Lectures on Monetary Policy, Inflation and the Business Cycle
Chapter 2 Exercises
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1. Marginal Costs and Decreasing Returns

Suppose that technology is given by $Y_t(i) = A_t \; N_t(i)^{1-\alpha}$ for $i \in [0,1]$ and where $0 < \alpha < 1$.

a) Derive a log-linear expression for a firm's marginal cost as a function of its own output, aggregate output and technology.

b) Derive an approximate log-linear expression for the economy's average marginal cost as a function of aggregate output and technology.

c) Discuss the role played by decreasing returns in variations of marginal costs in each case.

2. Introducing Government Purchases in the Simple Framework

Assume that the government purchases quantity $G_t(i)$ of good i, for all $i \in [0,1]$. Let $G_t \equiv \left[\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}} \; di\right]^{\frac{\epsilon}{\epsilon-1}}$ denote an index of public consumption, which the government seeks to maximize for any level of expenditures $\int_0^1 P_t(i) \; G_t(i) \; di$. We assume government expenditures are financed by means of lump-sum taxes.

a) Derive an expression for total demand facing firm i.

b) Derive a log-linear aggregate goods market clearing condition that is valid around a steady state with a constant public consumption share $S_G \equiv \frac{G}{Y}$.

c) Derive the corresponding expression for average real marginal cost as a function of aggregate output, government purchases, and technology. Discuss intuition.

d) How is the equilibrium relationship linking interest rates to current and expected output affected by the presence of government purchases?

3. Optimality Conditions under Alternative Utility Specifications

Derive the log-linearized optimality conditions of the household problem under the following alternative specifications of the period utility function:

a) The King-Plosser-Rebelo specification:

$$U\left(C_t, N_t\right) = \frac{1}{1-\sigma}\left[C_t \; (1-N_t)^{\nu}\right]^{1-\sigma}$$

b) The Chari-Kehoe-McGrattan specification:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{1}{1-\sigma}\left[C_t^{\nu} + \left(\frac{M_t}{P_t}\right)^{\nu}\right]^{\frac{1-\sigma}{\nu}} + \log(1-N_t)$$