Lectures on Monetary Policy, Inflation and the Business Cycle
Chapter 6 Exercises
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1. An Optimal Taylor Rule

Consider an economy with Calvo-type staggered price setting whose equilibrium dynamics are described by the system:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{t+1}\} - \rho) + \varepsilon_t$$

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t + u_t$$

where $\{\varepsilon_t\}$ and $\{u_t\}$ represent sequences of *i.i.d.*, mutually uncorrelated, demand and supply disturbances, with variances given by $var(\varepsilon)$ and $var(u)$.

Assume that the monetary authority decides to adopt a simple Taylor rule of the form

$$r_t = \rho + \phi\ \pi_t$$

a) Solve for the equilibrium processes for the output gap and inflation, as a function of the exogenous supply and demand shocks.

b) Determine the value of the inflation coefficient ϕ which minimizes the central bank's loss function:

$$\alpha\ var(\widetilde{y}_t) + var(\pi_t)$$

c) Discuss and provide intuition for the dependence of the optimal inflation coefficient on α and the variance ratio $\frac{var(\varepsilon)}{var(u)}$. What assumptions on parameter values would warrant an extremely aggressive response to inflation ($\phi \to +\infty$)? Explain.

2. Monetary Policy, Optimal Steady State Inflation and the Zero Lower Bound (40 points)

Consider the equilibrium conditions of a standard new Keynesian model:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\} - \rho) + e_t$$

and

$$\pi_t - \pi = \beta\ E_t\{(\pi_{t+1} - \pi)\} + \kappa\ \widetilde{y}_t + u_t$$

where $\widetilde{y}_t$ is the output gap, π_t denotes inflation, r_t is the nominal rate, and π is steady state inflation. The disturbances e_t and u_t represent demand and cost-push shocks, and are assumed to follow independent and serially uncorrelated normal distributions with zero mean and variances σ_e^2 and σ_u^2 respectively.

Assume that the loss function for the monetary authority is given by

$$\theta \ \pi + E_0 \sum_{t=0}^{\infty} \beta^t \ \left[\alpha \ \widetilde{y}_t^2 + (\pi_t - \pi)^2\right]$$

where the first term is assumed to capture the costs of steady state inflation.

(a) Derive the optimal policy under discretion (i.e., the time-consistent policy, resulting from period-by-period maximization) –including the choice of steady state inflation π–, subject to the constraint that the interest rate hits the zero-bound constraint with only with a 5 percent probability.

(b) Derive an interest rate rule that would implement the optimal allocation derived in (a) as the unique equilibrium.