

The New Keynesian Model.

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The purpose of this note is to put together a model in which money matters in the short run, in which the economy returns to the natural rate in the medium run, and in which we can think about the relation between nominal variables, in particular inflation, and real variables, in particular output.

The model has: Consumers who decide about how much to spend versus how much to save, how much of their wealth to keep in the form of money versus bonds, how much labor to supply; firms who set prices and decide how much to produce, and price decisions which are staggered over time.

The resulting model—and its associated loglinear approximation—is called the **New Keynesian Model**. While it falls short of what one would want to incorporate in a number of important dimensions, it is currently seen as the minimal model with which to think about fluctuations, the effects of shocks, and the role of policy.

The organization of the note is slightly different from previous notes. I start with two “ad-hoc” models of price staggering, the first one from Taylor, the second from Calvo. These models show most clearly the potential implications of price staggering. Then, I introduce price staggering in the model of monopolistic competition developed in topic 8, and derive the New Keynesian model. I end with a discussion of results.

1 Price staggering

The model of price setting introduced in topic 8 assumed that all prices were set simultaneously for one period. This is not particularly realistic, and we do not observe such coordination of price decisions. In most cases, staggering seems more appropriate.

What can price staggering deliver? Intuitively, if each price setter does not want to change his relative price very much, then, when it is his turn to change the price, he will change it only by a small amount. By symmetry, so will everybody else, when their turn comes. The price level will adjust slowly. The Taylor and Calvo models formalize this intuition in convenient and interesting ways.

1.1 Assumptions

Consider an economy composed of n producers/price setters, each selling a differentiated good. At any given time, the desired price of price setter i is given by:

$$p_i = p + ay - u$$

where p_i and p are the logs of the nominal price of producer i and of the price level respectively, y is the log of aggregate output, and u is a shock. The desired relative price increases with marginal cost, which is itself an increasing function of output y and of the shock u . All variables are measured as log deviations from the non stochastic stationary state.

(This specification is consistent with the derivation of the relative price chosen by producer i in the yeoman-farmer model of topic 7. In that model, the log linearization gave:

$$p_i = p + \left[\frac{1}{1 + \sigma(\beta - 1)} \right] [(\beta - 1)y - \beta z]$$

So, under that interpretation, the parameter a corresponds to $(\beta - 1)/(1 + \sigma(\beta - 1))$. a increases with β , and is positive if β is greater than one. In that interpretation also, the shock u corresponds to $-\beta(1 + \sigma(\beta - 1))z$ and is a technological shock.)

Under flexible prices, the equilibrium level of output (the second best level of output, as the economy still suffers from the distortion coming from the markup, itself due to the monopoly power of the firms), is given by:

$$\hat{y} = \frac{1}{a}u$$

so we can rewrite the price setting equation as:

$$p_i = p + a(y - \hat{y}) = p + ax$$

where x is defined as the output gap, the difference between actual output

and equilibrium (second best) output. (Note that, *given the output gap*, technological shocks or energy price changes do not appear anymore directly in the equation.)

1.2 The Taylor version

The Taylor formalization of price setting is based on two assumptions:

- Each period, $1/n$ of the price setters set their price for the current and the next $n - 1$ periods.
- Their price is fixed between readjustments.

This leads to the following two equations:

$$q_t = \frac{1}{n} \sum_{i=0}^{n-1} E p_{t+i}^*$$

$$p_t = \frac{1}{n} \sum_{i=0}^{n-1} q_{t-i}$$

The first equation states that q_t , the (log) price chosen in period t for periods t to $t + n - 1$ is equal to the average of the expected desired price for periods t to $t + n - 1$. Here and below, for any z , $E z_{t+i}$ stands for $E[z_{t+i}|\Omega_t]$. From above, the desired price for each period is given by

$$p_t^* = p_t + a x_t$$

The second equation simply states that the price level is a weighted average of all the prices currently in force.

For the case where $n = 2$, it is easy to derive the behavior of prices. Write:

$$q_t = 0.5 (p_t + E p_{t+1}) + 0.5 a (x_t + E x_{t+1})$$

and

$$p_t = 0.5(q_t + q_{t-1})$$

Replace p_t and Ep_{t+1} from the second equation in the first, and reorganize to get:

$$q_t = 0.5q_{t-1} + 0.5Eq_{t+1} + a(x_t + Ex_{t+1}) \quad (1)$$

or

$$(q_t - q_{t-1}) = (Eq_{t+1} - q_t) + 2a(x_t + Ex_{t+1}) \quad (2)$$

The first equation gives the (new) price as a function of itself lagged, itself expected, and the current and expected output gap. The second equation rewrites the relation as a relation between (new price) inflation, expected (new price) inflation, and the current and expected output gap.

Before discussing the implications of these two equations further, let us look at the Calvo version of price staggering.

1.3 The Calvo version

Assume that instead of being fixed for a fixed length of time, prices, once set, are readjusted with probability δ each period (a ‘‘Poisson’’ assumption which, as if often the case with Poisson, leads to nice aggregation)

In this case, the equations for individual prices and the price level can be written as:

$$q_t = \delta \sum_0^{\infty} (1 - \delta)^i Ep_{t+i}^*$$

$$p_t = \delta \sum_0^{\infty} (1 - \delta)^i q_{t-i}$$

where, as before:

$$p_t^* = p_t + ax_t$$

- The price q_t chosen in period t depends on all future expected desired prices, with weights corresponding to the probability that the price is still in place at each future date.
- The price level is in turn a weighted average of current and past individual prices, still in place today.

To solve, rewrite the two equations in recursive form as:

$$q_t = \delta(p_t + ax_t) + (1 - \delta)Eq_{t+1}$$

$$p_t = \delta q_t + (1 - \delta)p_{t-1}$$

Use the second equation to express q_t as a function of p_t and p_{t-1} and Eq_{t+1} as a function of Ep_{t+1} and p_t , and replace in the first equation to get:

$$p_t - (1 - \delta)p_{t-1} = \delta^2(p_t + ax_t) + (1 - \delta)(Ep_{t+1} - (1 - \delta)p_t)$$

Reorganizing:

$$p_t = \frac{1}{2}p_{t-1} + \frac{1}{2}Ep_{t+1} + \frac{1}{2} \frac{\delta^2}{1 - \delta} ax_t \quad (3)$$

Or, in terms of inflation:

$$(p_t - p_{t-1}) = (Ep_{t+1} - p_t) + \frac{\delta^2}{1 - \delta} a x_t \quad (4)$$

The coefficient on the output gap depends not only on a (the slope of the marginal cost curve), but also on δ , the parameter reflecting the proportion of prices being adjusted each period.

Note two differences with the Taylor formalization: The equations are in terms of the price level p_t rather than in terms of the new price q_t ; and only the current output gap appears, instead of the current and expected output gaps. These two characteristics make the Calvo model slightly more tractable and explain why it has become more widely adopted than the Taylor alternative.

1.4 Implications. Price versus inflation stickiness

Staggering leads to **price stickiness**. This is clear from equations (1) or (3), in which the price level at time t depends on the price level at time $t - 1$, and the expected price level at time $t + 1$, with equal weights.

To see this more clearly, take equation (3) and close the model by assuming $u_t = 0$ so $\hat{y}_t = 0$, and $y_t = m_t - p_t$, so

$$x_t = y_t - \hat{y}_t = m_t - p_t$$

In this case, the price level is given by:

$$p_t = bp_{t-1} + bEp_{t+1} + (1 - 2b)m_t$$

where $b \equiv (1 - \delta)/(2(1 - \delta) + (1 - \delta)^2 a)$

Solving this equation (say, by factorization) gives:

$$p_t = \lambda p_{t-1} + (1 - \lambda)^2 \sum \lambda^i E m_{t+i}$$

where $\lambda \equiv (1 - \sqrt{1 - b^2})/b \leq 1$

- Note that the closer a is to 0, the closer b is to $1/2$, and the closer λ is to 1: The flatter the marginal cost, the more price stickiness.
- Note also that the closer δ is to 0, the closer b is to $1/2$, and the closer λ is to 1: The less often prices are adjusted, the higher the price stickiness.

Consider the case where m_t follows a random walk with innovation ϵ_t . In this case, the price level follows:

$$p_t = \lambda p_{t-1} + (1 - \lambda)m_t$$

and so output is given by:

$$y_t = \lambda y_{t-1} + \epsilon_t$$

The closer λ is to one, the longer lasting the effects of an unanticipated shock in money on output.

There is however **no stickiness in inflation**. In the Calvo version, inflation is fully forward looking. This is clear from equation (4). In the Taylor model,

this is not exactly true; New price inflation is also fully forward looking, but inflation is a weighted average of current and lagged (new price) inflation.)

In the Calvo formalization, we can solve equation (4) forward to get:

$$\pi_t = \left[\frac{\delta^2}{1 - \delta} a \right] \lim_{k \rightarrow \infty} \sum_0^k E x_{t+i}$$

Inflation depends on current and expected output gaps, not (directly) on the past.

This is a strong result, and a warning that price stickiness does not necessarily imply inflation stickiness (stickiness in a level does not necessarily imply stickiness in a derivative).

It has important implications for monetary policy: Assume $x_t = m_t - p_t$ and suppose that money grows at rate g . Then verify that the solution to equation (4) is that $p_t = m_t$, so the inflation rate is equal to g and the output gap is equal to zero. Suppose that, at time t , the rate of money growth from t on decreases from g to $g' < g$. Then inflation from t on decreases from g to g' , and the output gap remains equal to zero. (Check that this is indeed the solution to equation (4). In words, disinflation is achieved without any output loss.

It raises however two questions:

Is it robust, or is it instead the result of the special assumptions underlying Calvo or Taylor staggering? The answer is that it is very robust. Staggering tends to generate little inflation inertia.

Is it consistent with reality? The evidence is it does not appear to be, that there is inflation inertia, i.e. direct dependence of inflation on past inflation. We shall come back to this later.

The next step is to introduce price staggering a la Calvo in the model we developed earlier.

2 The “New Keynesian” model

Let’s go back to the model developed in topic 8 and extend it to allow for staggered price setting a la Calvo.

2.1 Assumptions

Assume the economy is composed of a continuum of households, indexed by i , who maximize:

$$\max E \left[\sum_0^{\infty} \beta^k (U(C_{it+k}) + V(\frac{M_{it+k+1}}{\bar{P}_{t+k}}) - Q(N_{it+k})) \mid \Omega_t \right]$$

subject to:

$$C_{it} \equiv \left[\int_0^1 C_{ijt}^{\sigma-1/\sigma} dj \right]^{\sigma/(\sigma-1)} \quad \bar{P}_t = \left[\int_0^1 P_{jt}^{1-\sigma} dj \right]^{1/(1-\sigma)}$$

$$\int_0^1 P_{jt} C_{ijt} + M_{it+1} + B_{it+1} = W_t N_{it} + (1 + i_t) B_{it} + M_{it} + \Pi_{it} + X_{it}$$

Consumers derive utility from a consumption basket, real money balances, and leisure. The consumption basket is a CES function of different consumption goods. The price index associated with the consumption basket, the price level, is now denoted by a bar.

Consumers supply labor in a competitive labor market and so receive labor income $W_t N_{it}$. (This is a slightly different formalization than that used in topic 8. The yeoman-farmer approach taken in topic 8, together with price staggering, would lead to heterogeneity of consumers-workers. Here, with a competitive labor market, all consumer workers are the same.) Consumers equally own all the firms producing the individual consumption goods, and receive profits Π_{it} . The budget constraint is otherwise the same as before.

Individual consumption goods are produced by firms, according to a linear production technology:

$$Y_{jt} = N_{jt}Z_t$$

(Note an implication of introducing a competitive labor market. The firms take the wage and therefore marginal cost as constant, whereas before they took into account the marginal disutility of leisure).

Firms set prices a la Calvo, with probability δ of changing the price every period.

2.2 Derivation

The first order conditions for households are straightforward and familiar:

$$C_{ijt} = \left(\frac{P_{jt}}{\bar{P}_t}\right)^{-\sigma} C_{it}$$

$$U'(C_{it}) = E[\beta(1 + r_{t+1})U'(C_{it+1})|\Omega_t]$$

$$V'\left(\frac{M_{it+1}}{\bar{P}_t}\right)/U'(C_{it}) = \frac{i_{t+1}}{1 + i_{t+1}}$$

$$\frac{W_t}{\bar{P}_t} U'(C_{it}) = Q'(N_{it})$$

The new condition is the condition determining price setting for those firms which are allowed to change prices at time t . The price chosen by firm j at time t , P_{jt} is the solution to:

$$\max E\left[\sum_k \beta^k \frac{U'(C_{t+k})}{U'(C_t)} (1 - \delta)^k \left(\frac{P_{jt}}{\bar{P}_{t+k}} Y_{jt+k} - \frac{W_{t+k}}{\bar{P}_{t+k}} \frac{Y_{jt+k}}{Z_{t+k}}\right) \middle| \Omega_t\right]$$

subject to

$$Y_{jt+k} = \left(\frac{P_{jt}}{\bar{P}_{t+k}}\right)^{-\sigma} Y_{t+k}$$

Firms maximize the expected present value of profits they will get at the chosen price P_{jt} . Profit is equal to real revenues minus real costs. Profit at time $t + k$ is discounted at the marginal rate of substitution of consumers (where we can drop the index i as all consumers face the same maximization problem and thus have the same consumption), times the probability that the price chosen at time t is still in effect at time $t + k$, $(1 - \delta)^k$.

Maximization yields the following expression for P_{jt} :

$$P_{jt} = P_t = \frac{\sigma}{\sigma - 1} \frac{E[\sum_k A(k) (W_{t+k}/Z_{t+k})|\Omega_t]}{E[\sum_k A(k)|\Omega_t]}$$

where

$$A(k) \equiv \beta^k \frac{U'(C_{t+k})}{U'(C_t)} (1 - \delta)^k (\bar{P}_{t+k})^{\sigma-1} Y_{t+k}$$

So the price chosen by price setters is a weighted average of current and expected future marginal costs W_{t+k}/Z_{t+k} . This is very much like in Calvo, except for the fact that the weights are much more complex, involving expectations of products of random variables. Note that, in contrast to the Calvo formalization, future periods are also discounted by the discounted factor β^k .

The price level is given in turn by:

$$\bar{P}_t = [(1 - \delta)\bar{P}_{t-1}^{1-\sigma} + \delta P_t^{1-\sigma}]^{1/(1-\sigma)}$$

2.3 General equilibrium

In general equilibrium, $C_{it} = C_t = Y_t$, $N_t = Y_t/Z_t$, $M_{it} = M_t$, so the first order conditions become:

$$\text{IS : } U'(Y_t) = E[\beta(1 + r_{t+1})U'(Y_{t+1})|\Omega_t]$$

$$\text{LM : } V'(\frac{M_{t+1}}{\bar{P}_t})/U'(Y_t) = \frac{i_{t+1}}{1 + i_{t+1}}$$

$$\text{LS : } \frac{W_t}{\bar{P}_t} U'(Y_t) = Q'(Y_t/Z_t)$$

$$\text{PS : } P_t = \frac{\sigma}{\sigma - 1} \frac{E[\sum_k A(k) (W_{t+k}/Z_{t+k})|\Omega_t]}{E[\sum_k A(k)|\Omega_t]}$$

where

$$A(k) \equiv \beta^k \frac{U'(Y_{t+k})}{U'(Y_t)} (1 - \delta)^k (\bar{P}_{t+k})^{\sigma-1} Y_{t+k}$$

and

$$\bar{P}_t = [(1 - \delta)\bar{P}_{t-1}^{1-\sigma} + \delta P_t^{1-\sigma}]^{1/(1-\sigma)}$$

$$\text{PF : } N_t = Y_t/Z_t$$

LS stands for labor supply. PS stands for price setting. PF for production function.

Not a simple system, and we shall need to log linearize to make progress with dynamics. But it is easy to characterize the second best level of output (the level of output which would prevail absent nominal rigidities).

From the labor supply equation:

$$\frac{W_t}{\bar{P}_t} = Q'(Y_t/Z_t)/U'(Y_t)$$

From the price-setting equation, without nominal rigidities:

$$\bar{P}_t = \frac{\sigma}{\sigma - 1} \frac{W_t}{Z_t}$$

or rewriting:

$$\frac{W_t}{\bar{P}_t} = \frac{\sigma - 1}{\sigma} Z_t$$

Price setting determines the real wage paid by firms; the real wage is a decreasing function of the markup of firms, and an increasing function of productivity.

Putting the two together gives:

$$Q'(Y_t/Z_t)/U'(Y_t) = \frac{\sigma - 1}{\sigma} Z_t$$

This determines implicitly output as a function of the technological shock. If we assume that $U(\cdot)$ is log (as is required under separability to get a balanced growth path), then the equation becomes:

$$Q'(Y_t/Z_t) Y_t/Z_t = \frac{\sigma - 1}{\sigma}$$

This determines a unique value of Y_t/Z_t , or equivalently a unique value of N_t . So second best employment is constant, and second best output varies one for one with Z_t .

2.4 Log linearization

Log linearization around the non stochastic steady state gives:

$$\text{IS} : y_t = E y_{t+1} - a r_{t+1}$$

$$\text{LM} : m_{t+1} - \bar{p}_t = b y_t - c i_{t+1}$$

$$\text{LS} : w_t - \bar{p}_t = \gamma n_t + z_t$$

$$\text{PS} : p_t = (1 - \delta)\beta E p_{t+1} + (1 - \beta(1 - \delta))(w_t - z_t)$$

$$\bar{p}_t := (1 - \delta)\bar{p}_{t-1} + \delta p_t$$

$$\text{PF} : n_t = y_t - z_t$$

The last three equations can be combined to give a “Phillips curve relation”.

Use the labor supply relation to eliminate the wage in the price setting equation:

$$p_t = (1 - \delta)\beta E p_{t+1} + (1 - \beta(1 - \delta))(\bar{p}_t + \gamma n_t)$$

Denote the second best log deviations of output and employment by \hat{y}_t and \hat{n}_t . Define the output gap, i.e. the difference between actual and second

best log deviation of output $y_t - \hat{y}_t$ by x_t . From above $\hat{y}_t = z_t$ and $\hat{n}_t = 0$ so

$$x_t = y_t - \hat{y}_t = n_t - \hat{n}_t = n_t$$

Replacing in the previous equation gives:

$$p_t = (1 - \delta)\beta E p_{t+1} + (1 - \beta(1 - \delta))(\bar{p}_t + \gamma x_t)$$

Combining with the equation for the price level and reorganizing gives:

$$\text{PC : } \pi_t = \beta E \pi_{t+1} + \frac{\delta(1 - \beta(1 - \delta))}{1 - \delta} \gamma x_t$$

where $\pi_t \equiv p_t - p_{t-1}$. Inflation depends on itself expected, and on the output gap. If output is above its second best (or “natural”) level, then, given expected inflation, inflation increases. If output is below its natural level, then, given expected inflation, inflation decreases. As in the ad hoc Calvo model, inflation is forward looking, and there is no inflation inertia. Unlike the Calvo model, the coefficient in front of expected inflation is β rather than one. This implies a (small, as β is close to one) long run trade off between inflation and output.

2.5 The New Keynesian model.

Putting the essential equations together one last time:

$$\text{IS : } y_t = E y_{t+1} - a r_{t+1}; \quad r_{t+1} = i_{t+1} - E \pi_{t+1}$$

$$\text{LM : } \quad m_{t+1} - \bar{p}_t = b y_t - c i_{t+1}$$

$$\text{PC : } \pi_t = \beta E \pi_{t+1} + d x_t; \quad x_t = y_t - \hat{y}_t = y_t - z_t$$

These three equations constitute the New Keynesian model. This model has been heavily used, with appropriate extensions, to look at the effects of shocks, and the role of fiscal and monetary policy. The role of monetary policy is the subject of topic 10. The role of fiscal policy the subject of

topic 11. But, before turning to those, note some of the implications of this model (and compare them to the stylized facts described in topic 1)

- Expectations matter very much.
- Demand shocks, i.e. any shock which increases the demand for goods in the first equation, have an effect on output in the short run. Unless they also affect the natural level of output, their effect gradually goes away over time.
- Demand shocks may be shocks to nominal money, which affect the nominal interest rate and in turn the real interest rate. These may be shocks to the discount factor, a preference for the present relative to the future. These may be shocks which affect expectations of future income. These may be shocks coming from fiscal policy, from government spending or changes in taxation.
- In the long run, the economy returns to its natural (equivalently second best) level of output (equivalently, its natural level of employment; equivalently, its natural rate of unemployment). This natural level is not constant however. It varies with productivity shocks, and more generally, with the relative price of the inputs used in production (for example the relative price of oil if production uses oil and labor).
- Productivity shocks may well have effects in the short run, through their effect on demand. Higher expected productivity leads to higher expected income and higher consumption demand today.
- An increase in output above the natural level tends to be associated with higher inflation; a decrease in output below the natural level tends to be associated with lower inflation. The relation between inflation and the output gap is however quite different from the traditional Phillips curve relation.

Should we be satisfied with the model? The answer to any such question must be no. Among the pluses and the minuses, taking its size as given:

- Plus: The derivation of the model from first principles, so we can do welfare analysis right and derive optimal policies. The government is there to maximize expected utility.

- Plus: The model seems to capture in a simple way many of the stylized facts of fluctuations.
- Minus: The lack of inflation inertia. The model appears at odds with the evidence, both from disinflations, and from normal times.
- Minus: The model has a striking implication: There is no trade off between achieving a zero output gap (that is keeping output at the natural level) and stable inflation, even in the presence of productivity or price of oil shocks. This can be seen from equation (PC): If π is constant, then $x = 0$.

The implication is striking and implies for example that monetary policy should aim at keeping constant inflation even in the face of large oil price increases. This feels wrong, and suggests that something important is missing from the model.

(On these last two issues, see my paper with Gali)

There is obviously a long list of elements one would like to add to the model (but would increase its size, and complexity):

- Investment and capital accumulation.
- Government spending and taxes.
- Openness in goods and financial markets
- A non competitive labor market, so we can discuss unemployment rather than just employment or output.
- Nominal wage rigidities in addition to nominal price rigidities.
- Assumptions leading to non Ricardian equivalence, and thus richer implications of fiscal policy, of tax versus deficit finance.
- Heterogeneity, and the implications of incomplete markets.

All these extensions have been and are studied. Introducing them leads to larger models, and reliance on simulations. As it is, the model above is a very useful starting point, with a rich set of results already. Subject of the next notes.