Problem Set 3: Time and State Dependent Pricing

For this problem set I will ask you to refer to the exercises posted on the course website at:
http://web.mit.edu/14.461/www/part1/lecturesandex.htm

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Answer ALL of Problems 1, 2 and 3
and EITHER Problem 4 OR Problem 5

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The problem set is due on: Friday, October 14, 2005.

Problem 1: Indexation and the New Keynesian Phillips Curve

Section 4 exercises, Question 1.

Problem 2: Government Purchases and Sticky Prices

Section 4 exercises, Question 2.

Problem 3: A Simple Model with a Taylor Rule

Section 4 exercises, Question 3.

Problem 4: Optimal Price Setting and Equilibrium Dynamics in the Taylor Model

Section 4 exercises, Question 4.

Problem 5: State Dependent Pricing Formulation by Dotsey, King and Wolman (1999) and Dotsey and King (2005)

The aim of this question is to help build your intuition regarding state dependent models. It may be helpful to think about how this formulation relates to Gertler and Leahy (2005).

The setup is as follows. Within each period, some firms will adjust their price and some will not. In period t there is a distribution of firms summarised by $\{\theta_{jt}\}_{j=1}^{J}$, where fraction θ_{jt} of firms at time t last adjusted their prices j periods ago. In the absence of adjustment these firms charge P_{t-j}^* in period t. All adjusting firms will select P_t^*. In the model J and $\{\theta_{jt}\}_{j=1}^{J}$ are determined endogenously.

In period t a fraction α_{jt} of vintage j firms decide to adjust their price, so the total fraction of adjusting firms ω_{0t} satisfies:

$$\omega_{0t} = \sum_{j=1}^{J} \alpha_{jt}\theta_{jt} \tag{1}$$

The fraction of firms remaining with a price set at $t-j$ is $\omega_{jt} = (1-\alpha_{jt})\theta_{jt}$.

If the α_j are fixed through time, then the Calvo (1983) and Taylor (1980) models are included as special cases. Here α_{jt} will summarize an optimizing decision. In particular in every period t the firm draws a fixed cost ξ_t from a distribution $G(x)$ for $x \in (0, B)$, $B < \infty$. It must pay this fixed cost in labour if it wishes to adjust its price. $\xi_t \sim$ i.i.d. across firms and periods. The firm will decide whether to adjust its price after observing the fixed cost.

(a) In period t each j-type firm (a firm that last changed its price j periods ago) has a value function of the form:

$$v(p_t, \xi_t, s_t) = \max\{v_{jt}, v_{0t}\} \tag{2}$$

$$\text{where } v_{jt} = z(p_{jt}, s_t) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} v\left(p_{j+1,t}\frac{P_t}{P_{t+1}}, \xi_{t+1}, s_{t+1}\right) \tag{3}$$

$$v_{0t} = \max_{p_t^*}\left[z(p_t^*) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} v\left(p_t^*\frac{P_t}{P_{t+1}}, \xi_{t+1}, s_{t+1}\right)\right] - w_t\xi_t \tag{4}$$

where $p_{j,t}$ $\left(=\frac{P^*_{t-j}}{P_t}\right)$ is the relative price of a firm who last changed its price j periods ago, s_t is a state vector that governs the evolution of the firm's demand, $\frac{\lambda_{t+1}}{\lambda_t}$ is the ratio of marginal utility in the future to marginal utility today, and $z(p_{jt}, s_t) = \left[p_{jt} - \psi_{jt}\right] c_{jt}$ denotes real profits.

Interpret the value function formulation above. Why does this expression summarise the firm's decisions?

The value function is written in relative prices. Does this make the model a real rigidity model?

(b) Derive the FOC determining the optimal value of p_t^*.

Show that there is a cutoff level $\bar{\xi}(j)$ for the fixed cost such that all j-type firms will adjust their price in period t if and only if the value of ξ_t drawn is less than $\bar{\xi}(j)$.

Hint: $\xi_t \sim$ i.i.d. across periods.

(c) The Calvo (1983) model has been criticised by Wolman (1999) because it implies that there will always be some firms in the economy who have not changed their prices for a VERY long time. Is this true with the state dependent pricing model above? What condition is required to ensure that J is finite?

Hint: Consider sustained inflation in the general price level.

(d) The Euler equation derived in part (b) can be iterated forward to relate the optimal relative price p_t^* to current and expected future variables.

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \left\{(\omega_{j,t+j}/\omega_{0,t}) \cdot (\lambda_{t+j}/\lambda_t) \cdot \psi_{j,t+j} \cdot c_{t+j}\right\}}{\sum_{j=0}^{J-1} \beta^j E_t \left\{(\omega_{j,t+j}/\omega_{0,t}) \cdot (\lambda_{t+j}/\lambda_t) \cdot (P_{t+j}/P_t) \cdot c_{t+j}\right\}} \tag{5}$$

where $(\omega_{j,t+j}/\omega_{0,t}) = (1-\alpha_{j,t+j}) \cdot (1-\alpha_{j-1,t+j-1}) \cdots (1-\alpha_{1,t+1})$ is the probability of non-adjustment from t through $t+j$.

Note that I have assumed constant demand elasticity for all firms at all levels of output.

Show that according to (5), the optimal relative price is a fixed markup over real marginal cost $\left(p^* = \frac{\varepsilon}{\varepsilon-1}\psi\right)$ if real marginal cost and the price level are expected to be constant over time.

If these are not constant over time, we must weight the real marginal costs appropriately. Explain why real marginal costs are weighted according to the probability of non-adjustment.