

# Introducing nominal rigidities. A static model.

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May 2005

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\* 14.452. Spring 2005. Topic 7.

Why introduce nominal rigidities, and what do they imply? An informal walk-through.

- In the model we just saw, the price level (the price of goods in terms of money) behaved like an asset price.

$$M/P = CL(i) = CL(r + \pi^e)$$

So any change in the nominal interest rate, from either changes in the equilibrium real interest rate, or in the expected rate of inflation (itself from future changes in the nominal money supply) led to a change in the price level today.

This is particularly clear if we use the Cagan specification we saw earlier (with  $C$  and  $r$  constant), where we can express the log price level as:

$$p_t = \frac{1}{1 + \alpha} \left( \sum \left( \frac{\alpha}{1 + \alpha} \right)^i E[m_{t+i} | \Omega_t] \right)$$

- The price level is not an asset price. It is an aggregate of millions of individual prices, each of them set by a price setter, at discrete intervals in time. So, it is unlikely to adjust in the manner above.
- If  $P$  adjusts more slowly, then what will happen? If the equation above still holds, then the nominal interest rate will not move in the same way. An increase in  $M$  will lead to a decrease in the nominal interest rate, and likely the real interest rate.
- If the demand for goods is given by the same equations as before, the demand for goods will therefore move differently from before (go back to the FOC for consumers, or the q theory characterization for investment. Both depend on the sequence of current and anticipated real rates. )
- What will happen to output? This depends on how the price (wage) setters decide to respond to shifts in demand.

(The older fix price equilibrium line of research—Barro, Grossman, Malinvaud: Output will be given by the minimum of demand and supply at the given price. It died, and rightly so, because markets with price setters are unlikely to be competitive, and we have to understand what price setters do, and how they react if demand is not equal to what they expected.)

If they have monopoly power, they may want to accommodate these shifts so long as price exceeds marginal cost. So movements in demand, both positive or negative will have an effect on output, at least within some range (as long as  $MC < P$ ).

Much of the work of the last 20 years has gone into looking at the foundations for this story, and the implications for fluctuations, and for monetary and fiscal policy.

We shall proceed in three steps.

- First (this topic), look at a static model, in which these issues can be discussed (simplified version of Blanchard Kiyotaki). The new element here is the introduction of monopolistic competition in the goods market, so we can think about price setting.  
There are enough new steps and concepts that it is better to start with a static model. First, without nominal rigidities. Second, with nominal rigidities. Effects of nominal money, and effects on output and welfare.
- Second, put these nominal rigidities in the type of model we have developed until now, with  $C/S$ ,  $L/N$ , and  $C/(M/P)$  choices. Examine the effects of shocks, and compare to stylized facts seen in topic 1.
- Third, look at price setting more closely, examine the effects of price staggering, and derive one of the current workhorses, which incorporates all these aspects, known as the “New Keynesian” model. Then, reexamine implications for monetary and fiscal policy.

## 1 A one-period model of yeomen farmers

Think of an economy composed of a large number of households, each producing a differentiated good, and each consuming all goods. More specifically, a continuum of households and goods on [01].

Each household produces its good using its own labor (this way we integrate producers and suppliers of labor, and have to keep track only of prices, not wages and prices).

The utility function of a household  $i$  is given by:

$$U(C_i, \frac{M_i}{P}, N_i)$$

where:

$$C_i \equiv [\int_0^1 C_{ij}^{\sigma-1/\sigma} dj]^{\sigma/(\sigma-1)}$$
$$P = [\int_0^1 P_j^{1-\sigma} dj]^{1/(1-\sigma)}$$

The budget constraint is given by:

$$\int_0^1 P_j C_{ij} + M_i = P_i Y_i + \bar{M}_i$$

and the production function for producing good  $i$  is given by:

$$Y_i = Z N_i$$

Things to note about the model:

- We set it up as a one-period problem. Also, for the moment, no uncertainty. But will introduce both later on, first uncertainty about  $\bar{M}$  and  $Z$  later in these notes, and then, in topic 8, a dynamic version, with bonds and money.
- Each household enjoys a consumption basket, composed of all goods.

It needs money for transactions; this is formalized by putting money in the utility function rather than formalizing the exact structure of transactions and using CIA.

- Each household produces a differentiated good using labor and a constant returns technology.  $Z$  is the level of technology. We shall think of movements in  $Z$  as technological shocks. Each household faces a demand curve for its product, which we shall have to derive (the demand for the good by all other consumers.)
- The budget constraint is a short cut to a dynamic budget constraint.

It is easy to characterize the equilibrium of the model with a general utility function. But it is even easier to do it with the following utility

$$U(C_i, \frac{M_i}{P}, N_i) = (\frac{C_i}{\alpha})^\alpha (\frac{M_i/P}{1-\alpha})^{1-\alpha} - \frac{1}{\beta} N_i^\beta$$

Among the advantages of this specification will be a very simple relation between consumption and real money balances, and constant marginal utility of income.

To characterize the general equilibrium, proceed in 4 steps:

- Given spending on consumption, derivation of consumption demands for each good by each household.
- Derivation of the relation between aggregate consumption and aggregate real money balances.
- Derivation of the demand curve facing each household, and derivation of its pricing decision
- General equilibrium

For the moment, no nominal rigidities. Could solve all these steps simultaneously, but much less intuitive.

## 1.1 Demand for individual goods

Suppose household  $i$  depends to spend a nominal amount  $X_i$  on consumption. So it maximizes:

$$\max C_i \equiv \left[ \int_0^1 C_{ij}^{\sigma-1/\sigma} dj \right]^{\sigma/(\sigma-1)}$$

subject to:

$$\int_0^1 P_j C_{ij} dj = X_i$$

Then, with a bit of algebra (make sure you go through the steps), we get:

$$C_{ij} = \frac{X_i}{P} \left( \frac{P_j}{P} \right)^{-\sigma}$$

where  $P$  is the price index we wrote earlier, and  $C_i, P, X_i$  satisfy:

$$C_i P = X_i$$

so we can rewrite the consumption demand for good  $j$  as:

$$C_{ij} = C_i \left( \frac{P_j}{P} \right)^{-\sigma}$$

In words, we can think of the consumer taking a two-step decision. First, how much to consume of the consumption basket, at price  $P$ . This gives  $C_i$ .

Then, given that decision, he allocates demand to each good in proportion to its relative price. It is clear that, for later, we need  $\sigma > 1$  so the demand curves are sufficiently elastic.

## 1.2 The choice of money and consumption

Using what we just learned, we can rewrite the problem of the consumer as:

$$\max \left( \frac{C_i}{\alpha} \right)^\alpha \left( \frac{M_i/P}{1-\alpha} \right)^{1-\alpha} - \frac{N_i}{\beta}$$

subject to:

$$PC_i + M_i = P_i Y_i + \bar{M}_i$$

The change is in the budget constraint, where we use the fact that we can think of spending as the product of the consumption basket times its price index, the price level—so we are back to a familiar optimization problem.

Given income and initial money balances, we can solve for optimal consumption and money balances:

$$C_i = \alpha \frac{P_i Y_i + \bar{M}_i}{P}, \quad \frac{M_i}{P} = (1-\alpha) \frac{P_i Y_i + \bar{M}_i}{P}$$

People allocate their initial wealth in proportion  $\alpha$  and  $1-\alpha$  to consumption and real money balances.

- For future use, the following FOC between the two will be useful: Relation between real money balances and consumption (both endogenous):

$$C_i = \frac{\alpha}{1-\alpha} \frac{M_i}{P}$$

- This implies that the demand for good  $j$  by household  $i$  can be written as:

$$C_{ij} = C_i \left( \frac{P_j}{P} \right)^{-\sigma} = \frac{\alpha}{1-\alpha} \frac{M_i}{P} \left( \frac{P_j}{P} \right)^{-\sigma}$$

- Replacing  $C_i$  and  $M_i/P$  in the utility function gives an indirect utility function of the form:

$$\frac{P_i}{P} Y_i - (1/\beta) N_i^\beta + \frac{\bar{M}_i}{P}$$

This is where the special form of the utility function helps a bit. It basically implies constant marginal utility of income, so the problem of choosing output, employment, and prices looks like the conventional monopolist problem. (This will no longer be the case in the dynamic GE model we shall see later.)

### 1.3 Pricing and output decisions

Household  $i$  then chooses the price and the level of output of good  $i$ . To do so, it maximizes:

$$\max \frac{P_i}{P} Y_i - (1/\beta) Y_i^\beta Z^{-\beta}$$

where I have used the fact that  $N_i = Z^{-1} Y_i$ , and I ignore the last term in the utility function ( $\bar{M}_i/P$ ), which is given at the time of the maximization.

Integrating over households  $j$ , the demand for good  $i$  is given by:

$$Y_i = \int_0^1 C_{ji} dj = \frac{\alpha}{1-\alpha} \frac{M}{P} \left(\frac{P_i}{P}\right)^{-\sigma}$$

where  $M = \int_0^1 M_j dj$ . Using the fact that, in equilibrium, the money balances households want to hold must be equal to the nominal money stock, so  $M = \bar{M}$ , then:

$$Y_i = \frac{\alpha}{1-\alpha} \frac{\bar{M}}{P} \left(\frac{P_i}{P}\right)^{-\sigma}$$



Solving the maximization problem gives:

$$\frac{P_i}{P} = \frac{\sigma}{\sigma - 1} Y_i^{(\beta-1)} Z^{-\beta}$$

Price equals marginal cost times a markup. Solving for  $Y_i$  gives:

$$\frac{P_i}{P} = \left[ \frac{\sigma}{\sigma - 1} X^{(\beta-1)} Z^{-\beta} \right]^{1/(1+\sigma(\beta-1))}$$

where

$$X \equiv \frac{\alpha}{1 - \alpha} \frac{\bar{M}}{P}$$

An increase in  $\bar{M}/P$  leads to an increase in the relative price. The effect depends on  $\beta$  and  $\sigma$ . The closer  $\beta$  is to unity, the smaller the effect on the relative price.

Can characterize the equilibrium graphically. Demand is a function of relative price, and real money balances. Marginal revenue as well. Marginal cost is increasing in output. Draw marginal cost, marginal revenue and demand. Figure 8-1 in BF.

#### 1.4 General equilibrium

In general equilibrium, the relative price must be equal to 1. So, output for each household must be such that this holds:

$$1 = \frac{\sigma}{\sigma - 1} Y^{(\beta-1)} Z^{-\beta}$$

so:

$$Y = \left[ \frac{\sigma - 1}{\sigma} Z^\beta \right]^{\frac{1}{\beta-1}}$$

and:

$$N = \left[ \frac{\sigma - 1}{\sigma} Z \right]^{\frac{1}{\beta-1}}$$

So lower equilibrium output than under perfect competition. But only a small modification, for the presence of a markup. Output is lower.

Technological shocks increase both employment and output. The more so, the closer  $\beta$  is to one.

The price level must be such that the real money stock generates the right level of demand:

$$Y = \frac{\alpha}{1 - \alpha} \frac{\bar{M}}{P} \Rightarrow P = \frac{\alpha}{1 - \alpha} \frac{\bar{M}}{Y}$$

So this would seem like little progress: Output determined by: marginal cost plus markup equals price. Nominal money neutral. But in fact, much closer:

- First, a model with aggregate demand. An effect of real money balances. Clearly simplistic, but we know how to extend it. (And we shall do so in the dynamic version where real money will affect the interest rate, which in turn will affect aggregate demand).
- Second, a model with price setters. So we can look at how they set prices, and what determines the price level.
- Some intuition for price level determination. Consider an increase in nominal money, from  $M$  to  $M'$ .

Requires a proportional increase in  $P$ , no change in relative prices.

But nobody is in charge of the price level. Each price setter tries to adjust its relative price. If  $\beta$  not too far above 1, then relative prices increase only a little. And then a bit more, and so on, until the price level has adjusted.

Suggests that the adjustment may be slow, and that the speed depends on how much price setters want to adjust their relative price. Now ready to introduce nominal rigidities.

## 2 Yeomen farmers and nominal rigidities

Think of the households having to set nominal prices. Two arguments for why they may want to do this at discrete intervals.

- Menu costs. (Akerlof Mankiw) Small changes in prices (equivalently, small deviations of prices from optimum) have only a second order effect on profit.

But a small change in the price level has a first order effect on output and welfare. Why? Because of the initial wedge created by monopoly power. Back to diagram.

- Desired change in relative price may be small. Go back to the equation for  $P_i/P$  earlier. If marginal cost is relatively flat ( $\beta - 1$  close to zero), then want to change the relative price by little.

So modify the model as follows. Each household chooses the price of its product before knowing the realization of nominal money and productivity this period. Consumption decisions, and thus demand, are taken after observing the realization.

So return to the choice of the relative price by households.

$$\max E\left[\frac{P_i}{P}Y_i - \frac{1}{\beta}Y_i^\beta Z^{-\beta}\right]$$

subject to:

$$Y_i = \frac{\alpha}{1-\alpha} \frac{\bar{M}}{P} \left(\frac{P_i}{P}\right)^{-\sigma} \equiv X \left(\frac{P_i}{P}\right)^{-\sigma}$$

The difference is that  $\bar{M}$  and  $Z$  are now random variables. The FOC is given by:

$$E\left[X(1-\sigma)\left(\frac{P_i}{P}\right)^{-\sigma} + \sigma X^\beta Z^{-\beta}\left(\frac{P_i}{P}\right)^{-\beta\sigma-1}\right] = 0$$

Or, rearranging:

$$\frac{P_i}{P} = \left[ \frac{\sigma}{\sigma - 1} \frac{E[X^\beta Z^{-\beta}]}{E[X]} \right]^{1/(1+\sigma(\beta-1))}$$

The only difference from before is the presence of the expectation. But the principle is the same. The higher expected nominal money, the higher the relative price.

## 2.1 General equilibrium

In general equilibrium, all price setters must set prices so that the relative price is equal to 1. So, the price level is implicitly determined by:

$$1 = \frac{\sigma}{\sigma - 1} \frac{E[X^\beta Z^{-\beta}]}{E[X]}$$

where  $X \equiv (\alpha/(1 - \alpha))\bar{M}/P$ . Demand and output (as long as  $MC < P$ ) are given by:

$$Y = (\alpha/(1 - \alpha))\bar{M}/P$$

and employment is given by:

$$N = Z^{-1}Y$$

This gives us our basic set of results:

- Given the predetermined price level,  $\bar{M}/P$  moves with  $\bar{M}$  and so does consumption.
- Unanticipated movements in nominal money affect real money balances one for one and so affect consumption one for one.
- Demand affects output, so long as marginal cost is less than price—so suppliers willing to supply. Back to diagram.
- No systematic movement in relative prices (in real wages in a model with a labor market). Fits the data well.

- Welfare goes up and down with output. Indeed, higher than expected money is good. This again has many implications. Temptation to increase welfare by unexpectedly increasing money.
- Unanticipated technological shocks have no effect on demand and thus on output (this comes from the very strong constraint that demand depends only on real money balances. This will no longer be true in the more realistic dynamic model we shall see later).
- Unanticipated technological shocks decrease employment initially (i.e during the period during which prices are predetermined).

The model is too rough, but these results are appealing, given the evidence we saw in topic 1 (and then later in the review of evidence on technological shocks in topic 3). Nominal money seems to affect output and employment. Technological shocks seem to have a limited effect on output and perhaps to decrease employment initially.

## 2.2 A useful log linear version

The model is simple. Yet, the equations which characterize the solution are non trivial, given the interaction between non linearities and expectations. In such cases, it is typically very useful to derive a log linear version of the model, and use for example it to look at various policy experiments.

The only equation which presents a problem is the equation implicitly defining the price level. In general, it is not log linear. So we have to have to take a log linear approximation. Let me go through it step by step (so that you see log linear approximation at least once):

Take a log linear approximation around the steady state associated with given values of money  $M_0$  and technology  $Z_0$ .  $\bar{M}_0$ ,  $P_0$  and  $Z_0$  therefore satisfy:

$$1 = \frac{\sigma}{\sigma - 1} \frac{\left(\frac{\alpha}{1-\alpha} \frac{\bar{M}_0}{P_0}\right)^\beta Z_0^{-\beta}}{\frac{\alpha}{1-\alpha} \frac{\bar{M}_0}{P_0}}$$

Use lower case letters  $m$ ,  $p$  and  $z$  for log deviations from the values above.

Then:

$$E\left[\frac{\alpha}{1-\alpha} \frac{\bar{M}}{P}\right] \approx \frac{\alpha}{1-\alpha} \frac{\bar{M}_0}{P_0} E[1 + m - p]$$

and

$$E\left[\left(\frac{\alpha}{1-\alpha} \frac{\bar{M}}{P}\right)^\beta Z^{-\beta}\right] \approx \left(\frac{\alpha}{1-\alpha} \frac{\bar{M}_0}{P_0}\right)^\beta Z_0^{-\beta} E[1 + \beta(m - p) - \beta z]$$

So:

$$1 \approx \frac{\sigma}{\sigma - 1} \left(\frac{\alpha}{1-\alpha} \frac{\bar{M}_0}{P_0}\right)^\beta Z_0^{-\beta} E[1 + \beta(m - p) - \beta z] / \frac{\alpha}{1-\alpha} \frac{\bar{M}_0}{P_0} E[1 + m - p]$$

Or using the relation between  $\bar{M}_0$ ,  $P_0$ ,  $Z_0$ :

$$1 \approx \frac{E[1 + \beta(m - p) - \beta z]}{E[1 + m - p]}$$

or

$$p \approx Em - \frac{\beta}{\beta - 1} z$$

Under some further assumptions, an equation can sometimes be expressed as an exact log linear **relation** (not only a log linear approximation). This is the case here. Suppose that  $M$  (forget the bar for notational simplicity) is log normally distributed, so  $\log M$  is normal with mean  $Em$  and variance  $v$ . Assume, for simplicity that  $\log Z$  is constant and equal to zero. (Trivial

to extend, but note in this case that the covariance between  $\log M$  and  $\log Z$  will matter.)

In this case,

$$E[M] = \exp(Em + v/2)$$

$$E[M^\beta] = \exp(\beta Em + \beta^2 v/2)$$

Rewrite equation 1 as:

$$1 = \frac{\sigma}{\sigma - 1} \left( \frac{\alpha}{1 - \alpha} \right)^{(\beta - 1)} Z^{-\beta} P^{1 - \beta} \frac{E[M^\beta]}{E[M]}$$

Replace the two expectations by their expression above, and take logs:

$$0 = \log\left(\frac{\sigma}{\sigma - 1}\right) + (\beta - 1) \log\left(\frac{\alpha}{1 - \alpha}\right) - (\beta - 1)p + (\beta - 1)Em + (\beta^2 - 1)v/2$$

or

$$p = Em + \frac{1}{\beta - 1} \log\left(\frac{\sigma}{\sigma - 1}\right) + \log\left(\frac{\alpha}{1 - \alpha}\right) + (1 + \beta)v/2$$

Note this relation is between log levels of the price level and nominal money, not log deviations from steady state (so there are constant terms in the relation).

Once the log linearization is done, the log linear approximation of the model is given by:

$$p = Em - \frac{\beta}{\beta - 1} Ez$$

$$y = m - p = m - Em + \frac{\beta}{\beta - 1} Ez$$

$$n = y - z$$

where lower case letters indicate log deviations from steady state. Absent nominal rigidities (we shall call this level of output the second best level and denote it by a hat):

$$\hat{y} = \frac{\beta}{\beta - 1}z, \quad \hat{n} = \frac{1}{\beta - 1}z$$

So note that output responds to unexpected money, but not to unexpected technological shocks. The second best output does not respond to money, but responds to technological shocks.

Optimal monetary policy? Say, minimize distance from second best,  $y - \hat{y}$ . If central bank can adjust money after having observed  $z$ , then easy:

$$m - Em = \frac{\beta}{\beta - 1}(z - Ez) \Rightarrow y - \hat{y} = 0$$

Increase money in the face of positive productivity shocks, so as to increase demand in line with supply, and get employment to increase rather than decrease.

Why not do even better and try to further increase welfare and achieve first best  $y_{FB} = ct + \hat{y}$ . Can clearly do it ex-post by increasing  $m$  further. What is the problem? What will agents expect ex-ante? (This is the problem known as time inconsistency. More on this later. But, if you want more now, see for example BF 11-4 for an introduction)

Simple log linear model... but a rich story behind it.

Still: Many issues. Here, one period. Transmission of changes in real money to output through interest rates? More realistic transmission mechanism? More realistic price setting. So look at a dynamic version. Topics 8 and 9.