

Lectures on Monetary Policy, Inflation and the Business Cycle
Chapter 7-9 Exercises

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1. Optimal Monetary Policy in a Sticky Wage Economy

The representative firm is perfectly competitive and has access to a technology:

$$y_t = a_t + n_t$$

where y , n , a denote the logs of output, employment, and productivity, respectively. Prices are flexible. We assume

$$a_t = \rho a_{t-1} + \varepsilon_t$$

The labor supply satisfies:

$$w_t - p_t = \delta n_t$$

where w and p denote the log of the wage and price levels, respectively.

(a) Derive the equilibrium behavior of employment and output under the assumption of flexible wages and prices.

(b) Next we introduce sticky wages. Each period half the workers set the (log) nominal wage, which remains constant for two periods, according to:

$$w_t^* = \frac{1}{2} (p_t + E_t\{p_{t+1}\}) + \frac{\delta}{2} (n_t + E_t\{n_{t+1}\})$$

The average effective (log) wage paid by the firm in period t is thus

$$w_t = \frac{1}{2} (w_t^* + w_{t-1}^*)$$

Show that inflation evolves according to:

$$\pi_t = E_t\{\pi_{t+1}\} + \delta \tilde{n}_t + u_t$$

where $\tilde{n}_t \equiv n_{t-1} + E_{t-1}\{n_t\} + n_t + E_t\{n_{t+1}\}$ and $u_t \equiv -4a_t - (p_t - E_{t-1}\{p_t\})$.

(c) Suppose that aggregate demand is given by the Is equation:

$$y_t = -\frac{1}{\sigma} (r_t - E_t\{\pi_{t+1}\}) + E_t\{y_{t+1}\}$$

and assume that the optimal policy requires that the flexible wage allocation be replicated. Describe the equilibrium behavior of the interest rate, wage inflation, and price inflation under the optimal policy.

(d) Discuss your findings in light of the existing literature.

2. Labor Market Institutions as a source of Long Run Money Non-Neutrality

A perfectly competitive representative firm maximizes profits each period

$$P_t Y_t - W_t N_t$$

subject to a technology $Y_t = N_t^{1-\alpha}$. Assume that the desired labor supply is inelastic and equal to one. Equilibrium in the goods market is given by

$$Y_t = \frac{M_t}{P_t}$$

with the nominal money supply following an AR(1) process (in logs):

$$m_t = \rho_m m_{t-1} + \varepsilon_t$$

Derive the equilibrium process for (the log) of output y_t , employment n_t , prices p_t , and real wages $w_t - p_t$ under each of the alternative assumptions on the wage setting process:

- a) nominal wages are fully flexible and determined competitively.
- b) nominal wages are set in advance, so that the labor market clears in expectation (i.e., $E_{t-1}\{n_t\} = 0$).
- c) nominal wages are set in advance by a union, so that in expectation only currently employed workers are employed (i.e., $E_{t-1}\{n_t\} = n_{t-1}$)
- d) Discuss the empirical relevance of the three scenarios in light of their implied properties (comovements, persistence) for real wages, employment and output.

3. Monetary Policy and Real Wage Rigidities

Assume that the representative household's utility is given $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ with $U(C_t, N_t) = C_t - \frac{1}{2} N_t^2$, where C_t denotes consumption and N_t denotes hours worked. Let firms' technology be given by the production function $Y_t = A_t N_t$, where Y_t denotes output and A_t is an exogenous technology parameter. All output is consumed.

Firms set prices in a staggered fashion à la Calvo, which results in the inflation dynamics equation:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t$$

where $\widehat{mc}_t \equiv mc_t - mc$ represent the log deviations of real marginal cost from its level in the zero inflation steady state.

- (a). Derive an expression for the (log) of the *efficient* level of employment (which we will denote by n_t^*) as a function of (log) productivity a_t (i.e., the level of employment that a benevolent social planner would choose, given preferences and constraints).

(b). Assume that the (log) nominal wage w_t is set each period according to the schedule $w_t = p_t + \frac{1}{1+\delta}n_t$, where $\delta > 0$ (the same assumption is maintained for parts (c), (d) and (e) below). Compare the behavior of the equilibrium real wage under that schedule with the one that would be observed under competitive labor markets. In what sense the condition $\delta > 0$ can be interpreted as a “real rigidity”?

(c) Derive the implied (log) *natural* level of employment (denoted by \bar{n}_t), defined as the equilibrium level of employment under flexible prices (when all firms keep a constant (log) markup μ).

(d) Derive an expression for the real marginal cost \widehat{mc}_t as a function of the employment gap $\tilde{n}_t \equiv n_t - \bar{n}_t$.

(e) Derive the inflation equation in terms of the welfare-relevant employment gap $n_t - n_t^*$. Show how the presence of real wage rigidities ($\delta > 0$) generates a trade-off between stabilization of inflation and stabilization of the welfare-relevant employment gap.

(f) Suppose that the monetary authority has a loss function given by $E_0 \sum_{t=0}^{\infty} \beta^t [\alpha(n_t - n_t^*)^2 + \pi_t^2]$. Solve for the equilibrium process for inflation and output under the optimal monetary policy under discretion (time consistent solution), under the assumption of an i.i.d. technology process a_t . Explain the difference with the case of perfect competition in the labor market. (note: for simplicity you can assume that the frictionless markup μ is infinitesimally small when answering this question).

4. A Small Open Economy Model

Consider a small open economy where trade is always balanced (e.g., no international trade in assets is allowed). Hence,

$$p_t + c_t = p_{H,t} + y_t$$

where c_t denotes consumption, y_t is output, $p_{H,t}$ is the domestic price level, p_t is the CPI (all in logs). Assuming a constant price level in the rest of the world ($p_t^* = 0$), we can write

$$p_t = (1 - \alpha) p_{H,t} + \alpha e_t$$

and e_t is the nominal exchange rate

Let $s_t \equiv e_t - p_{H,t}$ denote the terms of trade. Under the assumption of a unit elasticity of substitution between foreign and domestic goods we have

$$s_t = y_t - y_t^*$$

where y_t^* is (log) output in the rest of the world (assumed to evolve exogenously). The domestic aggregate technology can be written as

$$y_t = a_t + n_t$$

where a_t is an exogenous technology process. We assume perfect competition in both goods and labor markets, with flexible prices and wages. The labor supply takes the form:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Finally, we assume a money demand function $m_t - p_t = c_t$

- a) Determine the equilibrium processes for output, consumption, the terms of trade, and the nominal exchange rate in the small open economy, as a function of productivity a_t , foreign output y_t^* , and the money supply under the assumption that the latter evolves exogenously. Discuss the implications of assuming $\sigma = 1$.
- b) How would your answer have to be modified if a fixed nominal exchange rate regime was in place?
- c) Discuss, in words, how some of the results in a) and b) would change qualitatively in the presence of imperfect competition and sticky prices.