A static model with nominal rigidities.

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A few clarifications on log linearizations and log linear relations. And a correction—with apologies to those of you who spent time trying to replicate. (The correction is indicated by a star below).

From the static model, with prices set in advance, we get:

$$1 = \frac{\sigma}{\sigma - 1} \frac{E[X^{\beta}\, Z^{-\beta}]}{E[X]}$$

where $X \equiv (\alpha/(1-\alpha))\bar{M}/P$.

Demand and output (as long as $MC < P$) are given by:

$$Y = (\alpha/(1-\alpha))\bar{M}/P$$

and employment is given by:

$$N = Z^{-1} Y$$

Can we log linearize? The only problem equation is the first one.

In general, it is not log linear. So we have to have to take a log linear

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approximation.

Take a log linear approximation around the steady state associated with given values of money M_0 and technology Z_0. $\bar{M}_0$, P_0 and Z_0 therefore satisfy:

$$1 = \frac{\sigma}{\sigma - 1} \frac{\left(\frac{\alpha}{1-\alpha}\frac{\bar{M}_0}{/P_0}\right)^{\beta} {Z_0}^{-\beta}}{\frac{\alpha}{1-\alpha}\frac{\bar{M}_0}{P}}$$

Use lower case letters m, p and z for log deviations from the values above. Then:

$$E[\frac{\alpha}{1-\alpha}\frac{\bar{M}}{P}] \approx \frac{\alpha}{1-\alpha}\frac{\bar{M}_0}{P_0}E[1+m-p]$$

and

$$E[(\frac{\alpha}{1-\alpha}\frac{\bar{M}}{P})^{\beta} Z^{-\beta}] \approx (\frac{\alpha}{1-\alpha}\frac{\bar{M}_0}{P_0})^{\beta} {Z_0}^{-\beta} E[1+\beta(m-p)-\beta z]$$

So:

$$1 \approx \frac{\sigma}{\sigma-1}(\frac{\alpha}{1-\alpha}\frac{\bar{M}_0}{P_0})^{\beta} {Z_0}^{-\beta} E[1+\beta(m-p)-\beta z] \; / \; \frac{\alpha}{1-\alpha}\frac{\bar{M}_0}{P_0}E[1+m-p]$$

Or using the relation between $\bar{M}_0, P_0, Z_0$:

$$1 \approx \frac{E[1+\beta(m-p)-\beta z]}{E[1+m-p]}$$

or

$$p \approx Em - \frac{\beta}{\beta - 1} z$$

Under some further assumptions, the first equation can be expressed as an exact log linear **relation** (not only a log linear approximation). Suppose that M (forget the bar for notational simplicity) is log normally distributed, so $\log M$ is normal with mean Em and variance v. Assume, for simplicity that $\log Z$ is constant and equal to zero. (Trivial to extend, but note in this case that the covariance between $\log M$ and $\log Z$ will matter.)

In this case,

$$E[M] = \exp(Em + v/2)$$
$$E[M^\beta] = \exp(\beta Em + \beta^2 v/2)$$

Rewrite equation 1 as:

$$1 = \frac{\sigma}{\sigma - 1}(\frac{\alpha}{1-\alpha})^{(\beta-1)} Z^{-\beta} P^{1-\beta} \frac{E[M^\beta]}{E[M]}$$

Replace the two expectations by their expression above, and take logs:

$$0 = \log(\frac{\sigma}{\sigma-1}) + (\beta - 1)\log(\frac{\alpha}{1-\alpha} - (\beta-1)p + (\beta-1)Em + (\beta^2 - 1)v/2$$

or

$$p = Em + \frac{1}{\beta - 1}\log(\frac{\sigma}{\sigma-1}) + \log(\frac{\alpha}{1-\alpha} + (1+\beta)v/2$$

Note this relation is between log levels of the price level and nominal money, not log deviations from steady state (so there are constant terms in the relation).

* The correction. What is not correct in the notes is the log linear approximation for $(p_i - p)$ on p13, which is missing the term $(1 + \sigma(\beta - 1))$. (the

corresponding expression in Blanchard-Fischer on p385 is correct). But, given that in equilibrium, $p_i - p = 0$, that mistake does not matter for what we did above.

This expression should be:

$$(1 + \sigma(\beta - 1))(p_i - p) = (\beta - 1)(Em - p) - \beta z$$

or

$$(p_i - p) = \frac{\beta - 1}{1 + \sigma(\beta - 1)}(Em - p) - \frac{\beta}{1 + \sigma(\beta - 1)}$$

This correction also leads to a correction of notation in the Taylor-Calvo models we saw on monday. The starting equation, in terms of the log output gap, takes the form:

$$(p_i - p) = \frac{\beta - 1}{1 + \sigma(\beta - 1)}\, x$$

rather than, as written in class $p_i - p = (\beta - 1)x$. This changes nothing of substance, but must indeed have been confusing.