14.461: Advanced Macroeconomics I
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Recitation 3: The Taylor Principle of Monetary Policy

The Taylor Principle states that the (eventual) response of the nominal interest rate to a change in the inflation rate must be more than one for one. Here we explore a little more.

1. Separable Utility

1.1 Taylor Principle

The flexible-price equilibrium conditions in the notes have the following form:

$$\bar{y}_t = E_t\{\bar{y}_{t+1}\} - \frac{1}{\sigma}(\hat{r}_t - E_t\{\pi_{t+1}\}) \tag{1}$$

$$\bar{y}_t = \gamma + \psi_a a_t \tag{2}$$

$$m_t - p_t = \bar{y}_t - \eta \hat{r}_t \tag{3}$$

(1) is the Euler equation together with goods market clearing. (2) is the markup = - real marginal cost condition. (3) is the money market clearing condition. The central bank uses a simple Taylor rule of the form:

$$r_t = \rho + \phi_\pi \pi_t \tag{4}$$

In the lecture notes we combine this rule with (1) to yield:

$$\phi_\pi \pi_t = E_t\{\pi_{t+1}\} + \sigma E_t\{\Delta \bar{y}_{t+1}\}$$

Iterate forward to solve for π_t:

$$\pi_t = \sigma \sum_{k=1}^{\infty} \left(\frac{1}{\phi_\pi}\right)^k E_t\{\Delta \bar{y}_{t+k}\} \tag{5}$$

It is tempting to say that the condition for a determinate price level is $\phi_\pi > 1$ because this ensures that the expression on the right hand side of equation (5) converges. However, this is in general incorrect. Convergence of the sum in (5) depends upon the behaviour of $E_t\{\Delta \bar{y}_{t+k}\}$ as $k \to \infty$. In the lecture notes we assume the AR(1) technology shock process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

For $\rho_a \in [0, 1)$ the price level is determinate when $\phi_\pi > 1$. We show this below.

1.2 Blanchard-Kahn Method

Conditions for determinacy can be derived using the Blanchard-Kahn method. We assume that the central bank allows money growth at the rate consistent with its implementation of the Taylor rule (the actual growth in money supply can be calculated using equation (3)). The Taylor rule (4) and the output equation (2) can be substituted into equation (1). Together with the technology shock process we obtain the system of equations:

$$E_t\{\pi_{t+1}\} = \phi_\pi \pi_t - \sigma \psi_a (\rho_a - 1) a_t \tag{6}$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}^a \tag{7}$$

In matrix form:

$$E_t \begin{bmatrix} \pi_{t+1} \\ a_{t+1} \end{bmatrix} = \begin{bmatrix} \phi_\pi & -\sigma\psi_a(\rho_a - 1) \\ 0 & \rho_a \end{bmatrix} E_t \begin{bmatrix} \pi_t \\ a_t \end{bmatrix}$$

The characteristic equation is $(\phi_\pi - \lambda)(\rho_a - \lambda) = 0$ and the solutions are $\lambda = \phi_\pi$ or $\lambda = \rho_a < 1$.

a_t is a predetermined variable and π_t is a non-predetermined variable. In order to have determinacy we require $\phi_\pi > 1$.

2. Non-Separable Utility

In the lecture we covered equilibrium conditions when the utility function is non-separable in consumption and real balances. This recitation handout introduces monetary policy into the system.

2.1 Log-Linearized Equilibrium Conditions

From lecture notes:

$$\bar{y}_t = E_t\{\bar{y}_{t+1}\} - \frac{1}{\sigma}(\hat{r}_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta\hat{r}_{t+1}\}) \tag{8}$$

$$\bar{y}_t = \gamma + \psi_a a_t - \psi_r \hat{r}_t \tag{9}$$

$$m_t - p_t = \bar{y}_t - \eta\hat{r}_t \tag{10}$$

The central bank follows a Taylor rule:

$$r_t = \rho + \phi_\pi \pi_t \tag{11}$$

We can substitute equation (11) into equation (8) to obtain:

$$\phi_\pi \pi_t = E_t\{\pi_{t+1}\} + \sigma E_t\{\Delta\bar{y}_{t+1}\} + \omega\phi_\pi E_t\{\Delta\pi_{t+1}\}$$

Iterate forward to solve for π_t:

$$\pi_t = \frac{\sigma}{1+\omega\phi_\pi}\sum_{k=1}^{\infty}\left(\frac{1+\omega\phi_\pi}{\phi_\pi(1+\omega)}\right)^k E_t\{\Delta\bar{y}_{t+k}\} \tag{12}$$

Again, it is tempting to conclude from (12) that the condition required for determinacy is:

$$\frac{1+\omega\phi_\pi}{\phi_\pi(1+\omega)} < 1$$

$$\Leftrightarrow \phi_\pi > 1$$

However, we cannot comment on determinacy before we know the behaviour of $E_t\{\Delta\bar{y}_{t+k}\}$. In particular, remember that the inflation rate depends upon expected future output growth, and in addition expected future output growth now depends upon expected future inflation rates through the Euler equation (8). Expected future inflation affects expected future nominal interest rates through the Taylor rule (11). Therefore, it is not clear whether $\phi_\pi > 1$ is enough for the system to be determinate.

2.2 Blanchard-Kahn Method

The central bank allows the money growth at the rate consistent with its implementation of the Taylor rule. Substituting (11) into (8) and rearranging:

$$E_t\{\pi_{t+1}\}(1+\omega\phi_\pi) = \phi_\pi(1+\omega)\pi_t - \sigma E_t\{\Delta\bar{y}_{t+1}\}$$

Substitute (9) into this expression. Use the technology shock process as before and eliminate all expressions for the nominal interest rate using the Taylor rule. Rearranging terms we obtain the system of equations:

$$E_t\{\pi_{t+1}\} = A\pi_t - Ba_t \tag{13}$$

$$a_{t+1} = \rho_a a_t + \varepsilon^a_{t+1} \tag{14}$$

where $A = \frac{\phi_\pi(1+\omega) - \sigma\psi_r\phi_\pi}{1+\omega\phi_\pi - \sigma\psi_r\phi_\pi}$ and $B = \frac{\sigma\psi_a(\rho_a - 1)}{1+\omega\phi_\pi - \sigma\psi_r\phi_\pi}$.

In matrix form:

$$E_t\begin{bmatrix}\pi_{t+1}\\ a_{t+1}\end{bmatrix} = \begin{bmatrix}A & -B\\ 0 & \rho_a\end{bmatrix} E_t\begin{bmatrix}\pi_t\\ a_t\end{bmatrix}$$

The characteristic equation is $(A-\lambda)(\rho_a-\lambda)=0$ and the solutions are $\lambda = A$ or $\lambda = \rho_a < 1$.

a_t is a predetermined variable and π_t is a non-predetermined variable. In order to have determinacy we require $A > 1$. This is equivalent to the condition that $\phi_\pi > 1$.

You might be wondering why we use the Blanchard-Kahn method when the result ($\phi_\pi > 1$) appears to be the same as with the incorrect method used in section 2.1. In fact, the coincidence of the results is an artefact of the technology shock process assumed. This ensures that $E_t\{\Delta\bar{y}_{t+k}\}$ is well behaved. In the general case, only the Blanchard-Kahn method will be correct.

3. Modified Taylor Principle with Inertial Interest Rates

This section is taken from the Woodford (2003) book, page 95-6. To make your forays into the book less confusing, I will use Woodford's notation. The interest rate exhibits some inertia and the partial adjustment dynamics are captured in the equation:

$$\hat{\imath}_t = \bar{\imath}_t + \rho(\hat{\imath}_{t-1} - \bar{\imath}_{t-1}) + \phi_\pi \pi_t \tag{15}$$

where $\phi_\pi \geq 0$ and $\rho \in [0,1)$ measures the degree of intrinsic inertia in the central bank's adjustment of its operating target. Since $\rho < 1$ we may solve the modified Taylor rule backward and obtain a finite value for $\hat{\imath}_t$:

$$\hat{\imath}_t = \bar{\imath}_t + \phi_\pi \sum_{j=0}^{\infty} \rho^j \pi_{t-j}$$

Let $\bar{\pi}_t \equiv (1-\rho)\sum_{j=0}^{\infty}\rho^j \pi_{t-j}$ and $\Phi_\pi \equiv \frac{\phi_\pi}{1-\rho}$. The Taylor rule may be rewritten:

$$\hat{\imath}_t = \bar{\imath}_t + \Phi_\pi \bar{\pi}_t \tag{16}$$

From Proposition 2.7 in the book (page 95), the equilibrium is determinate iff $\Phi_\pi > 1 \Leftrightarrow \phi_\pi > 1-\rho$. In other words, the *contemporaneous* response of the nominal interest rate to the rate of inflation need not be more than one for one. The reason is as follows. In this model, interest rates have inertia. Therefore, if the interest rate $\hat{\imath}_t$ is moved from its target value $\bar{\imath}_t$, the deviation from the target value will persist over a number of periods. This persistence will stabilise the economy in the future. Therefore, the modified Taylor principle reads as follows: The *eventual* increase in the nominal interest rate as a result of a *sustained* increase in the inflation rate is more than one for one.

Empirical application of the Taylor principle requires a more nuanced understanding of inertia in the interest rate adjustment process.