From draft of Chapter 2 in "Lectures on monetary Policy..." by Jordi Galí.

2 Introducing Money (*)

The model developed above abstracts from the existence of an asset we can refer to as *money*, and which can be used as a means of payment or as a store of value. Instead money in the model above serves only a role of numéraire, i.e. of unit of account in which prices, wages and securities' payoffs are stated. While in much of the analysis in subsequent chapters we will keep the assumption of a *cashless economy*, it is useful to understand how the basic framework can incorporate other roles for money and, in particular, how it can generate a demand for money holdings. The discussion in the following section focuses on models that achieve the previous objective by assuming that real balances are an argument the utility function. In subsequent sections we discuss justifications for that assumption, as well as alternative models.

2.1 Money in the Utility Function

The introduction of money in the utility function requires that we modify the househols problem described above in two ways. problem facing. First, preferences are given now by a discounted sum of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

where M_t denotes monetary holdings in period t and P_t is the price index defined above.

The flow budget constraint now takes the form (once optimal allocation of expenditures is accounted for):

$$P_t \, C_t + R_t^{-1} B_t + M_t \leq B_{t-1} + M_{t-1} + W_t \, N_t - T_t$$

Letting $\mathcal{A}_t \equiv B_{t-1} + M_{t-1}$ denote total financial wealth at the beginning of the period t (i.e. before consumption and portfolio decisions are made), we can rewrite the previous flow budget constraint as:

$$P_t \, C_t + R_t^{-1} \mathcal{A}_{t+1} + (1 - R_t^{-1}) \, M_t \leq \mathcal{A}_t + W_t \, N_t - T_t$$

while the corresponding solvency constraint takes the form:

$$\lim_{T \to \infty} E_t \left(\prod_{j=t}^{T} R_j^{-1} \right) \mathcal{A}_{T+1} \geq 0$$

The previous representation of the budget constraint can be thought of as equivalent to that of an economy in which all financial assets (represented by $\mathcal{A}_t$) yield a nominal return R_t , and where agents can purchase the utility-yielding "services" of money balances at an implicit price $(1 - R_t^{-1})$. The latter, which is increasing in R_t should be thought of as the opportunity cost of holding one's financial wealth in terms of monetary assets. *

Two of the optimality conditions of the household problem are the same as those obtained for the cashless model, i.e. (10) and (11), with the marginal utility terms being defined on the augmented set of arguments.$(C_t, \frac{M_t}{P_t}, N_t)$. In addition, we will have the following optimality condition:

$$\frac{U_{m,t}}{U_{c,t}} = 1 - R_t^{-1} \tag{22}$$

where $U_{m,t} \equiv \frac{\partial U\left(C_t, \frac{M_t}{P_t}, N_t\right)}{\partial(M_t/P_t)} > 0$.

If we specify the utility function to have the functional form

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

the above optimality condition takes tes the form:

$$\frac{M_t}{P_t} = \left(\frac{C_t^\sigma}{1 - R_t^{-1}}\right)^{\frac{1}{\nu}} \tag{23}$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and inversely related to the nominal interest rate, as in conventional specifications.

We can easily derive the following log-linear approximation around a steady state, up to an uninteresting constant (see Appendix 1):

$$m_t - p_t = \frac{\sigma}{\nu}\, c_t - \eta\, \widehat{r}_t \tag{24}$$

where $\eta \equiv \frac{1}{\nu(R-1)}$ is the implied interest semi-elasticity of money demand.

In many applications it is often assumed $\nu = \sigma$, thus implying a unit elasticity with respect to consumption. Under that assumption, in the basic model without capital accumulation or government purchases, we obtain a conventional linear demand for real balances which we will just assume throughout much of the discussion below.

$$m_t - p_t = y_t - \eta\, \widehat{r}_t$$