14.461: Advanced Macroeconomics I
Suman S. Basu, MIT

Recitation 4: Alternative Specifications for Price Setting

The nature of the price setting process directly affects the form of the derived Phillips curve. This handout covers 2 models which have been quite influential in the monetary policy literature. The first is the Rotemberg (1982, then 1996) model of price-setting subject to quadratic adjustment costs. The second model is the recent contribution of Mankiw and Reis (2002) which replaces sticky prices with sticky information.

1. Rotemberg (1996): Pricing with Quadratic Adjustment Costs

This handout does not purport to work through this paper. Rather, we only sketch the derivation of the log-linearized first order condition for the consumer's problem. We derive the price level at time t. Then we do something that is NOT in the paper: we show that the Phillips Curve implied by the model is identical in form to the New Keynesian Phillips Curve (NKPC) derived from the Calvo setup of the lecture notes.

1.1 Model Summary

Households both produce and consume goods. Each of N households produces a differentiated good j according to the production function:

$$Z_t^{j,s} = X_t F(H_t^j); \qquad F' > 0, F'' < 0 \tag{1}$$

Aggregate demand for the good is

$$Z_t^{j,d} = Y_t d\left(\frac{P_t^j}{P_t}\right),\ d(1) = 1 \tag{2}$$

The household solves the following program:

$$\text{Max } U^i = E_t \sum_k \beta^k \left\{ C_{t+k}^j - X_t v(H_{t+k}^j) - \frac{cX_t}{2}\left[\log(P_{t+k}^j) - \log(P_{t+k-1}^j)\right]^2 \right\} \tag{3}$$

$$\text{s.t. } P_t C_t^j \leq M_t^j \tag{4}$$

$$M_{t+1}^j = P_t^j X_t F(H_t^j) + M_t^j - P_t C_t^j + T_{t+1}^j \tag{5}$$

Improvements in technology X_t raise both the utility costs of working and the utility costs of raising prices. The cost of changing prices is quadratic in the magnitude of the adjustment. The Cash in Advance (CIA) and Intertemporal Budget Constraint (IBC) are shown.

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QUESTION: IN ROTEMBERG (1982), FIRMS FACE ADJUSTMENT COSTS. HERE CONSUMERS DO. ARE THESE EQUIVALENT? SUPPOSE CONSUMERS AND PRODUCERS ARE DIFFERENT PEOPLE. ARE THE TWO SPECIFICATIONS EQUIVALENT THEN?

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Now to begin solving the model. Demand for goods equals the supply: so from (1) and (2) we obtain:

$$H_t^j = F^{-1}\left(\frac{M_t}{P_t X_t} d\left(\frac{P_t^j}{P_t}\right)\right) \tag{6}$$

This condition, together with (5) and the fact that (4) is binding, yields the program:

$$\text{Max } U^i = E_t \sum_k \beta^k \left\{ \begin{array}{c} \frac{P_{t+k-1}^j}{P_{t+k}} \frac{M_{t+k-1}}{P_{t+k-1}} d\left(\frac{P_{t+k-1}^j}{P_{t+k-1}}\right) + \frac{T_{t+k}^j}{P_{t+k}} - X_t v\left(F^{-1}\left(\frac{M_{t+k}}{P_{t+k}X_{t+k}} d\left(\frac{P_{t+k}^j}{P_{t+k}}\right)\right)\right) \\ -\frac{cX_t}{2}\left[\log(P_{t+k}^j) - \log(P_{t+k-1}^j)\right]^2 \end{array} \right\} \tag{7}$$

The only decision at t that affects utility is the choice of P_t^j. We take the FOC of the above expression with respect to P_t^j and evaluate the expression at the symmetric equilibrium. Then we log-linearize about the steady state, where hours and M_t/P_tX_t are constant. The log-linearized version of the FOC is

$$E_t \left[\begin{array}{c} \beta\left[c(1+g) - \frac{M}{PX}\frac{1+d'}{1+\pi}\right](p_{t+1} - p_t) + \left[c\pi(1-\beta(1+g)) - \frac{d'}{F'}\left(\frac{M}{PX}\right)^2 \frac{v''F' - v'F''}{F'^2}\right](m_t - p_t - x_t) \\ -c(p_t - p_{t-1}) + \beta c\pi(1+g)(x_t - x_{t-1}) \end{array} \right] = 0 \tag{8}$$

Equation (8) is a second order difference equation in p_t whose characteristic equation has 2 roots. The solution is unique and nonexplosive if one of the roots is inside, and one outside, the unit circle. The unique solution relating the nonpredetermined variable to the predetermined variable will be:

$$p_t = \alpha p_{t-1} + (1-\alpha)(1-\delta)E_t\left\{\sum_{\tau=0}^{\infty} \delta^\tau (m_{t+\tau} - x_{t+\tau})\right\} \tag{9}$$

where α is the root smaller than 1 and $1/\delta$ is the other root, exceeding 1.

1.2 Phillips Curve

So far, the exposition has followed the paper. However, now I use the price setting equation (9) in order to derive the Phillips Curve, which is not in the article. We may rewrite (9) in the form:

$$p_t = \alpha p_{t-1} + (1-\alpha)p_t^*, \text{ where } p_t^* = (1-\delta)E_t\left\{\sum_{\tau=0}^{\infty}\delta^\tau(m_{t+\tau} - x_{t+\tau})\right\}$$

$$\Leftrightarrow \pi_t = (1-\alpha)(p_t^* - p_{t-1}) \tag{10}$$

. Once the equation has been written in this form we may apply similar techniques as in the lecture notes, although obviously the notation will be different.

$$\begin{aligned} p_t^* - p_{t-1} &= (1-\delta)E_t\left\{\sum_{\tau=0}^{\infty}\delta^\tau(m_{t+\tau} - x_{t+\tau})\right\} - p_{t-1} \\ &= (1-\delta)E_t\left\{\sum_{\tau=0}^{\infty}\delta^\tau\left[(m_{t+\tau} - p_{t+\tau} - x_{t+\tau}) + (p_{t+\tau} - p_{t-1})\right]\right\} \\ &= (1-\delta)E_t\sum_{\tau=0}^{\infty}\delta^\tau g_{t+\tau} + (1-\delta)E_t\sum_{\tau=0}^{\infty}\delta^\tau(p_{t+\tau} - p_{t-1}) \end{aligned}$$

$$= (1-\delta)E_t\sum_{\tau=0}^{\infty}\delta_{t+\tau}^\tau g_{t+\tau} + E_t\sum_{\tau=0}^{\infty}\delta^\tau\pi_{t+\tau} \tag{11}$$

In a more compact form:

$$p_t^* - p_{t-1} = \delta E_t\left\{p_{t+1}^* - p_t\right\} + (1-\delta)g_t + \pi_t \tag{12}$$

Substitute (12) into (10) and rearrange:

$$\pi_t = \frac{\delta}{\alpha} E_t \pi_{t+1} + \frac{(1-\alpha)(1-\delta)}{\alpha} g_t \tag{13}$$

What is z_t? We need to interpret this term.

$$\begin{aligned} g_t &= m_t - p_t - x_t \\ &= y_t - x_t \text{ from (CIA) and market clearing} \\ &= \log\{F(H_t)\} \text{ from (1)} \end{aligned}$$

This term can be normalised so that it equals 0 at the steady state (where hours H_t is constant). Then g_t exceeds 0 when hours exceed the steady state level of hours, and is less than 0 when hours are less than the steady state level. From the production function (1), output exceeds the steady state output when hours exceed steady state hours. To a first order approximation we may relate the output gap $\tilde{y}_t$to the variable g_t thus:

$$\tilde{y}_t = \chi g_t \tag{14}$$

Substituting (14) into (13) we derive the Phillips Curve:

$$\pi_t = \overset{\wedge}{\beta} E_t \pi_{t+1} + \overset{\wedge}{\kappa} \tilde{y}_t \tag{15}$$

where $\overset{\wedge}{\beta} = \frac{\delta}{\alpha}$ and $\overset{\wedge}{\kappa} = \frac{(1-\alpha)(1-\delta)}{\alpha\chi}$.

This confirms that the price setting model with adjustment costs yields a Phillips Curve of the same form as the New Keynesian Phillips Curve (NKPC). However, the interpretation of the coefficients on the Phillips Curve has changed.

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QUESTION: AS WE CHANGE α, δ AND χ HOW DO THE COEFFICIENTS ON THE NKPC CHANGE? WHAT IS THE INTUITION?

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2. Mankiw and Reis (2002): Sticky Information versus Sticky Prices

Note: This section follows Question 5 parts (a)-(d) from the Chapter 4 Exercises posted on the class website. The solutions were provided by Professor Gali and I have proof-read them.

2.1 Question 5 Parts (a) - (d)

The Mankiw-Reis Model: Inflation Dynamics under Predetermined Prices

Consider our benchmark framework with monopolistic competition. Suppose that each period a fraction of firms $1-\theta$ gets to choose *a path of future prices* for their respective goods (a "price plan"), while the remaining fraction θ keep their current price plans. We let $\{P_{t,t+k}\}_{k=0}^{\infty}$ denote the price plan chosen by firms that get to revise that plan in period t. Firm's technology is given by $Y_t(i) = \sqrt{A_t} N_t(i)$. Consumer's period utility is assumed to take the form $U(C_t, N_t) = C_t - \frac{N_t^2}{2}$, where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$. The demand for real balances is assumed to be proportional to consumption with a unit velocity, i.e. $\frac{M_t}{P_t} = C_t$. All output is consumed.

(a) Let $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$ denote the aggregate price index. Show that, up to a first order approximation, we will have:

$$p_t = (1-\theta)\sum_{j=0}^{\infty}\theta^j p_{t-j,t} \tag{16}$$

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Using the definition of the price index, we have

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} di \\ &= (1-\theta)\sum_{j=0}^{\infty}\theta^j \left(\frac{P_{t-j,t}}{P_t}\right)^{1-\varepsilon} \\ &= (1-\theta)\sum_{j=0}^{\infty}\theta^j \exp\{(1-\varepsilon)(p_{t-j,t}-p_t)\} \end{aligned}$$

A first order approximation of the latter expression about the symmetric equilibrium with constant prices ($p_{t-j,t} = p_t$, all t) yields the desired result.

$$\begin{aligned} 1 &\approx 1 + (1-\varepsilon)(1-\theta)\sum_{j=0}^{\infty}\theta^j (p_{t-j,t} - p_t) \\ &\Leftrightarrow \; p_t \approx (1-\theta)\sum_{j=0}^{\infty}\theta^j p_{t-j,t} \end{aligned}$$

QUESTION: WHY DON'T WE CANCEL OUT THE $(1-\theta)$ AS WE DID THE $(1-\varepsilon)$ IN THE EXPRESSION ABOVE?

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(b) A firm i, revising its price plan in period t will seek to maximize

$$\sum_{k=0}^{\infty}\theta^k E_t \left\{ Q_{t.t+k} Y_{t+k}(i) \left(P_{t,t+k} - \frac{W_{t+k}}{\sqrt{A_{t+k}}} \right) \right\}$$

Derive the first order condition associated with that problem, and show that it implies the following approximate log-linear rule for the price plan:

$$p_{t,t+k} = \mu + E_t\left\{mc^n_{t+k}\right\} \tag{17}$$

for $k = 0, 1, 2, ...$ where $mc^n_t = w_t - \frac{1}{2}a_t$ is the nominal marginal cost.

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The firm faces a demand for its good with a constant elasticity ε, which implies the first order condition:

$$\begin{aligned} 0 &= E_t \left\{ Q_{t,t+k} Y_{t+k}(i) \left(\frac{P_{t,t+k}}{P_{t+k}} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k} \right) \right\} \\ &= E_t \left\{ Q_{t,t+k} Y_{t+k}(i) \left(\exp(p_{t,t+k} - p_t) - \frac{\varepsilon}{\varepsilon - 1}\exp(mc_{t+k}) \right) \right\} \end{aligned}$$

for $k = 0, 1, 2, 3...$ where $MC_{t+k} = \frac{W_{t+k}}{\sqrt{A_{t+k}}P_{t+k}}$. Linearizing around a perfect foresight steady state (with zero inflation), and letting $\mu \equiv \log \frac{\varepsilon}{\varepsilon-1}$, we have

$$p_{t,t+k} - p_t = \mu + E_t \{mc_{t+k}\}$$

or, equivalently, $p_{t,t+k} = \mu + E_t \left\{mc^n_{t+k}\right\}$.
$MC^n_t = \frac{W_t}{\sqrt{A_t}}$ so $mc^n_t = w_t - \frac{1}{2}a_t$.

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QUESTION: INTUITIVELY, WHY IS THE FOC ABOVE NOT OF THE SAME FORM AS THE ONE IN THE LECTURE NOTES $\left(0 = \sum\limits_{k=0}^{\infty} \theta^k E_t \left\{Q_{t,t+k} Y_{t+k}(P^*_t) \left(P^*_t - \frac{\varepsilon}{\varepsilon-1} MC^n_{t+k}\right)\right\}\right)$?

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(c) Using the optimality conditions for the consumer's problem, and the labour market clearing condition show that (i) the *natural* level of output satisfies $\overline{y}_t = -\mu + a_t$, and (ii) the (log) real marginal cost (in deviation from its perfect foresight steady state value) equals the output gap, i.e.

$$\overset{\wedge}{mc}_t = \widetilde{y}_t$$

for all t, where $\widetilde{y}_t \equiv y_t - \overline{y}_t$.

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In equilibrium, (log) real marginal cost is given by

$$\begin{aligned} mc_t &= (w_t - p_t) - \frac{1}{2}a_t \\ &= n_t - \frac{1}{2}a_t \text{ from } w_t - p_t = n_t \\ &= y_t - a_t \text{ from } y_t = \frac{1}{2}a_t + n_t \end{aligned}$$

Under flexible prices, $mc_t = -\mu$ for all t. Hence, $\overline{y}_t = -\mu + a_t$. Using the fact that $\overset{\wedge}{mc}_t \equiv mc_t - mc = mc_t + \mu$ it follows that $\overset{\wedge}{mc}_t = \widetilde{y}_t$.

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(d) Using (16) and (17) show how one can derive the following equation for inflation:

$$\pi_t = \frac{1-\theta}{\theta}\widetilde{y}_t + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j E_{t-j}\left\{\Delta\widetilde{y}_t + \pi_t\right\} \tag{18}$$

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Using (16) and (17):

$$\begin{aligned} p_t &= (1-\theta)\sum_{j=0}^{\infty}\theta^j\left(\mu + E_{t-j}\{mc^n_t\}\right) \\ &= (1-\theta)\sum_{j=0}^{\infty}\theta^j E_{t-j}\left\{p_t + \widetilde{y}_t\right\} \\ &= (1-\theta)(p_t + \widetilde{y}_t) + (1-\theta)\sum_{j=0}^{\infty}\theta^{j+1}E_{t-1-j}\left\{p_t + \widetilde{y}_t\right\} \end{aligned}$$

An analogous equation for period $t-1$ is given by:

$$p_{t-1} = (1-\theta)\sum_{j=0}^{\infty}\theta^j E_{t-1-j}\left\{p_{t-1} + \widetilde{y}_{t-1}\right\}$$

Multiplying both sides of the latter expression by θ, and subtracting it from the expression for p_t above, yields the desired inflation equation.

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QUESTION: INTERPRET THE STICKY INFORMATION PHILLIPS CURVE. DOES IT SURVIVE THE McCALLUM (1998) CRITIQUE?

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2.2 Impulse Responses for the Sticky Information versus Sticky Price Phillips Curve

Overleaf I have included impulse responses to 2 shocks, both examined in the Mankiw-Reis (2002) paper. Question 5 parts (e)-(f) are left to you as an exercise and will be briefly covered in the next recitation.

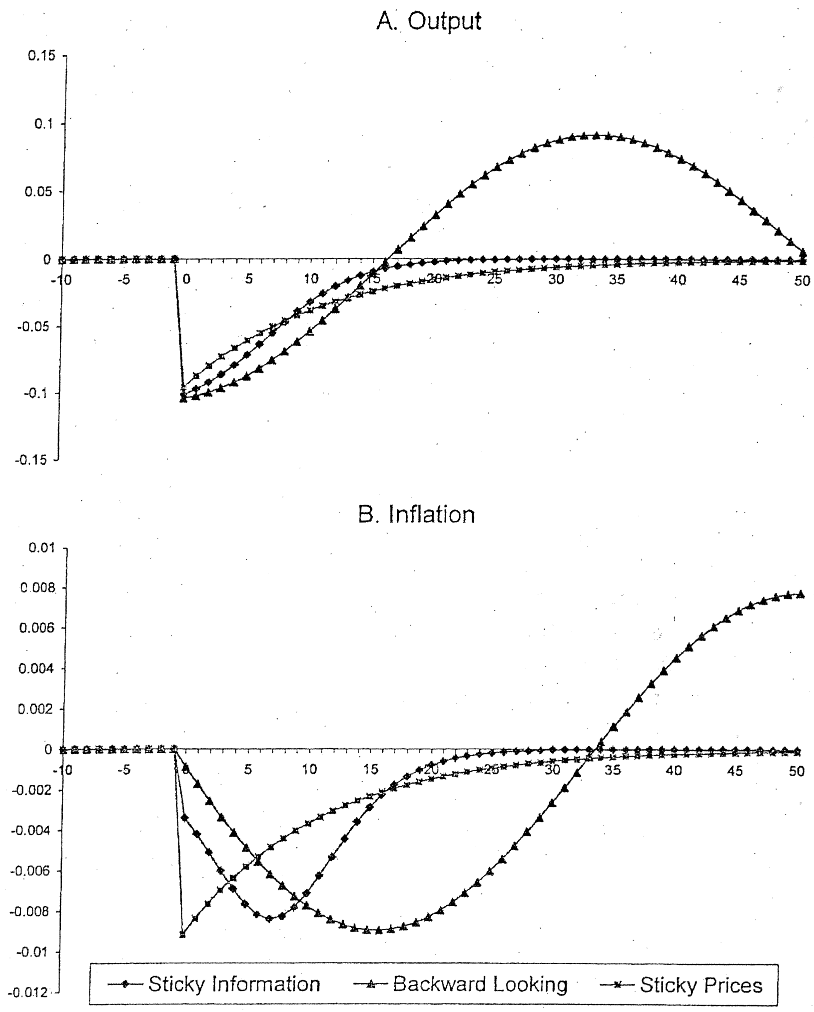

FIGURE I
Dynamic Paths after a 10 Percent Fall in the Level of Aggregate Demand at Time 0

The impact of the fall in demand on output is close to zero at sixteen quarters. The backward-looking model generates an oscillatory pattern, whereas the other two models yield monotonic paths. Otherwise, the models seem to yield similar results.

Differences among the models become more apparent, however, when we examine the response of inflation in the bottom of Figure I. In the sticky-price model, the greatest impact of the fall

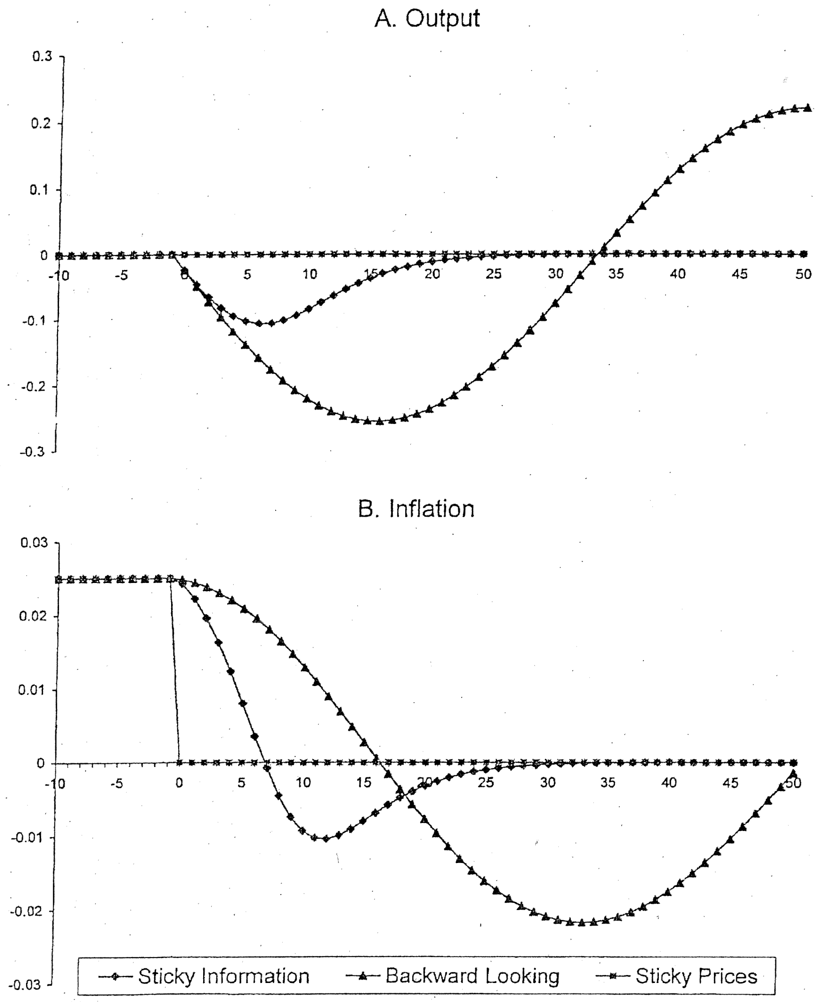

FIGURE II
Dynamic Paths Given an Unanticipated Fall in the Growth Rate of Aggregate Demand at Date 0

place, most price setters are still marking up prices based on old decisions and outdated information. As a result of this inertial behavior, inflation is little changed one or two quarters after the disinflation has begun. With a constant money supply and rising prices, the economy experiences a recession, which reaches a trough six quarters after the policy change. Output then gradu-