Lectures on Monetary Policy, Inflation and the Business Cycle
Chapter 5 Exercises
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1. The Inflation Bias and the New Keynesian Phillips Curve

Suppose that inflation π_t is determined by a New Keynesian Phillips Curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t \tag{1}$$

where $\widetilde{y}_t$ is the output gap. We assume that the latter can be chosen by the central bank, through the appropriate choice of instrument settings.

The central bank's objective function is:

$$E_0\{\sum_{t=0}^{\infty} \beta^t [\alpha \; (\widetilde{y}_t - \widetilde{y}^*)^2 + \pi_t^2]\} \tag{2}$$

where $\widetilde{y}^* > 0$ is a (constant) positive target for the output gap.

(a) Suppose that the central bank re-optimizes each period, taking as given private sector's expectations ("time-consistent solution"). Determine the resulting equilibrium levels for inflation and the output gap.

(b) How would that outcome change if the central bank could (credibly) commit to deliver a certain path for the output gap $\{\widetilde{y}_t\}_{t=0}^{\infty}$ ("commitment")

2. Optimal Monetary Policy with Price-Setting in Advance

Consider a version of the classical economy developed in class, where the representative consumer maximizes $E_0 \sum_{t=0}^{\infty} \beta^t \; U\left(C_t, \frac{M_t}{P_t}, N_t\right)$ subject to a sequence of dynamic budget constraints

$$P_t \; C_t + M_t + R_t^{-1} B_t \leq M_{t-1} + B_{t-1} + W_t \; N_t - T_t$$

and with a period utility given by:

$$U(C_t, \frac{M_t}{P_t}, N_t) = \log C_t + \log \frac{M_t}{P_t} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{3}$$

Firms are monopolistically competitive, each producing a differentiated good whose demand is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$. Each firm has access to the linear production function

$$Y_t(i) = A_t \; N_t(i) \tag{4}$$

where productivity evolves according to:

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a) \exp\{\varepsilon_t\}$$

with $\{\varepsilon_t\}$ being an i.i.d., normally distributed process with mean 0 and variance σ_ε^2.

The money supply varies exogenously according to the process

$$\frac{M_t}{M_{t-1}} = (1 + \gamma_m) \exp\{u_t\} \tag{5}$$

where $\{u_t\}$ is an i.i.d., normally distributed process with mean 0 and variance σ_u^2.

Finally, we assume that all output is consumed, so that in equilibrium $Y_t = C_t$ for all t.

a) Derive the optimality conditions for the problem facing the representative consumer.

b) Assume that firms are monopolistically competitive, each producing a differentiated good. Each period, after observing the shocks, firms set the price of their good in order to maximize current profit

$$Y_t(i) \left(P_t(i) - \frac{W_t}{A_t} \right)$$

subject to the demand schedule. Derive the optimality condition associated with the firm's problem.

c) Show that the equilibrium levels of aggregate employment, output, and inflation are given by

$$N_t = \left(1 - \frac{1}{\varepsilon} \right)^{\frac{1}{1+\varphi}} \equiv \Theta$$

$$Y_t = \Theta \ A_t$$

$$\pi_t = (\gamma_m - \gamma_a) + u_t - \varepsilon_t$$

d) Discuss how utility depends on the two parameters describing monetary policy, γ_m and σ_u^2 (recall that the nominal interest rate is constrained to be non-negative, i.e., $r_t \geq 0$ for all t). Show that the optimal policy must satisfy the Friedman rule and discuss alternative ways of supporting that rule in equilibrium.

e) Next let us assume that each period firms have to set the price in advance, i.e., before the realization of the shocks. In that case they will choose a price in order to maximize the discounted profit

$$E_{t-1} \left\{ Q_{t-1,t} \ Y_t(i) \left(P_t(i) - \frac{W_t}{A_t} \right) \right\}$$

subject to the demand schedule $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$, where $Q_{t-1,t} \equiv \beta \frac{C_{t-1}}{C_t} \frac{P_{t-1}}{P_t}$ is the stochastic discount factor. Derive the first order condition of the firm's problem and solve (exactly) for the equilibrium levels of employment, output and real balances.

2

f) Evaluate expected utility at the equilibrium values of output, real balances and employment:

g) Consider the class of money supply rules of the form (5) such that $u_t = \phi_\varepsilon \ \varepsilon_t + \phi_v \ \nu_t$, where $\{\nu_t\}$ is a normally distributed i.i.d. process with zero mean and unit variance, and independent of $\{\varepsilon_t\}$ at all leads and lags. Notice that within that family of rules, monetary policy is fully described by three parameters: γ_m, ϕ_ε, and ϕ_v. Determine the values of those parameters that maximize expected utility, subject to the constraint of a non-negative nominal interest rate. Show that the resulting equilibrium under the optimal policy replicates the flexible price equilibrium analyzed above.

3. Inflation Targeting with and without Noisy Data

Consider a model economy whose output gap and inflation dynamics are described by the system:

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \kappa \ \widetilde{y}_t \tag{6}$$

$$\widetilde{y}_t = -\frac{1}{\sigma} \ (r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t) + E_t\{\widetilde{y}_{t+1}\} \tag{7}$$

where $\overline{rr}_t$ denotes the natural rate of interest. The latter is assumed to follow the exogenous process

$$\overline{rr}_t - \rho = \rho_r \ (\overline{rr}_{t-1} - \rho) + \varepsilon_t$$

where $\{\varepsilon_t\}$ is white noise and $\rho_r \ \in [0, 1)$.

Suppose that inflation is measured with some i.i.d. error ξ_t , i.e., $\pi_t^o = \pi_t + \xi_t$ where π_t^o is measured inflation. Assume that the central bank now follows the rule

$$r_t = \rho + \phi_\pi \ \pi_t^o \tag{8}$$

a) Solve for the equilibrium processes for inflation and the output gap under the rule (8) (hint: to simplify you may want to assume $\rho_r = 0$).

b) Describe the behavior of inflation, the output gap, and the nominal rate when ϕ_π approaches infinity.

c) Determine the size of the inflation coefficient that minimizes the variance of actual inflation.