Problem Set 3: Time and State Dependent Pricing

For this problem set I will ask you to refer to the exercises posted on the course website at:

Answer ALL of Problems 1, 2 and 3
and EITHER Problem 4 OR Problem 5

The problem set is due on: Friday, October 14, 2005.

Problem 1: Indexation and the New Keynesian Phillips Curve
Section 4 exercises, Question 1.

Problem 2: Government Purchases and Sticky Prices
Section 4 exercises, Question 2.

Problem 3: A Simple Model with a Taylor Rule
Section 4 exercises, Question 3.

Problem 4: Optimal Price Setting and Equilibrium Dynamics in the Taylor Model
Section 4 exercises, Question 4.

Problem 5: State Dependent Pricing Formulation by Dotsey, King and Wolman (1999) and
Dotsey and King (2005)

The aim of this question is to help build your intuition regarding state dependent models. It may be helpful
to think about how this formulation relates to Gertler and Leahy (2005).

The setup is as follows. Within each period, some firms will adjust their price and some will not. In
period \( t \) there is a distribution of firms summarised by \( \{ \theta_{jt} \}_{j=1}^J \), where fraction \( \theta_{jt} \) of firms at time \( t \) last
adjusted their prices \( j \) periods ago. In the absence of adjustment these firms charge \( P_t \).
All
adjusting firms will select \( P_t^* \). In the model \( J \) and \( \{ \theta_{jt} \}_{j=1}^J \) are determined endogenously.

In period \( t \) a fraction \( \alpha_{jt} \) of vintage \( j \) firms decide to adjust their price, so the total fraction of adjusting
firms \( \omega_{0t} \) satisfies:

\[
\omega_{0t} = \sum_{j=1}^{J} \alpha_{jt} \theta_{jt}
\]  

(1)

The fraction of firms remaining with a price set at \( t - j \) is \( \omega_{jt} = (1 - \alpha_{jt}) \theta_{jt} \).

If the \( \alpha_{t} \) are fixed through time, then the Calvo (1983) and Taylor (1980) models are included as special
cases. Here \( \alpha_{jt} \) will summarize an optimizing decision. In particular in every period \( t \) the firm draws a fixed
cost \( \xi_t \) from a distribution \( G(x) \) for \( x \in (0, B) \), \( B < \infty \). It must pay this fixed cost in labour if it wishes to
adjust its price. \( \xi_t \sim i.i.d. \) across firms and periods. The firm will decide whether to adjust its price after
observing the fixed cost.

(a) In period \( t \) each \( j \)-type firm (a firm that last changed its price \( j \) periods ago) has a value function of the form:

\[
v(p_t, \xi_t, s_t) = \max \{ v_{jt}, v_{0t} \}
\]  

(2)

where \( v_{jt} = \max \{ z(p_{jt}, s_t) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} v \left( p_{j+1,t}, \frac{P_t^*}{P_t}, \xi_{t+1}, s_{t+1} \right) \} \)  

(3)

\[
v_{0t} = \max_{p_t^*} \left[ z(p_t^*) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} v \left( \frac{P_t^*}{P_{t+1}}, \xi_{t+1}, s_{t+1} \right) \right] - w_t \xi_t
\]  

(4)
where $p_{jt} = \frac{p_{t+j}}{p_t}$ is the relative price of a firm who last changed its price $j$ periods ago, $s_t$ is a state vector that governs the evolution of the firm’s demand, $\frac{\lambda_{t+j}}{\lambda_t}$ is the ratio of marginal utility in the future to marginal utility today, and $z(p_{jt}, s_t) = [p_{jt} - \psi_{jt}] \zeta_{jt}$ denotes real profits.

Interpret the value function formulation above. Why does this expression summarise the firm’s decisions?

The value function is written in relative prices. Does this make the model a real rigidity model?

(b) Derive the FOC determining the optimal value of $p_t^*$. Show that there is a cutoff level $\xi(j)$ for the fixed cost such that all $j$-type firms will adjust their price in period $t$ if and only if the value of $\xi_t$ drawn is less than $\xi(j)$. Hint: $\xi_t \sim$ i.i.d. across periods.

(c) The Calvo (1983) model has been criticised by Wolman (1999) because it implies that there will always be some firms in the economy who have not changed their prices for a VERY long time. Is this true with the state dependent pricing model above? What condition is required to ensure that $J$ is finite? Hint: Consider sustained inflation in the general price level.

(d) The Euler equation derived in part (b) can be iterated forward to relate the optimal relative price $p_t^*$ to current and expected future variables.

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \{ (\omega_{j,t+j}/\omega_{0,t}) \cdot (\lambda_{t+j}/\lambda_t) \cdot \psi_{j,t+j} \cdot c_{t+j} \}}{\sum_{j=0}^{J-1} \beta^j E_t \{ (\omega_{j,t+j}/\omega_{0,t}) \cdot (\lambda_{t+j}/\lambda_t) \cdot (P_{t+j}/P_t) \cdot c_{t+j} \}}$$

where $(\omega_{j,t+j}/\omega_{0,t}) = (1 - \alpha_{j,t+j}) \cdot (1 - \alpha_{j-1,t+j-1}) \cdots (1 - \alpha_{1,t+1})$ is the probability of non-adjustment from $t$ through $t+j$.

Note that I have assumed constant demand elasticity for all firms at all levels of output.

Show that according to (5), the optimal relative price is a fixed markup over real marginal cost \( p^* = \frac{\varepsilon}{\varepsilon - 1} \psi \) if real marginal cost and the price level are expected to be constant over time.

If these are not constant over time, we must weight the real marginal costs appropriately. Explain why real marginal costs are weighted according to the probability of non-adjustment.