Lectures on Monetary Policy, Inflation and the Business Cycle
Chapter 4 Exercises
Jordi Galí

1. Indexation and the New Keynesian Phillips Curve

Consider the Calvo model of staggered price setting with the following modification: in the periods between price re-optimizations firms adjust mechanically their prices according to some indexation rule. Formally, a firm that re-optimizes its price in period t (an event which occurs with probability $1-\theta$) sets a price P_t^* in that period. In subsequent periods (i.e., until it re-optimizes prices again) its price is adjusted according to one of the following two alternative rules:

Rule #1: full indexation to steady state inflation Π :

$$P_{t+k|t} = P_{t+k-1|t} \ \Pi$$

Rule #2: partial indexation to past inflation (assuming zero inflation in the steady state)

$$P_{t+k|t} = P_{t+k-1|t} \ (\Pi_{t+k-1})^{\omega}$$

for $k = 1, 2, 3, \ldots$and

$$P_{t,t} = P_t^*$$

and where $P_{t+k|t}$ denotes the price effective in period $t+k$ for a firm that last re-optimized its price in period t, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the aggregate gross inflation rate, and $\omega \in [0,1]$ is an exogenous parameter that measures the degree of indexation (notice that when $\omega = 0$ we are back to the standard Calvo model, with the price remaining constant between re-optimization period).

Suppose that all firms have access to the same constant returns to scale technology and faces a demand schedule with a constant price elasticity ϵ.

The objective function for a firm re-optimizing its price in period t (i.e., choosing P_t^*) is given by

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ [P_{t+k|t} \ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})] \right\}$$

subject to a sequence of demand contraints, and the rules of indexation described above. $Y_{t+k|t}$ denotes the output in period $t+k$ of a firm that last re-optimized its price in period t, $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the usual stochastic discount factor for nominal payoffs, Ψ is the cost function, and θ is the probability of not being able to re-optimize the price in any given period. For each indexation rule:

a. Derive the first order condition of the above problem, which determines the optimal price P_t^*.

b. Log-linearize the first-order condition around the corresponding steady state and derive an expression for p_t^* (i.e., the approximate log-linear price setting rule).

c. Using the definition of the price level index $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}}$ derive a log-linear expression for the evolution of inflation π_t as a function of the average price adjustment term $p_t^* - p_{t-1}$.

d. Combine the results of (b) and (c) to derive an inflation equation of the form:

$$\widehat{\pi}_t = \beta\ E_t\{\widehat{\pi}_{t+1}\} + \lambda\ \widehat{mc}_t$$

where $\widehat{\pi}_t \equiv \pi_t - \pi$ in the case of rule #1, and

$$\pi_t = \gamma_b\ \pi_{t-1} + \gamma_f\ E_t\{\pi_{t+1}\} + \lambda\ \widehat{mc}_t$$

in the case of rule #2 .

2. Government Purchases and Sticky Prices

Consider the Calvo staggered price setting model analyzed in class. The consumer's log-linearized Euler equation takes the form:

$$c_t = -\frac{1}{\sigma}\ (\widehat{r}_t - E_t\{\pi_{t+1}\}) + E_t\{c_{t+1}\}$$

where c_t is consumption, r_t is the nominal rate, and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t+1$ (as in class, lower case letters denote the logs of the original variable). The consumer's log-linearized labor supply is given by:

$$w_t - p_t = \sigma\ c_t + \varphi\ n_t$$

where w_t denotes the nominal wage, p_t.is the price level, and n_t is employment.

Firms' technology is given by:

$$y_t = n_t$$

The time between price adjustments is random, which gives rise to an inflation equation:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - \overline{y}_t$ is the output gap.(with $\overline{y}_t$ representing the natural level of output). We assume that in the absence of constraints on price adjustment firms would set a price equal to a constant markup over marginal cost given by μ (in logs).

Suppose that the government spends on goods a fraction τ_t of output, which varies exogenously. Government purchases are financed through lump-sum taxes.(remark: we ignore the possibility of capital accumulation or the existence of an external sector).

a. Derive a log-linear version of the goods market clearing condition, of the form $y_t = c_t + g_t$.

b. Derive an expression for (log) real marginal cost mc_t as a function of y_t and g_t .

c. Determine the behavior of the natural level of output $\overline{y}_t$ as a function of g_t and discuss the mechanism through which a fiscal expansion leads to an increase in output when prices are flexible.

d. Assume that $\{g_t\}$ follows a simple AR(1) process with autoregreesive coefficient $\rho_g \in [0, 1)$. Derive the new IS equation:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t)$$

together for an expression for the natural rate $\overline{rr}_t$ as a function of g_t.

3. A Simple Model with a Taylor Rule

Consider the basic New Keynesian model described by the equilibrium conditions:

Phillips curve:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

IS equation:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t\}$$

Policy rule

$$r_t = \phi_\pi\ \pi_t$$

Natural real rate

$$\overline{rr}_t = \rho_r\ \overline{rr}_{t-1} + \varepsilon_t^r$$

where π_t denotes inflation, $\widetilde{y}_t$ is the output gap, r_t is the nominal rate, and $\overline{rr}_t$ is the natural real rate.

1) Show that the equilibrium behavior of inflation and the output gap is given by

$$\pi_t = \rho_r\ \pi_{t-1} + \psi_\pi\ \varepsilon_t^r$$

$$x_t = \rho_r\ \pi_{t-1} + \psi_x\ \varepsilon_t^r$$

where ψ_π and ψ_x are coefficients to be determined.

2) Discuss the role of ϕ_π and other parameter in determining the volatility of inflation.

3) Notice that the persistence in inflation (as measured by its first-order autocorrelation) is independent of the monetary policy parameter ϕ_π. Can you think of modifications in the basic model that would generate such a dependence?

4. Optimal Price Setting and Equilibrium Dynamics in the Taylor Model

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(i) = A_t \ N_t(i)$$

where A_t represents the level of technology, and $a_t \equiv \log A_t$ evolves exogenously according to some stationary stochastic process.

Each period a fraction $\frac{1}{N}$ of firms reset their prices, which will remain effective for N periods. Hence a firm i setting a new price P_t^* in period t will seek to maximize

$$\sum_{k=0}^{N-1} E_t \left\{ Q_{t,t+k} \ \left(P_t^* \ Y_{t+k}(i) - TC_{t+k}(Y_{t+k}(i)) \right) \right\}$$

subject to

$$Y_{t+k}(i) \leq (P_t^*/P_{t+k})^{-\varepsilon} \ C_{t+k} \equiv Y_{t+k}^d(P_t^*)$$

where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ is the usual stochastic discount factor.

a) Show that P_t^* must satisfy the first order condition:

$$\sum_{k=0}^{N-1} E_t \left\{ Q_{t,t+k} \ Y_{t+k}^d(P_t^*) \ \left[P_t^* - \left(\frac{\varepsilon}{\varepsilon - 1} \right) \ MC_{t+k}^n \right] \right\} = 0$$

b) Derive the following log-linearized optimal price setting rule (around a zero inflation steady state):

$$p_t^* = \mu + \sum_{k=0}^{N-1} \omega_k \ E_t \left\{ mc_{t+k}^n \right\}$$

where $\omega_k \equiv \frac{\beta^k (1-\beta)}{1-\beta^N}$ and $\mu \equiv \log \left(\frac{\varepsilon}{\varepsilon - 1} \right)$. Show that in the limiting case of $\beta = 1$ (no discounting) we can rewrite the above equation as

$$p_t^* = \mu + \frac{1}{N} \sum_{k=0}^{N-1} E_t \left\{ mc_{t+k}^n \right\}$$

Discuss and provide intuition for the difference with the analogous equation for the Calvo model.

c) Recalling the expression for the aggregate price index $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon}\ di\right]^{\frac{1}{1-\epsilon}}$, show that around a zero inflation steady state the (log) price level will satisfy:

$$p_t = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} p_{t-k}^*$$

d) Consider the particular case of $N = 2$ and $\beta = 1$, and assume that the consumer's marginal rate of substitution between labor and consumption is given by $\sigma c_t + \varphi n_t$. Assume also that all output is consumed. Show that in this case we can write:

$$p_t^* = \frac{1}{2}\ p_{t-1}^* + \frac{1}{2}\ E_t\{p_{t+1}^*\} + \delta\ (\widetilde{y}_t + E_t\{\widetilde{y}_{t+1}\})$$

where $\delta \equiv \sigma + \varphi$.

e) Assume that money demand takes the simple form $m_t - p_t = y_t$ and that both m_t and a_t follow (independent) random walks, with innovations ε_t^m and ε_t^a, respectively. Derive a closed-form expression for the output gap, employment, and the price level as a function of the exogenous shocks.

f) Discuss the influence of δ on the persistence of the effects of a monetary shock, and provide some intuition for that result.

5. The Mankiw-Reis Model: Inflation Dynamics under Predetermined Prices

Consider our benchmark framework with monopolistic competition. Suppose that each period a fraction of firms $1-\theta$ gets to choose *a path of future prices* for their respective goods (a "price plan"), while the remaining fraction θ keep their current price plans. We let $\{P_{t,t+k}\}_{k=0}^{\infty}$ denote the price plan chosen by firms that get to revise that plan in period t. Firm's technology is given by $Y_t(i) = \sqrt{A_t}\ N_t(i)$. Consumer's period utility is given assumed to take the form $U(C_t, N_t) = C_t - \frac{N_t^2}{2}$, where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$. The demand for real balances is assumed to be proportional to consumption with a unit velocity, i.e., $\frac{M_t}{P_t} = C_t$. All output is consumed.

a) Let $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$ denote the aggregate price index. Show that, up to a first order approximation, we will have:

$$p_t = (1-\theta) \sum_{j=0}^{\infty} \theta^j\ p_{t-j,t} \tag{1}$$

b) A firm i, revising its price plan in period t will seek to maximize

$$\sum_{k=0}^{\infty} \theta^k\ E_t \left\{ Q_{t,t+k} Y_{t+k}(i) \left(P_{t,t+k} - \frac{W_{t+k}}{\sqrt{A_{t+k}}} \right) \right\}$$

Derive the first order condition associated with that problem, and show that it implies the following approximate log-linear rule for the price plan:

$$p_{t,t+k} = \mu + E_t\{mc^n_{t+k}\} \tag{2}$$

for $k = 0, 1, 2, ...$where $mc^n_t = w_t - \frac{1}{2}a_t$ is the nominal marginal cost.

c) Using the optimality conditions for the consumer's problem, and the labor market clearing condition show that the *natural* level of output satisfies $\overline{y}_t = -\mu + a_t$, and (ii) the (log) real marginal cost (in deviation from its perfect foresight steady state value) equals the output gap, i.e.

$$\widehat{mc}_t = \widetilde{y}_t$$

for all t, where $\widetilde{y}_t \equiv y_t - \overline{y}_t$.

d) Using (1) and (2) show how one can derive the following equation for inflation:

$$\pi_t = \frac{1-\theta}{\theta}\ \widetilde{y}_t + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j\ E_{t-j}\{\Delta\widetilde{y}_t + \pi_t\} \tag{3}$$

e) Suppose that the money supply follows a random walk process $m_t = m_{t-1} + u_t$, where $m_t \equiv \log M_t$ and $\{u_t\}$ is white noise. Determine the dynamic response of output, employment, and inflation to a money supply shock. Compare the implied response to the one we would obtain under the standard new Keynesian Phillips curve, where $\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$. (hint: use the fact that in equilibrium $y_t = m_t - p_t$ to substitute for $\widetilde{y}_t$ in (3), in order to obtain a difference equation for the (log) price level)

f) Suppose that technology is described by the random walk process $a_t = a_{t-1} + \varepsilon_t$, where where $a_t \equiv \log A_t$, and $\{\varepsilon_t\}$ is white noise. Determine the dynamic response of output, the output gap, employment, and inflation to a technology shock. Compare the implied response to the one we would obtain under the standard new Keynesian Phillips curve, where $\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$. (hint: same as above).

6. Wage-Setting, Time-Varying Market Power, and the New Phillips Curve

The technology available to a firm producing a differentiated good is given by

$$Y_t(i) = N_t(i)$$

where

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\eta_t}}\ dj\right]^{\frac{\eta_t}{\eta_t - 1}}$$

and where $N_t(i,j)$ denotes the quantity of type-j labor employed by firm i in period t. We assume that η_t follows an exogenous stochastic process.

Household j is specialized in the supply of type of labor j. Each period the household sets the wage nominal wage for type j labor, $W_t(j)$, in order to maximize

$$U(C_t(j), N_t(j)) = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\varphi}}{1+\varphi}$$

subject to a budget constraint $P_t C_t(j) = W_t(j) N_t(j) + X_t$ (where X represents other terms not affecting the wage setting decision), and a demand schedule for its labor to be derived below.

a) Show that the quantity of type j labor demanded by the typical firm will be given by

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\eta_t} N_t(i)$$

where $W_t \equiv \left[\int_0^1 W_t(j)^{1-\eta_t}\, dj\right]^{\frac{1}{1-\eta_t}}$ is an aggregate wage index.

b) Derive the household's optimal wage setting rule. Show that it has a log-linear representation of the form:

$$w_t(j) = \mu_t^w + p_t + \sigma\ c_t(j) + \varphi\ n_t(j)$$

where $\mu_t^w \equiv \log\left(\frac{\eta_t}{\eta_t - 1}\right)$

c) Derive the associated Phillips curve when firms set prices a la Calvo. (hint: think about the appropriate definition of the natural rate of output to be used).