**Lectures on Monetary Policy, Inflation and the Business Cycle**
**Chapter 3 Exercises**

Jordi Galí

## 1. Alternative Interest Rules for the Classical Economy

Consider the simple classical economy described in the text, in which the following equilibrium relationships must be satisfied

$$\overline{y}_t = E_t\{\overline{y}_{t+1}\} - \frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}\} - \rho\right)$$

and

$$\begin{aligned} r_t - E_t\{\pi_{t+1}\} &= \overline{rr}_t \\ &= \rho + \sigma E_t\{\Delta \overline{y}_{t+1}\} \end{aligned}$$

and where $\overline{y}_t$ and, as result, $\overline{rr}_t$, evolve according to a stochastic process independent of monetary policy. Next we analyze, in turn, three alternative monetary policy rules and their implications.

When relevant, we assume that the money market clearing condition takes the form

$$m_t - p_t = y_t - \eta\, \widehat{r}_t + \varepsilon_t^m$$

where $\varepsilon_t^m$ is a stochastic money demand disturbance.

a) *Inflation Targeting.*

(i) Derive an interest rate rule that would guarantee full stabilization of inflation, i.e. $\pi_t = \pi^*$ for all $t$ where $\pi^*$ is an inflation target assumed to be "close to" zero (so that the log-linearized equilibrium conditions remain valid).

(ii) Determine the behavior of money growth that is consistent with the strict inflation targeting policy analyzed in (i).

(iii) Explain why a policy characterized by a constant rate of money growth $\Delta m_t = \pi$ will generally not succeed in stabilizing inflation in that economy.

b) *An Interest Rate Peg.*

Derive an interest rate rule that yields a unique equilibrium implying a constant nominal interest rate $r_t = r^*$, all $t$

c) *Price Level Targeting.*

(i) Consider the interest rate rule

$$r_t = \rho + \phi_p \ (p_t - p^*)$$

where $\phi_p > 0$, and $p^*$ is a (constant) target for the (log) price level. Determine the equilibrium behavior of the price level under this rule. (hint: you may find it useful to introduce a new variable $\widehat{p}_t \equiv p_t - p^*$ –the deviation of the price level from target–to ease some of the algebraic manipulations).

(ii) Consider instead the money targeting rule

$$m_t = p^*$$

Determine the equilibrium behavior of the price level under this rule.

(iii) Show that the money targeting rule considered in (ii) can be combined with the money market clearing condition and rewritten as a price-level targeting rule of the form

$$r_t = \rho + \psi \ (p_t - p^*) + u_t$$

where $\psi$ is a coefficient and $u_t$ is a stochastic process to be determined.

(iv) Suppose that the central bank want to minimize the volatility of the price level. Discuss the advantages and disadvantages of the interest rate rule in (i) versus the money targeting rule in (ii) in light of your findings above.

**2. Nonseparable Preferences and Money Superneutrality**

Assume that the representative consumer's period utility is given by:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{1}{1-\sigma}\left[(1-\vartheta)\ C_t^{1-\nu} + \vartheta\ \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1-\sigma}{1-\nu}} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

a) Derive the optimality conditions of the associated consumer's problem.

b) Assume that the representative firm has access to a simple technology $Y_t = N_t$ and that the monetary authority keeps a constant money growth $\gamma_m$. Derive the economy's steady state equilibrium under the assumption of perfect competition.

c) Discuss the effects on inflation and output of a permanent change in the rate of money growth $\gamma_m$, and relate it to the existing evidence.

**3. Optimal Monetary Policy in a Classical Economy**

Consider a version of the classical economy with money in the utility function, where the representative consumer maximizes $E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right)$ subject to the sequence of dynamic budget constraints

$$P_t C_t + M_t + R_t^{-1} B_t \leq M_{t-1} + B_{t-1} + W_t \; N_t - T_t$$

Assume a period utility given by:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \log C_t + \log \frac{M_t}{P_t} - \frac{N_t^{\ 1+\varphi}}{1+\varphi} \tag{1}$$

Suppose there is a representative perfectly competitive firm, producing the single consumption good. The firm has access to the linear production function $Y_t(i) = A_t \; N_t(i)$, where productivity evolves according to:

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a) \; \exp\{\varepsilon_t^a\}$$

with $\{\varepsilon_t^a\}$ is an i.i.d. random process, normally distributed, with mean 0 and variance $\sigma_a^2$.

The money supply varies exogenously according to the process

$$\frac{M_t}{M_{t-1}} = (1+\gamma_m) \; \exp\{\varepsilon_t^m\} \tag{2}$$

where $\{\varepsilon_t^m\}$ is an i.i.d., normally distributed process with mean 0 and variance $\sigma_u^2$ . We assume that $\{\varepsilon_t^m\}$ evolves exogenously, outside the control of the monetary authority (e.g., could reflect shocks in the monetary multiplier that prevent the monetary authority from fully controlling the money supply.). Finally, we assume that all output is consumed, so that in equilibrium $Y_t = C_t$ for all $t$.

a) Derive the optimality conditions for the problem of households and firms.

b) Determine the equilibrium levels of aggregate employment, output, and inflation (Hint: show that a constant velocity $\frac{P_t Y_t}{M_t} = V$ for all $t$ is a solution)

c) Discuss how utility depends on the two parameters describing monetary policy, $\gamma_m$ and $\sigma_u^2$ (recall that the nominal interest rate is constrained to be non-negative, i.e., $R_t \geq 1$ for all $t$). Show that the optimal policy must satisfy the Friedman rule ($R_t = 1$ all $t$) and discuss alternative ways of supporting that rule in equilibrium.