Problem Set 1: Exercises on Blanchard-Kahn (1980)

PROBLEM 1: Theory

[14.452, Spring 2004]

We consider a variable x_t which satisfies the rational expectations equation:

$$x_t = \alpha x_{t-1} + \beta E_t x_{t+1} + \varepsilon_t \qquad (1)$$

where ε_t is white noise and $\alpha\beta \neq 1/4$. We want to find a solution to this equation.

(1). Assume that the solution is of the following form

$$x_t = \sum_{i=0}^{\infty} a_i \varepsilon_{t-i}$$

Show that the sequence of coefficients must satisfy

$$a_0 = \beta a_1 + 1 \qquad (2)$$

$$a_i = \alpha a_{i-1} + \beta a_{i+1} \qquad (3)$$

(2). Show that the solutions to (3) in C form a two-dimensional vector space, which can be parameterised by

$$a_i = A_1 \lambda_1^{\,i} + A_2 \lambda_2^{\,i}$$

where A_1, A_2 are two arbitrary complex numbers, and (λ_1, λ_2) are the solution to

$$\beta\lambda^2 - \lambda + \alpha = 0$$

(3). What conditions must the roots satisfy to have a *unique* solution to (3) such that
(i) (2) is satisfied.
(ii) the implied time series x_t is stationary?

(4). Let $X_t = \begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix}$

Let t be an arbitrary date and define $Y^{(s)} = E_t X_{t+s}$ for any $s \geq 0$. Show that for $s \geq 1$,

$$Y^{(s+1)} = MY^{(s)} \qquad (2)$$

where M is a 2×2 matrix. What are the coefficients of M? What is the characteristic equation which can be used to compute its eigenvalues? Show that M can be written $M = P^{-1}DP$ such that P and D are matrices with complex coefficients and D is diagonal, and can be picked up such that D_{11} is the eigenvalue with the highest modulus.

(5). Given an initial value x_{t-1}, and a value of the shock ε_t, under what conditions on D is there a unique value of x_t, such that the corresponding sequence of vectors $\{Y^{(1)}, Y^{(2)}, \ldots\}$ is bounded? Show that if these conditions are satisfied, then the first coefficient of $Y^{(1)}$, which is also $E_t x_{t+1}$, must be such that

$$\left(PY^{(1)}\right)_1 = 0 \tag{3}$$

(6). How would you go about extending the method if:
(i) the shocks ε_t were correlated
(ii) a stationary, exogenous random variable z_t appeared on the RHS of (1)?

PROBLEM 2: Matlab Simulations

Look at the Matlab code blanchardkahneconomy.m posted on the website. Check that you understand what the commands are doing in the code.

(1). Keep the initial value of p_0 fixed. Change p_1 from 0 to the following values: (-0.5, 0.5, 0.001) and verify that p_t does NOT converge to 0. Explain the mechanisms driving the variables in the transition (Note: The exchange rate is fixed).

(2). Now change p_0. Verify that the code does not have p_t converging even when it should. Why? In this case the problem is relatively clear. Look out for this problem in the future when you are writing code.

(3). Write the flexible exchange rate model on the recitation handout in matrix form ready to apply the Blanchard-Kahn (1980) method.
(You need not write any code in this section. If you wish you may do so.)

Further elaborations on Matlab code are to be found at:
Harald Uhlig's toolkit: http://www.wiwi.hu-berlin.de/wpol/html/toolkit.htm
Paul Klein's webpage: http://www.ssc.uwo.ca/economics/faculty/klein/
(some of this code is written with/by Chris Sims)
Schmitt-Grohe and Uribe write code for higher order approximations (higher than first order).