A dynamic model with nominal rigidities.

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In topic 7, we introduced nominal rigidities in a simple static model. It is time to reintroduce dynamics. These notes reintroduce the C/S, N/L, and C/(M/P) choices we studied in the earlier models. The next set of notes will examine richer price setting structures, and their implications.

1 A dynamic GE model of yeomen farmers

One would like to construct a dynamic GE model which had:

- Non trivial investment and consumption decisions, as in the model examined in topic 4. (A rich IS)
- A rich description of how monetary policy determines the short term nominal interest rate, along the lines of topic 6 (A rich LM)
- A theory of price determination, which expanded on the model we have just seen. (A rich AS).

A model that did all this could be constructed. But at some pain, and clearly requiring numerical simulations in the end. So, need a simpler benchmark model. Here is one, variations of which can be found in the literature.

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1.1 The optimization problem

The economy is composed of yeomen farmers, who maximize the following objective function:

$$\max E[\sum_0^\infty \beta^k (U(C_{it+k}) + V(\frac{M_{it+k+1}}{P_{t+k}}) - Q(N_{it+k})\ )|\ \Omega_t]$$

subject to:

$$C_{it} \equiv [\int_0^1 C_{ijt}{}^{\sigma-1/\sigma} dj]^{\sigma/(\sigma-1)} \qquad P_t = [\int_0^1 P_{jt}{}^{1-\sigma} dj]^{1/(1-\sigma)}$$

$$\int_0^1 P_{jt}C_{ijt} + M_{it+1} + B_{it+1} = P_{it}Y_{it} + (1+i_t)B_{it} + M_{it} + X_{it}$$

$$Y_{it} = Z_t N_{it}$$

where k now denotes time, and the rest of the notation is standard.

In other words: Each household produces a differentiated product, using labor. It derives disutility from work, and utility from a consumption basket, and from real money balances.

It can save either in the form of bonds, or in the form of money. Bonds pay interest. Money does not.

A number of remarks

- Utility is separable in consumption, money balances, and leisure.
- Utility of money depends on end of period money balances, divided by the price level this period.

  Would look less strange if we denoted end of period balances by M_t

rather than M_{t+1}, so utility would depend on M_t/P_t rather than M_{t+1}/P_t.

But the assumption would be the same. It role is to deliver a relation between the demand for nominal money, the current price level, and the interest rate (M_{t+1}, P_t, i_{t+1}). (The formalization we saw earlier (money in the utility function, topic 6) gives a relation between the demand for nominal money, the price level **next period**, and the interest rate $(M_{t+1}, P_{t+1}, i_{t+1})$.) (The problem is not deep. It would go away in continuous time, where people would continuously rebalance their portfolios)

- There is no capital in the model. (Constant returns to labor). So, demand will be equal to consumption. Bonds are nominal bonds. They can be thought as inside bonds (in zero net supply, and so equal to zero in equilibrium), or government bonds, perhaps introduced in open market operations.

The structure of the solution is very much the same as before:

- Given spending on consumption, derivation of consumption demands for each good by each household.
- Derivation of consumption, real money balances and bond holdings. The relation between aggregate consumption and aggregate real money balances.
- Derivation of the demand curve facing each household, and derivation of its pricing decision
- General equilibrium

1.2 Demand for individual goods

Going through the same steps as in the static model gives the demand by household i for good j in period t:

$$C_{ijt} = C_{it}\ (\frac{P_{jt}}{P_t})^{-\sigma}$$

where, as before:

$$\int_0^1 P_{jt} C_{ijt} = P_t C_{it}$$

So that, for later use, aggregating over households, the demand for good j in period t is given by:

$$Y_{jt} = C_t (\frac{P_{jt}}{P_t})^{-\sigma}$$

1.3 Consumption and real money balances

Using the results above, the problem of the household can be rewritten as:

$$\max\ E[\sum_0^\infty \beta^k (U(C_{it+k}) + V(\frac{M_{it+k+1}}{P_{t+k}}) - Q(N_{it+k})\ )|\ \Omega_t]$$

subject to the budget constraint:

$$P_t C_{it} + M_{it+1} + B_{it+1} = P_{it} Y_{it} + (1 + i_t) B_{it} + M_{it} + X_{it}$$

and the demand and production functions:

$$Y_{it} = C_t (\frac{P_{it}}{P_t})^{-\sigma} \qquad\qquad Y_{it} = N_{it}$$

Let $\lambda_{t+k}\beta^k$ be the Lagrange multiplier associated with the budget constraint at $t+k$. (Replace N_{it} by Y_{it} in the objective function, and Y_{it} by the expression for demand, in the budget constraint, so only one constraint is left).

Look first at the FOC associated with the choices for consumption, real money balances, :

$$C_{it}\ :\ U'(C_{it}) = \lambda_t P_t$$

$$M_{it+1}\ :\ V'(\frac{M_{it+1}}{P_t}) = (\lambda_t - \beta E[\lambda_{t+1}\ |\Omega_t])P_t$$

$$B_{it+1}\ :\ \lambda_t = \beta(1+i_{t+1})E[\lambda_{t+1} \mid \Omega_t]$$

which we can reduce to two conditions (this should be familiar by now):

An intertemporal condition

$$U'(C_{it}) = E[\ \beta\ (1+r_{t+1})U'(C_{it+1}) \mid \Omega_t]$$

An intratemporal condition

$$V'(\frac{M_{t+1}}{P_t})/U'(C_{it}) = \frac{i_{t+1}}{1+i_{t+1}}$$

The interpretation is as before:

- The tilting smoothing condition for consumption, and the role of the real interest rate. (From the derivation, you can see that the real interest rate is the realized real rate, and thus random as of time t. It cannot be taken out of the expectation).

- The choice between real money balances and consumption, which depends on the nominal interest rate.

There is one FOC left, for the choice of the relative price, and the associated level of output and employment. Let's turn to it.

1.4 Pricing and output decisions

Replacing Y_{it} by the demand function in the budget constraint, differentiating with respect to P_{it}, and using the fact that $\lambda_t = U'(C_{it})/P_t$, gives:

$$\frac{P_{it}}{P_t} = (\frac{\sigma}{\sigma - 1}) \frac{Q'(Y_{it})}{U'(C_{it})Z_t}$$

Each household sets the price of its product as a markup over marginal cost. The markup is equal to $\sigma/(\sigma - 1)$. The marginal cost is equal to the disutility of work, divided by marginal utility.

1.5 General equilibrium

In symmetric general equilibrium:

$$Y_{it} = C_{it} = C_t = Y_t \ , N_t = Y_t {Z_t}^{-1}$$

So collecting equations:

$$IS: \quad U'(Y_t) = E[\ \beta(1 + r_{t+1})U'(Y_{t+1}) \mid \Omega_t]$$

$$LM: \quad V'(\frac{M_{t+1}}{P_t})/U'(Y_t) = \frac{i_{t+1}}{1 + i_{t+1}}$$

$$AS: \qquad 1 = \frac{\sigma}{\sigma - 1} \frac{Q'(N_t)}{U'(N_t Z_t) Z_t} \qquad Y_t = N_t Z_t$$

This gives us a nice characterization in terms of an IS relation, an LM relation, and an AS (aggregate supply) relation. But not much action:

We get full dichotomy: The AS relation fully determines Y_t. For example, if $U(.) = \log(.)$, then it follows that

$$1 = (\sigma/(\sigma - 1))Q'(N_t) \ \Rightarrow \ N_t = \bar{N}, \ Y_t = \bar{N} Z_t$$

. Then, roughly speaking, the IS determines the real interest rate consistent with goods market equilibrium. And the LM determines in turn the price level consistent with financial markets equilibrium.

This is clearest if we take a log linearization:

$$IS: \qquad y_t = -a r_{t+1} + E y_{t+1}$$

$$LM: \qquad m_{t+1} - p_t = b y_t - c(r_{t+1} + E p_{t+1} - p_t)$$

$$AS: \qquad y_t = d z_t$$

where I have ignored the constant terms, and d reflects the effect of z on y, and so depends on the shape of the functions $U(.)$ and $Q(.)$.

Note that:

- Output is determined by the AS alone.
- Expected output is equal to $E z_{t+1}$ and so the IS relation determines the ex-ante real interest rate r_{t+1} given current and expected output.
- Given current and future expected y_t and r_t, the LM equation gives a relation between p_t, m_{t+1} and $E p_{t+1}$, which can be solved recursively

forward to get the price level as a function of current and expected nominal money.

(Can you draw the IS/LM AD/AS relations of the undergraduate textbook? What do you draw them conditional on (i.e which expectations of the future, which policy variables?))

2 Equilibrium with nominal rigidities

Now introduce nominal rigidities. Assume prices are chosen at the beginning of each period, before the realization of money and productivity. What is changed? Only the price equation:

The individual price setting equation becomes:

$$\frac{P_{it}}{P_t} = \frac{\sigma}{\sigma - 1} \frac{E[Q'(N_{it}) C_t Z_t^{-1} | \Omega_{t-1}]}{E[U'(C_{it}) C_t | \Omega_{t-1}]}$$

Note the set on which we condition expectations. At the time the price decisions are taken, aggregate consumption, individual consumption, individual output, are not known.

In general equilibrium, the relative price must be equal to one, and $Y_{it} = C_{it} = Y_t = C_t$, and $Y_t = Z_t N_t$ so:

$$1 = \frac{\sigma}{\sigma - 1} \frac{E[Q'(N_t) N_t | \Omega_{t-1}]}{E[U'(Z_t N_t) N_t Z_t | \Omega_{t-1}]}$$

This determines the expected level of employment (output), and by implication, the price level, call it $\bar{P}_t$, which supports this allocation. (Not so easy to actually characterize this equilibrium price level here. The price level does not just depend on the distribution of M_{t+1} as before, but also on the distribution of M_{t+2} etc.)

2.1 The implied IS-LM-AS model

Collecting equations once more:

$$IS: \qquad U'(Y_t) = E[\ \beta(1 + r_{t+1})U'(Y_{t+1}) \mid \Omega_t]$$

$$LM: \qquad V'(\frac{M_{t+1}}{P_t})/U'(Y_t) = \frac{i_{t+1}}{1 + i_{t+1}}$$

$$AS \qquad \bar{P}_t \mid 1 = \frac{\sigma}{\sigma - 1}\frac{E[Q'(N_t)N_t|\Omega_{t-1}]}{E[U'(Z_tN_t)Z_tN_t|\Omega_{t-1}]}$$

Now that the price level is predetermined, the causality runs as follows. Given the price level, changes in money affect the nominal interest rate. Changes in the nominal interest rate leads, given expectations of inflation, to changes in the real interest rate, which given expectations of future output, lead to changes in demand and thus in output today.

This is again clearest when we use the log linearization:

$$IS: \quad y_t = -a(i_{t+1} - Ep_{t+1} + p_t) + Ey_{t+1}$$

$$LM: \quad m_{t+1} - p_t = by_t - ci_{t+1}$$

$$AS: \quad p_t = \bar{p}_t \mid Ey_t = dEz_t$$

(Draw the IS/LM and AS/AD again. How do they look? What do you draw them conditional on?)

Consider a simple exercise: The effects of unexpected permanent technological shock.

Suppose that at time t, prices are set based on the assumptions that money m_t will be always equal to m, and productivity z_t will always be equal to 0 (an innocuous normalization). Suppose that at time t, z_t unexpectedly increases to $z > 0$ and is expected to remain at z forever. Assume that expectations of money are not affected, so remain equal to m.

What happens at time t clearly now depends on what is expected to happen in the future. From $t + 1$ on, expectations are given by (make sure you understand why):

$$Ey_{t+i} = Ez, \; Ei_{t+1+1} = 0, \; Ep_{t+i} = m - bdz$$

Today's price level, p_t, set before the increase in productivity is equal to m. So the IS-LM gives:

$$IS: \quad y_t = -a(i_{t+1} + bdz) + dz$$

$$LM: \quad 0 = by_t - ci_{t+1}$$

So:

$$y_t = (\frac{1 - ab}{1 + (ab/c)}) \; dz$$

So output is likely to go up today (if $ab < 1$) but less than it would under flexible prices—as the fraction is less than one. Can you explain both aspects of the result?

Turn to optimal monetary policy. Suppose the central bank wants to achieve $y - \hat{y} = 0$.

- Suppose it can react after having observed the realization of z. What is the optimal money rule?

- Suppose it has no informational advantage over price setters and so cannot react to the realization of z this period, but can adjust money next period in response to z this period? What is then the optimal money rule? Can it achieve $y = \hat{y}$?