi - \pi^e)$$

This is a line in the (π, π^e) space. For a given x, $d\pi^e/d\pi = -(1 - \alpha\beta)/\alpha\beta$, so if $\alpha\beta < 1$ the line is downward sloping. If $\alpha\beta > 1$ upward sloping.

- If $\alpha\beta < 1$, then the equilibrium is stable. Start with $x > 0$, and $\pi = 0$. Then converge to $\pi = \pi^e = x$.

Effects of an increase in x? Jump in π above the new x, then back to new x over time. (From A to B, and then over time to C.) Why?

- If $\alpha\beta > 1$, then not. Why?

Cagan estimated α and β, found $\alpha\beta < 1$. Hyperinflation was the result of money growth, not a bubble.

Seignorage

What is the maximum revenue the government can get from money creation (called **seignorage**:

$$S \equiv \frac{dM/dt}{P} = \frac{dM/dt}{M}\frac{M}{P} = x\ \exp(-\alpha\pi^e)$$

So, in steady state:

$$S = x\ \exp(-\alpha x)$$

So $x^* = 1/\alpha$

Much lower than the growth rates of money observed during hyperinflation.

But just a steady state result. Can clearly get more in the short run, when π^e has not adjusted yet. This suggests looking at different dynamics: Given seignorage, dynamics of money growth and inflation.

Seignorage, money growth and inflation

Start from:

$$S = x \exp(-\alpha\pi^e)$$

For a given S, draw the relation between π^e and x in π^e, x space. Concave. Can cross the 45 degree line twice, once if tangent, not at all if no way to generate the required seignorage in steady state.

Which equilibrium is stable? Using the equation for adaptive expectations and the money demand relation in derivative form:

$$d\pi^e/dt = \beta(\pi - \pi^e) = \beta(x + \alpha d\pi^e/dt - \pi^e)$$

Or:

$$d\pi^e/dt = \beta/(1 - \alpha\beta)\ (x - \pi^e)$$

If two equilibria, lower one is stable. Start from it, and suppose S increases so no equilibrium. See Figure.

Then, money growth and inflation will keep increasing. This appears to capture what happens during hyperinflations.

Nominal Money Growth and Seignorage, during hyperinflations.

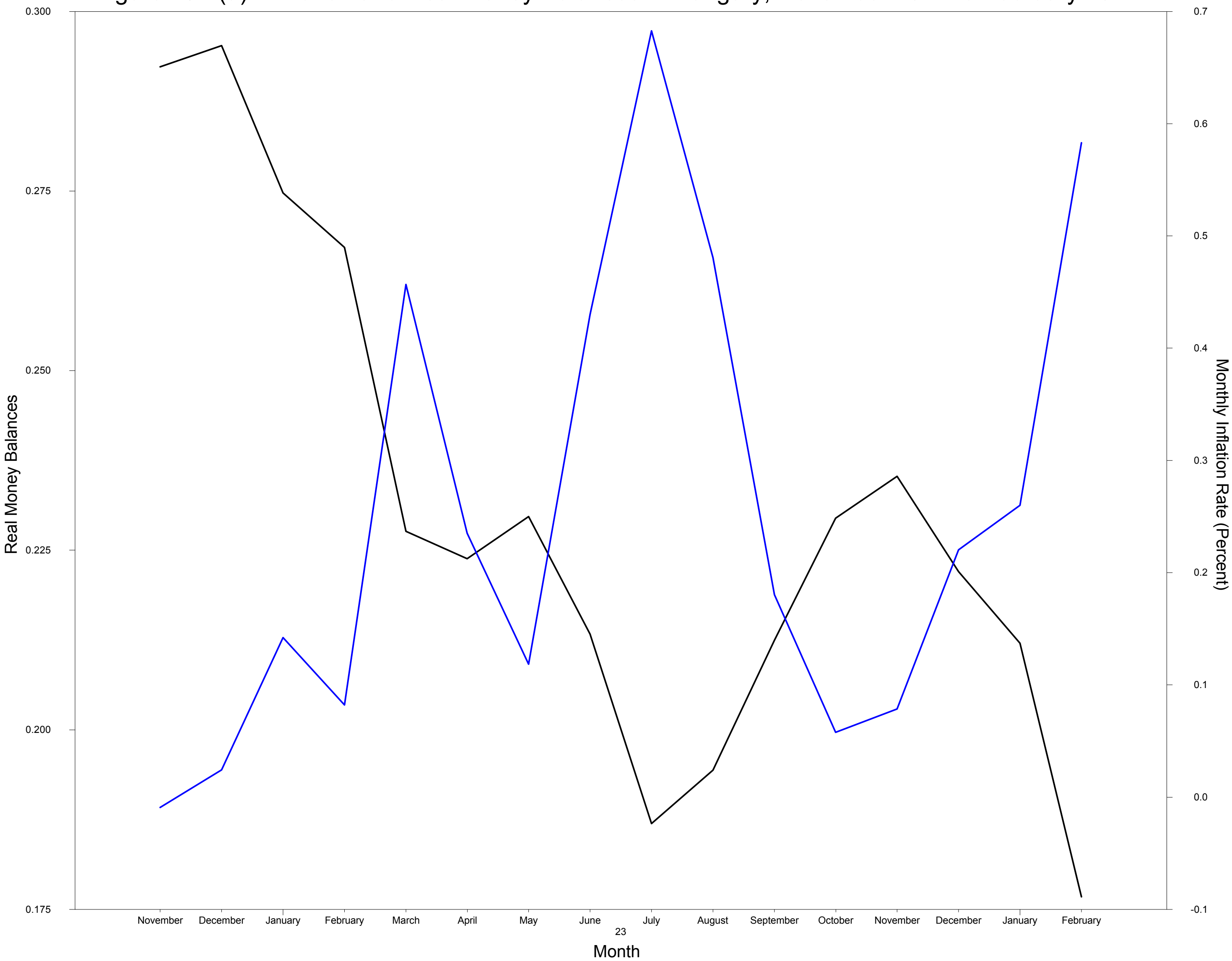
	Rate of Money Growth Maximizing Seignorage (% per month)	Implied Seignorage (% of output)	Actual Rate of Money Growth (% per month)
Austria	12	13	31
Germany	20	14	314
Greece	28	11	220
Hungary 1	12	19	33
Hungary 2	32	6	12,200
Poland	54	4.6	72
Russia	39	0.5	49

Monthly rate of nominal money growth, in percent. Source: Philip Cagan, "The Monetary Dynamics of Hyperinflation," in Studies in the Quantity Theory of Money, Milton Friedman ed. (Chicago: University of Chicago Press, 1956).

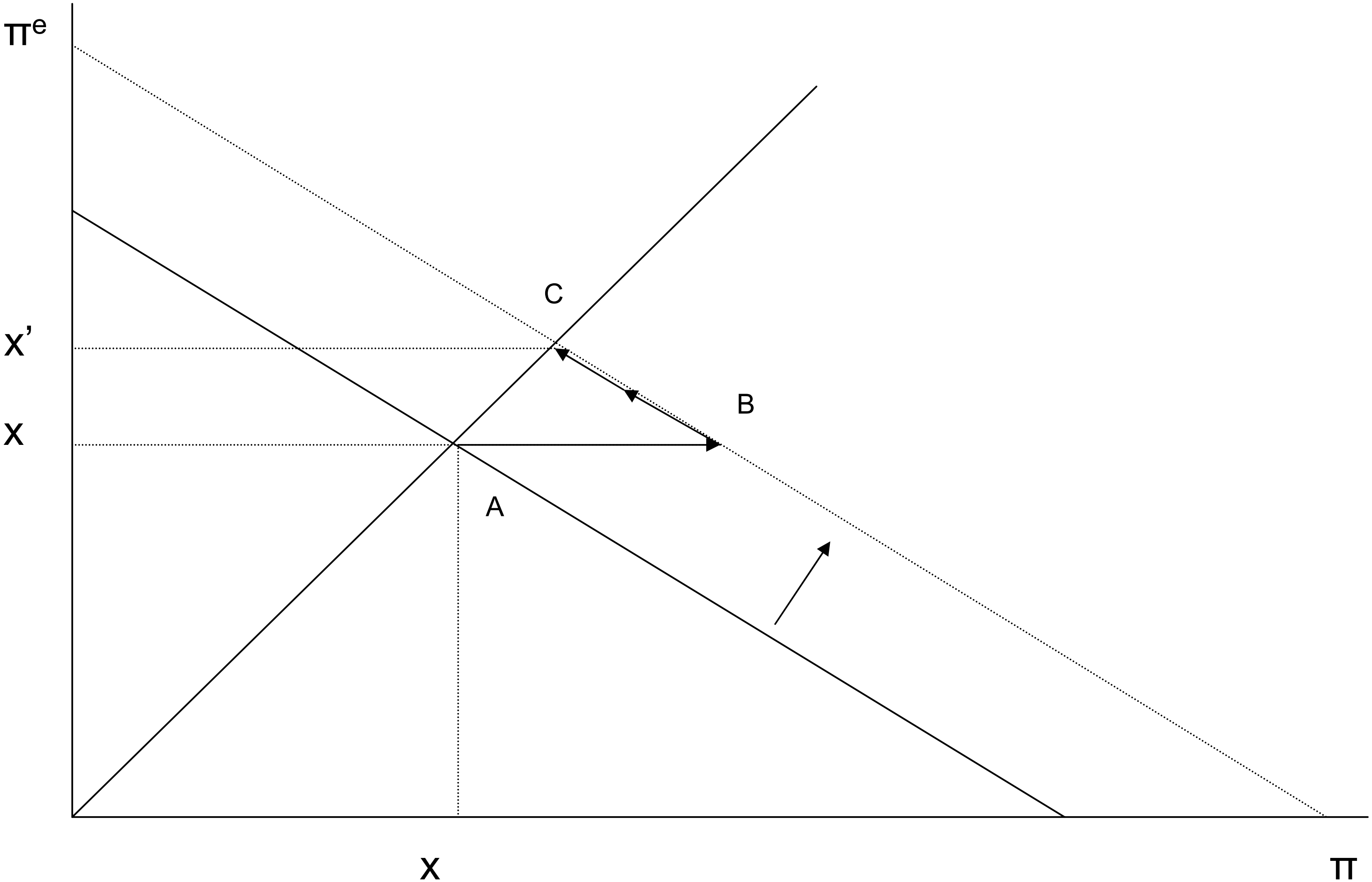
Some other issues

- Adaptive or rational expectations? (see BF 5-1) Recent work by Sargent.
- Fiscal policy, and the effects of inflation on the need for seignorage. (See Dornbusch et al)
- Unpleasant monetarist arithmetic? (see BF 10-2)

Figure 23-1(a). Inflation and Real Money Balances in Hungary, November 1922 to February 1924



Adjustment to an increase in money growth, x, when stability condition is satisfied.



Adjustment to an increase in S