Recitation 1: Blanchard-Kahn (1980)

Blanchard-Kahn (1980) illustrates how to solve linear difference models under rational expectations. This handout outlines the main steps.

Model

X [n × 1] = Predetermined variables (Function of variables known at time t).
P [m × 1] = Non-predetermined variables (Function of any variable in Ω_{t+1}).
Z [k × 1] = Exogenous variables.

(1) Structural model
(Restriction: All agents at a given time have the same information.)

(2) Rational expectations
(Excludes possibility that agents know endogenous but not exogenous variables.)

(3) Exogenous variables don't blow up too fast
(Rules out exponential growth of the time t expectation of Z_{t+i}.)

The above characterisation is for first differences, but second differences may be nested in a similar structure. Just add lagged variables to the column matrices of variables on the left and right hand sides of (1).

Solution

A is transformed into Jordan canonical form: $A = C^{-1}JC$.
J is further decomposed:

C, C^{-1}, γ are decomposed accordingly.

PROPOSITION 1: $\bar{m} = m$ (i.e. No. of $\{|\lambda_i| > 1\}$ = No. of non-predetermined variables)
$\Rightarrow \exists$ Unique solution.

Intuition: We are examining stability around a stochastic steady state. If we are not at the steady state, is there a unique path we should aim to be on (a saddlepath) so that we will approach the steady state?
$\overline{m} = 1 \Rightarrow 1$ eigenvalue outside the unit circle.
$m = 1 \Rightarrow 1$ variable we are free to set.
Then we *can* make sure we don't explode away from the steady state by setting the variable appropriately.
$y_t = \lambda^t y_0$. $|\lambda| > 1$ then set $y_0 = 0$.
Can you see why the actual values of the non-predetermined variables in X_t need not always be zero?
Can you see why we can argue in terms of the no. of predetermined variables not exactly which variables have to be predetermined?

PROPOSITION 2: $\overline{m} > m$ (i.e. No. of $\{|\lambda_i| > 1\}$ > No. of non-predetermined variables)
$\Rightarrow$ Solution explodes.

PROPOSITION 3: $\overline{m} < m$ (i.e. No. of $\{|\lambda_i| > 1\}$ < No. of non-predetermined variables)
$\Rightarrow$ Infinite no. of solutions!
What does this last proposition mean in practical terms: Do you abandon your model or can you add something to it?

<u>**Example**</u>

Small open economy characterised by the following equations.

$$y_t = -\alpha(r_t - E_t\pi_{t+1}) + \beta q_t + d_t \quad (1)$$
$$\pi_t = \gamma y_t + u_t \quad (2)$$

where

$$\pi_t \equiv p_t - p_{t-1}$$
$$q_t \equiv p^*_t + e_t - p_t$$

FIXED EXCHANGE RATE

The following restrictions apply

$$r_t = r^* = 0 \quad (3)$$
$$e_t = 0 \quad (4)$$
$$\text{Set } p^*_t = 0 \quad (5)$$

The system reduces to the following:

$$y_t = \alpha E_t(p_{t+1} - p_t) - \beta p_t + d_t$$
$$p_t - p_{t-1} = \gamma y_t + u_t$$

Substituting the second equation into the first and taking expectations:

$$\frac{1}{\gamma}p_t - \frac{1}{\gamma}p_{t-1} = \alpha E_t p_{t+1} - \alpha p_t - \beta p_t$$

$$\Rightarrow \qquad E_t p_{t+1} = \left[1 + \frac{\beta}{\alpha} + \frac{1}{\alpha\gamma}\right]p_t - \frac{1}{\alpha\gamma}p_{t-1}$$

This allows us to set up the matrix in the following way:

Find the eigenvalues:

This system has one predetermined variable and one non-predetermined variable. If one of the eigenvalues exceeds 1 in absolute value, then the solution is unique.
Note: What is the mechanism?

FLEXIBLE EXCHANGE RATE

In addition to (1) and (2), the following equations also apply:

$$r_t = \phi\pi_t \qquad (6)$$
$$r_t = E_t(e_{t+1} - e_t) \qquad (7)$$

Equation (6) is a Taylor rule equation, of a type we will see in future lectures. Equation (7) is the interest parity condition.

The problem set asks you to look at this model a little more closely.