1. Interpreting Cross-Country Correlations

Suppose money demand is given by

\[ m_t^i - p_t^i = y_t^i - \eta r_t^i \]

where \( m_t \) is the (log) money supply, \( p_t \) is the (log) price level, \( y_t \) is (log) output, and \( r_t \) is the nominal interest rate. Superscript \( i \) refers to the country.

a) Show that in a steady state with a constant real interest rate

\[ \Delta m_t^i = \pi_t^i + \Delta y_t^i \]

where \( \pi_t^i = p_t^i - p_{t-1}^i \) and where variables without \( t \) subscript refer to steady state values.

b) Assume that steady state output growth is determined exclusively from non-monetary factors in all countries, and that steady state values are approximated accurately by long-run averages. Show that average output growth must be correlated with either average inflation or average money growth across countries.

c) Determine the sign of those correlations if central banks follow an inflation targeting strategy (determining inflation objective \( \pi_t^i \) exogenously).

d) Determine the sign of those correlations if central banks follow a monetary targeting strategy (specify a money growth target \( \Delta m_t^i \) exogenously).

e) Relate your answers to the empirical evidence in McCandless and Weber (1995).

2. Interpreting Money-Output and Money-Interest Rate Correlations

Consider the following stylized model of output and interest rate determination. Equilibrium in the goods market is represented by the condition:

\[ y_t = d_t - \theta r_t \]

where \( y_t \) is (log) output, \( r_t \) is the interest rate, and \( d_t \) is an exogenous aggregate demand shock, with variance \( \sigma_d^2 \). In addition, equilibrium in the money market implies:

\[ m_t = y_t - \eta r_t + \nu_t \]

where \( m_t \) denotes the (log) money supply, and \( \nu_t \) is an exogenous money demand shock with variance \( \sigma_v^2 \), and orthogonal to \( d_t \). For simplicity we are implicitly assuming that prices are constant.

a) Determine the equilibrium level of output as a function of \( m_t, d_t, \) and \( \nu_t \).

b) Suppose that \( m_t \) follows an exogenous process with variance \( \sigma_m^2 \). Determine the implied correlations between (i) money and output and (ii) money and the interest rate.
c) Suppose that the monetary authority pegs the interest rate at a constant level ($r_t = 0$ for all $t$), with the money supply being entirely endogenous). Determine the implied correlation between money and output, and discuss the factors affecting its size.

d) Suppose that the monetary authority adjusts the money supply in order to stabilize output (say, $y_t = 0$ all $t$) Determine the implied correlation between money and the interest rate, and discuss the factors affecting its size.

e) Discuss the difficulties in testing for the presence of a liquidity effect and/or money non neutralities in light of the previous results.

3. Comovements vs. Structural Relationships

Equilibrium in the goods market takes the form

$$y_t = -\alpha r_t + \varepsilon_t^d$$

where $\varepsilon_t^d$ is an i.i.d. demand shock and $r_t$ is the monetary policy instrument (e.g., the interest rate). Parameter $\alpha \geq 0$ measures the effectiveness of monetary policy.

The interest rate is determined according to the policy rule:

$$r_t = \beta y_t + \varepsilon_t^m$$

where $\varepsilon_t^m$ is an i.i.d. monetary policy shock. Parameter $\beta \geq 0$ measures the degree of activism in trying to stabilize output fluctuations.

Discuss the difficulties in trying to learn about $\alpha$ and $\beta$ using the information contained in the time series for $\{r_t\}$ and $\{y_t\}$ only, and possible ways of overcoming that problem.