Recitation 6: Rotemberg and Woodford (1999)
Interest Rate Rules in an Estimated Sticky Price Model

How well do "simple" monetary policy rules perform in sticky price models? And how should central banks formulate such rules? The 1999 NBER Conference went some way into addressing these issues, and the key findings are presented in the volume "Monetary Policy Rules," edited by John B. Taylor. You should read the introduction to the volume. This recitation handout covers the main points of the contribution from Rotemberg and Woodford (chapter 2). You may regard it as a generalization of the table on the last page of the lecture handout, "Optimal Monetary Policy in the Baseline Sticky Price Model."

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1. Differences from the Baseline Framework in Lectures

The model is outlined below. Differences from the model covered in lectures are highlighted in **bold**.

1.1 IS Equation

- There is a **continuum of households** indexed by i, $i \in [0,1]$. Each household produces a single good c_t^i but consumes the composite good C_t^i (for the composite good, the superscript i refers to the consumer not the good). The utility of household i at time t is given by:

$$E_t \sum_{t=1}^{\infty} \beta^{T-t} \left[u\left(C_T^i; \xi_T\right) - v\left(y_T^i; \xi_T\right) \right]$$

where β is the discount rate, y_t^i is household i's production of its own good, and ξ_t is a **vector of preference or technological disturbances.** The composite consumption good is a Dixit-Stiglitz (1977) aggregator of the continuum of individual consumption goods:

$$C_T^i = \left[\int_0^1 c_T^i(z)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)}$$

where $c_t^i(z)$ is the quantity purchased of good z, and we assume that $\theta > 1$. This yields total demand for differentiated good as:

$$y_t^z = Y_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta} \quad \text{where } P_t = \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{1/(1-\theta)}$$

- **Households must choose their index of purchases C_t^i at date $t-2$.** This is introduced to account for the fact that US GDP responds to monetary policy after 2 quarters, and is discussed further in Rotemberg and Woodford (1997). Household optimization then requires:

$$E_t \left[u_C \left(C_{t+2}^i; \xi_{t+2} \right) \right] = E_t \left(\lambda_{t+2}^i P_{t+2} \right) \tag{1}$$

where λ_t^i is the Lagrange multiplier and marginal utility of income of household i. Assuming a borrowing constraint that never binds in equilibrium, the marginal utility of income must satisfy:

$$\lambda_t^i = \beta R_t E_t \lambda_{t+1}^i \tag{2}$$

where R_t is the gross return on a riskless bond purchased at date t.

- We assume **complete insurance markets** so that all households have the same marginal utility of income at any time.

  QUESTION: Why do complete insurance markets ensure this? And why do we want this property that the marginal utility of income is constant across households?

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- We have government expenditure satisfying:

$$C_t = Y_t - G_t \tag{3}$$

  Government expenditure G_t is determined at date $t-1$, i.e. after consumer expenditure but before the central bank sets the interest rate for date t.

- Log-linearizing equations (1) and (2) yield:

$$-\widetilde{\sigma} E_{t-2}\left(\hat{C}_t - \bar{C}_t\right) = E_{t-2}\overset{\wedge}{\lambda}_t \tag{4}$$

$$\overset{\wedge}{\lambda}_t = \hat{R}_t - \pi_{t+1} + E_t\overset{\wedge}{\lambda}_{t+1} \tag{5}$$

  where $\widetilde{\sigma} = -\frac{u_{CC}\bar{C}}{u_C}$.

- Log-linearizing the market clearing condition and substituting into the system of equations we derive the IS equation:

$$\hat{Y}_t = \hat{G}_t - \frac{1}{\sigma}E_{t-2}\sum_{T=t}^{\infty}\left(\hat{R}_T - \pi_{t+1}\right) \tag{6}$$

  where $\sigma = \frac{\bar{Y}}{\bar{C}}\widetilde{\sigma}$.

1.2 Phillips Curve / AS Equation

- Price-setting follows Calvo (1983) with some modifications. At the end of any period, fraction $1-\alpha$ of firms get to choose new prices. **Of these, a fraction γ start charging the new price at the beginning of the next period. The remaining fraction $1-\gamma$ of firms must wait until the following period to charge the new price (they must post the new price a quarter in advance).** These delays account for the fact that the largest response of inflation to a monetary policy shock takes place after 2 quarters.

  Prices are chosen to maximise the contributions to expected utility resulting from sales revenue on the one hand, and the disutility of output supply on the other, at each of the future dates and states at which the chosen price applies:

$$\max \Phi_{t-1}(p) \equiv E_{t-1}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\lambda_T(1-\tau)pY_T(p/P_T)^{-\theta} - v(Y_T(p/P_T)^{-\theta};\xi_T)\right] \tag{7}$$

- Substituting in the consumer demand and log-linearizing the optimization decision of each type of firm (γ and $1-\gamma$), then aggregating, we derive the Phillips Curve/AS equation:

$$\pi_t = (1-\psi)E_{t-2}\pi_t + \psi\left\{\kappa E_{t-1}\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{Y}_T - \hat{Y}_T^S\right) - \frac{\kappa}{\omega+\sigma}\left[E_{t-1}\sum_{T=t}^{\infty}\left(\hat{R}_T - \pi_{t+1}\right) - E_{t-2}\sum_{T=t}^{\infty}\left(\hat{R}_T - \pi_{t+1}\right)\right]\right\} \tag{8}$$

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where $\psi = \frac{\gamma\alpha}{1-\gamma(1-\alpha)}$, $\kappa = \frac{(1-\alpha)(1-\alpha\beta)(\omega+\sigma)}{\alpha(1+\omega\theta)}$, and $\omega = \frac{v_{yy}\bar{Y}}{v_y}$.

Because prices are set in advance, expectations of future output increases relative to Y^S also raise prices. In addition, if the long term interest rate at t is higher than had been expected at $t-1$, then inflation declines, because the upward revision increases the returns households expect to earn from their revenues. As a result, they raise revenue by cutting prices.

1.3 Monetary Policy Rule

- Interest rates are characterized according to a feedback rule, an extension of Taylor (1993):

$$r_t - r^* = \sum_{j=0}^{m_\pi} a_j(\pi_{t-j} - \pi^*) + \sum_{j=0}^{m_Y} b_j \hat{Y}_{t-j} + \sum_{j=1}^{m_R} c_j(r_{t-j} - r^*) \tag{9}$$

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2. VAR Approach

Rotemberg and Woodford (1999) estimate a recursive model of the state vector:

$$Z_t = \left[\hat{r}_t, \overset{\wedge}{\pi}_{t+1}, \hat{Y}_{t+1} \right]'$$

The estimated system is:

$$\bar{Z}_t = B\bar{Z}_{t-1} + U\bar{e}_t \tag{10}$$

where the vector $\bar{Z}_t$ is the transpose of $\left[Z_t', Z_{t-1}', Z_{t-2}' \right]$.
The restrictions on the VAR are:

- The interest rate in period t responds to inflation and output in period t, while these variables only react to lagged interest rates.

- $e_{1,t}$ is independent of the real disturbances so it is purely a monetary policy shock.

The structural parameters are then derived by minimizing the discrepancy between the estimated responses of output, inflation and the interest rate to the monetary disturbance $e_{1,t}$ and the responses predicted by the theoretical model when we use the monetary policy rule given by (9).

The authors also use calibration to obtain numerical values for a range of parameters on the basis of other evidence.

The question in this paper is: How would the US economy perform if it were subject to structural disturbances whose properties are the same as those that have affected it in the past while, at the same time, the way interest rates are set the central bank is different?

QUESTION: Does this empirical strategy suffer from the Lucas (1976) critique of econometric policy evaluation?

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3. Welfare Loss

The average level of welfare is given by:

$$W = E\left(u\left(C_t; \xi_t\right) - \int_0^1 v\left(y_t(z); \xi_t\right) dz\right)$$

Following Rotemberg and Woodford (1998), the authors take a second order Taylor approximation of the utility-based welfare measure around the steady state values of the variables that affect utility. Furthermore, it is assumed that the government not only uses an output subsidy to correct for the distortion created by monopolistic competition, but that the value of the subsidy varies with the particular monetary policy rule in order to keep the natural rate value of output equal to the welfare-maximizing value.

The derived loss measure is:

$$L = \text{var}\left\{\pi_t\right\} + (\psi^{-1} - 1)\text{var}\left\{\pi_t - E_{t-2}\pi_t\right\} + \Lambda \text{var}\left\{E_{t-2}\left(\hat{Y}_T - \hat{Y}_T^S\right)\right\} \tag{11}$$

The paper looks at the performance of monetary policy rules according to 2 approaches:

- The parameter values that minimize L.

- The performance of the rule in terms of the derived volatility of variables of interest about the natural rate values.

  The rationale for looking at the latter is that the former places too much weight on the particular functional form assumed for consumer utility.

4. Consequences of Simple Policy Rules

The tables and figure attached summarize the evidence. The categories of rules considered are listed below.

4.1 Simple "Taylor Rules"

$$\hat{r}_t = a\hat{\pi}_t + b\hat{Y}_t \tag{12}$$

where $\hat{r}_t = r_t - r^*$ and $\hat{\pi}_t = \pi_t - \pi^*$.

4.2 Lagged Interest Rate Rules

$$\hat{r}_t = a\hat{\pi}_t + b\hat{Y}_t + c\hat{r}_{t-1} \tag{13}$$

where we now allow c to be greater than 0.

4.3 Rules Using Only Lagged Data

$$\hat{r}_t = a\hat{\pi}_{t-1} + b\hat{Y}_{t-1} + c\hat{r}_{t-1} \tag{14}$$

QUESTION: Why might such a rule be implemented even if the central bank has privileged access to new contemporaneous information?

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4.4 Price Level Targeting Rules

$$\hat{r}_t = a\hat{P}_t + b\hat{Y}_t + c\hat{r}_{t-1} \tag{15}$$

A specification that nests price level and inflation targeting rules is:

$$\hat{r}_t = a_0\hat{P}_t + a_1\hat{P}_{t-1} + b\hat{Y}_t + c\hat{r}_{t-1} \tag{16}$$

Table 2.1   Statistics for Several Policy Rules

	a	b	c	var$\{R\}$	var$\{Y\}$	var$\{\pi\}$	var$\{\Delta p^\infty\}$	β^∞	var $\{\pi - E\pi\}$	var $\{E(Y - Y^s)\}$	L	π^*	$L + \pi^{*2}$
Contemporaneous-data Taylor rules													
A_0	3.00	0.80	1.00	6.24	7.24	0.66	4.00	3.82	0.27	8.77	1.32	2.66	8.38
B_0	1.20	1.00	1.00	6.41	2.73	1.82	16.05	5.44	0.48	11.69	2.80	2.74	10.29
C_0	1.50	0.50	0.00	17.14	3.87	7.34	121.14	11.14	0.81	13.86	8.72	6.37	49.29
D_0	1.50	1.00	0.00	16.26	1.79	6.76	100.77	9.80	0.83	14.63	8.18	6.13	45.73
E_0	1.22	0.06	1.28	1.88	13.47	0.38	1.47	−1.86	0.19	11.37	1.09	0.11	1.10
F_0	2.88	0.02	0.00	8.65	15.42	1.04	16.77	6.38	0.37	10.35	1.86	3.66	15.26
Lagged-data Taylor rules													
A_1	3.00	0.80	1.00	5.89	10.32	0.66	4.20	3.84	0.28	6.56	1.23	2.50	7.47
B_1	1.20	1.00	1.00	5.99	3.30	1.83	16.30	5.43	0.49	9.58	2.71	2.54	9.18
C_1	1.50	0.50	0.00	13.83	4.63	5.97	94.12	10.04	0.78	11.17	7.19	5.42	36.55
D_1	1.50	1.00	0.00	14.86	3.32	5.85	83.99	9.08	0.81	11.69	7.12	5.73	39.91
E_1	1.27	0.08	1.28	1.87	13.92	0.37	1.12	−1.35	0.19	12.18	1.12	0.10	1.13
Price level targeting rules													
G_0	0.26	0.07	1.03	2.00	11.14	0.34	0.00	0.00	0.23	11.93	1.11	0.21	1.16
G_1	0.38	0.10	0.85	2.04	11.35	0.34	0.00	0.00	0.25	12.57	1.15	0.24	1.21
Estimated historical U.S. policy													
H	—	—	—	7.64	4.79	2.28	26.19	5.60	0.66	12.14	3.43	3.26	14.06
Optimal policy													
I	—	—	—	1.93	11.30	0.39	1.77	−2.19	0.20	7.57	0.93	0.15	0.95
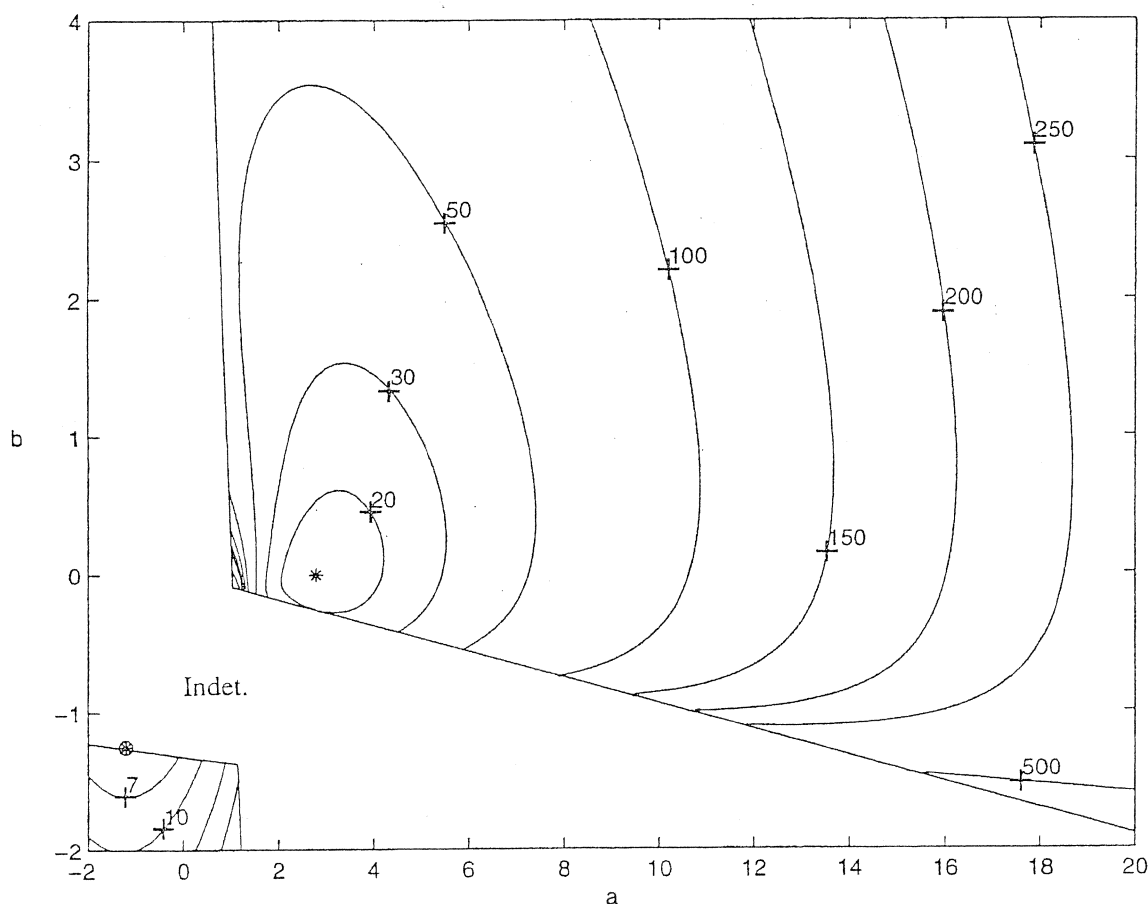
Fig. 2.4   Simple Taylor rules: $L + \pi^{*2}$ as a function of a and b

values (i.e., a stationary equilibrium always exists), but equilibrium is indeterminate in the region labelled "Indet." Indeterminacy arises, for example, when b is zero and a is small and positive. This indeterminacy implies, among other things, that inflation can vary simply as a result of changes in expectations. A "sunspot" can lead inflation at t to rise, for example. The real interest rate would then fall (because the nominal interest rate responds little) and the resulting increase in output means that expected future inflation is lower than current inflation. Thus the change in the expected future path of inflation that is required to justify the initial change in inflation is consistent with expected future inflation converging back to the target inflation rate π^*. In this case, a stationary rational expectations equilibrium is possible in which such fluctuations occur simply because they are expected to.

If, instead, a is large and positive, no such equilibrium is possible. Any increase in inflation above its unique saddle-path value is matched by increases in real interest rates that imply that output must fall. This, in turn, implies that expected future inflation rates must be higher than current inflation, given the nature of our AS curve. Thus inflation must be expected to explode, and since
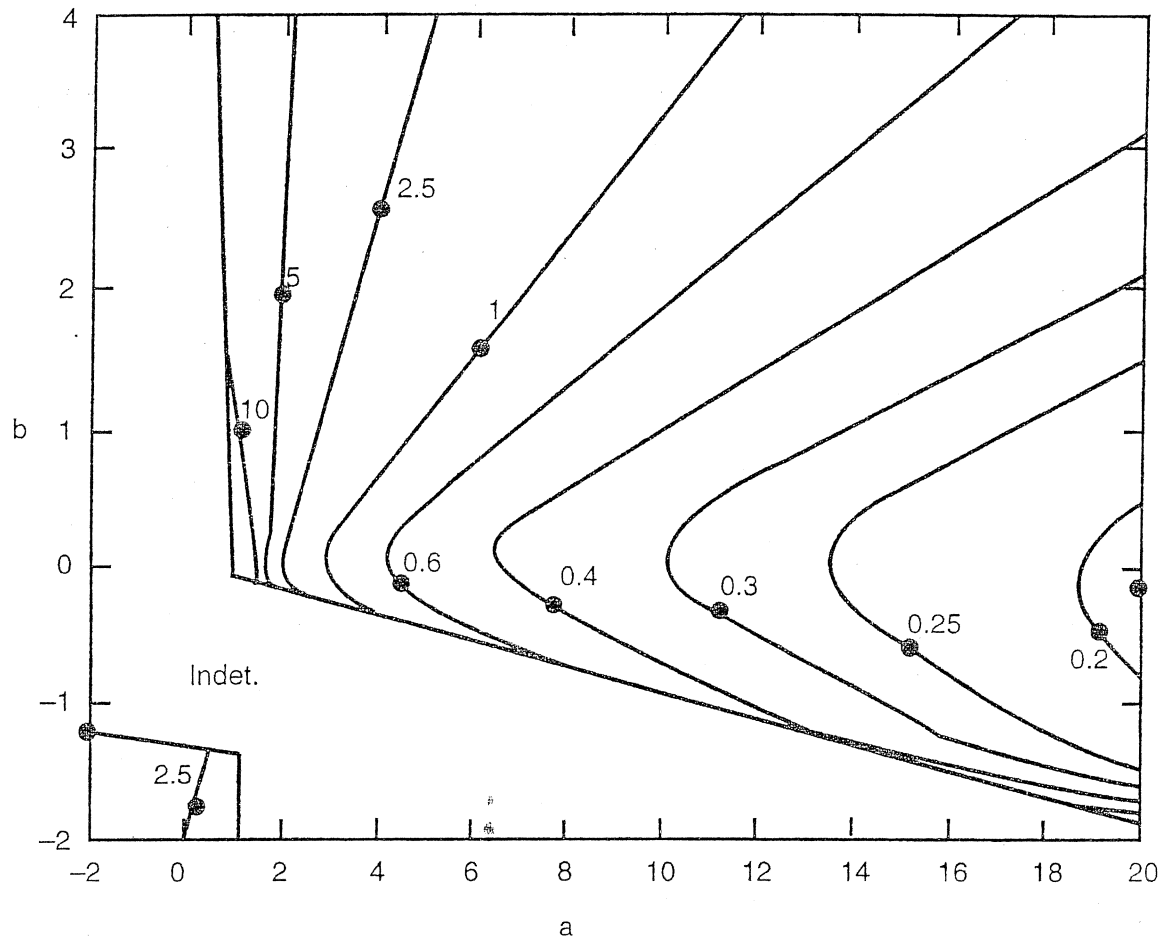
Fig. 2.5 Simple Taylor rules: var $\{\hat{\pi}\}$ as a function of a and b

Figure 2.4 presents contour lines for the value of our loss measure $L + \pi^{*2}$ in the regions where equilibrium is determinate. Policy F_0 appears as a star on this figure, at the point of a local minimum of the loss measure. However, the region of determinate equilibria with negative a and b also contains a local minimum. This point, which is shown with a star inside a circle, is actually the global minimum value. Nonetheless, we have chosen to present the local minimum F_0 in table 2.1, on the ground that restricting attention to values $a > 0$ corresponds to rules that are more similar to the Taylor and Henderson-McKibbin proposals. In addition, once we consider more general families of rules, we do find that the best rules involve tightening monetary policy (i.e., raising the funds rate) in response to inflation increases, as conventional wisdom (at least since the work of Wicksell [1907]) would indicate.

Similar contour plots for other statistics reported in table 2.1 provide further insight into why our loss measure varies with a and b as it does. Figure 2.5 shows the contour plots of the variance of inflation, while figure 2.6 shows the contour plots for the variance of $\hat{Y} - \hat{Y}^s$. These figures are essentially identical
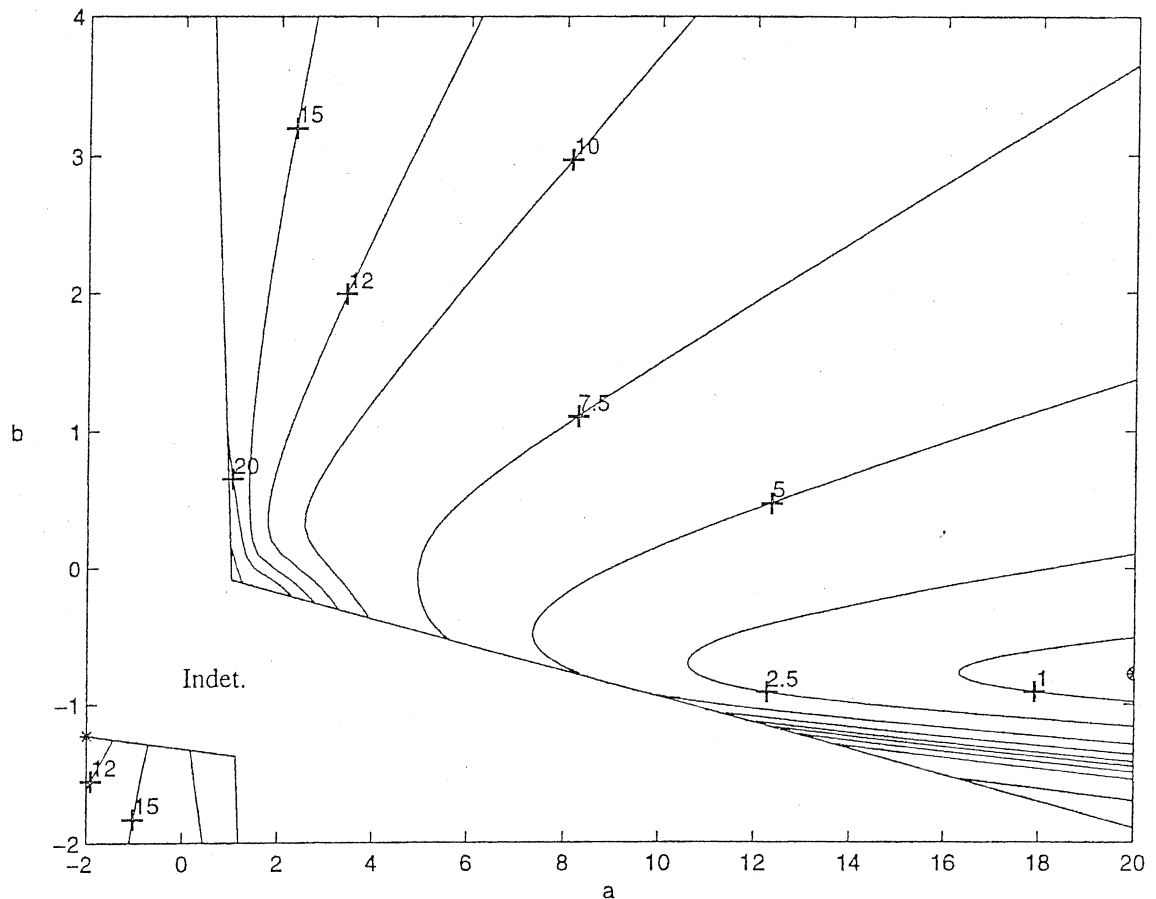
Fig. 2.6   Simple Taylor rules: var $\{\hat{Y} - \hat{Y}^s\}$ as a function of a and b

relates inflation to departures of $\hat{Y}$ from $\hat{Y}^s$. For the ranges considered in our figures, a wheel marks the global optimum for the performance criterion being considered. Thus the figures show that these variances become as small as possible when a is at its maximum possible value of 20 while b is set to a small negative number. Making a big contributes to stabilization because it ensures that interest rates rise a lot when either $\hat{G}$ rises or $\hat{Y}^s$ falls. This ensures that inflation does not rise much in either case and that, at least after the demand for output adjusts to changes in real rates, output does not rise in the former case while it declines substantially in the latter.

As figure 2.7 indicates, the rule that minimizes L by setting a equal to 20 leads to very variable interest rates. This is in part due to the delays in the response of output to interest rates. These delays imply that changes in $\hat{G}_t$ that become known at $t - 1$ inevitably change output at t since C_t is predetermined. This leads firms to raise their prices at t unless long-term real interest rates rise unexpectedly. With c equal to zero, this means that prices can only be stabilized if the nominal interest rate at t rises a great deal. The resulting variability of interest rates then requires a high average inflation rate for interest rates never to be negative. This high inflation is so costly, at least relative to the benefits of the additional stabilization that is possible with a high value of a, that the contour plots for the variance of the interest rate are essentially identi-
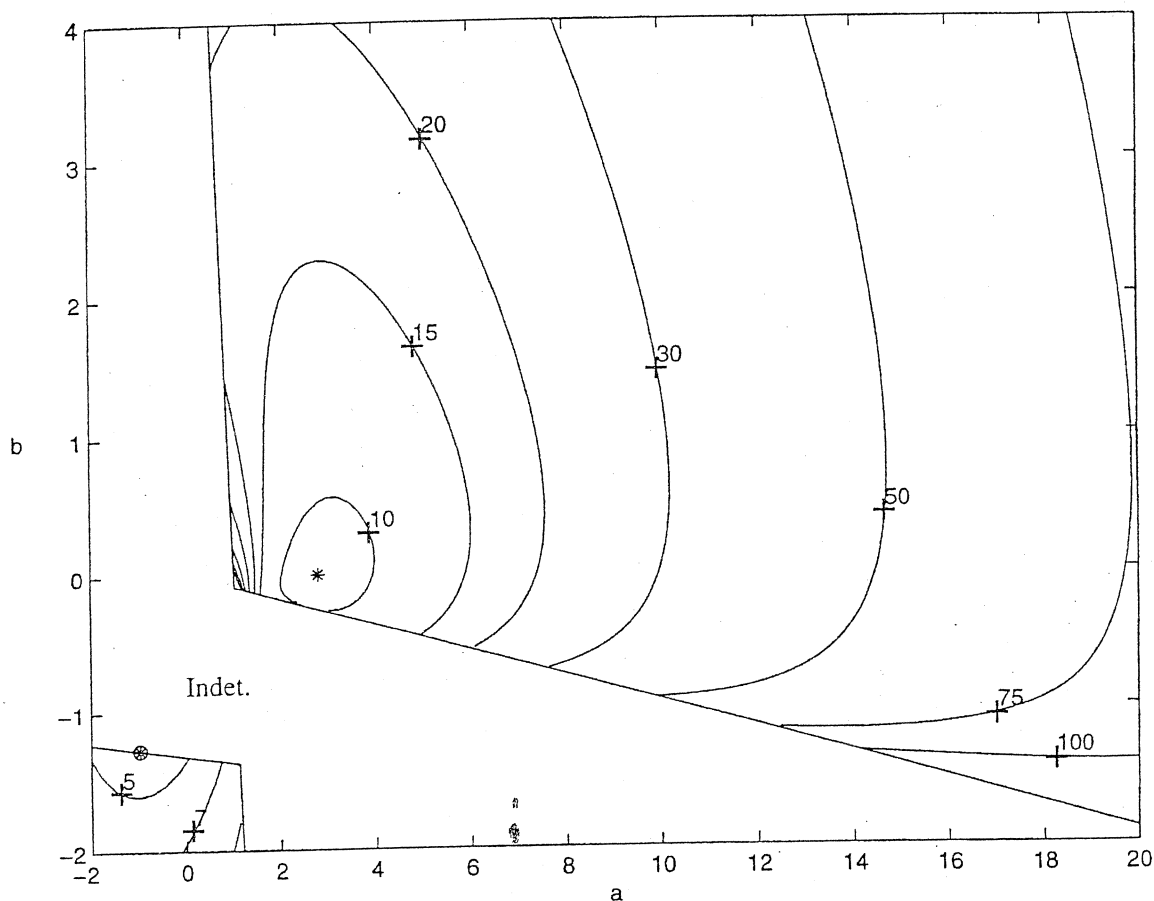
Fig. 2.7   Simple Taylor rules: $\mathrm{var}\{\hat{R}\}$ as a function of a and b

interest rates has a sufficiently stable inflation to be quite desirable as far as total welfare is concerned.

It is interesting to note that the stabilization of output requires a quite different set of parameters. This is demonstrated in figure 2.8, which gives the contour plots for the variance of output. This variance is reduced by keeping a small and positive while making b very large. Not surprisingly, output is stabilized if the real interest rate is raised significantly by the central bank whenever output rises, while it is lowered when output declines. What is interesting here is that the effects of the policy parameters on the variance of $\hat{Y} - \hat{Y}^s$, which are essentially the same as the effects on L, are very different from the effects on the variance of Y. The reason is that the VAR of Rotemberg and Woodford (1997) identifies large short-run fluctuations in $\hat{Y}^s$. As long as these are treated as variations in the welfare-maximizing level of output, setting b large is not desirable, and indeed, stabilization of $\hat{Y} - \hat{Y}^s$ requires that b be negative at least when a is 20. Even higher values of a reduce the variance of $\hat{Y} - \hat{Y}^s$ still further. Obviously, the result that the stabilization of $\hat{Y}$ relative to $\hat{Y}^s$ requires very different policies from those that stabilize output relative to trend is very sensitive to the assumption that our estimate of $\hat{Y}^s$ is indeed the welfare-maximizing level of output. This conclusion would presumably change dra-
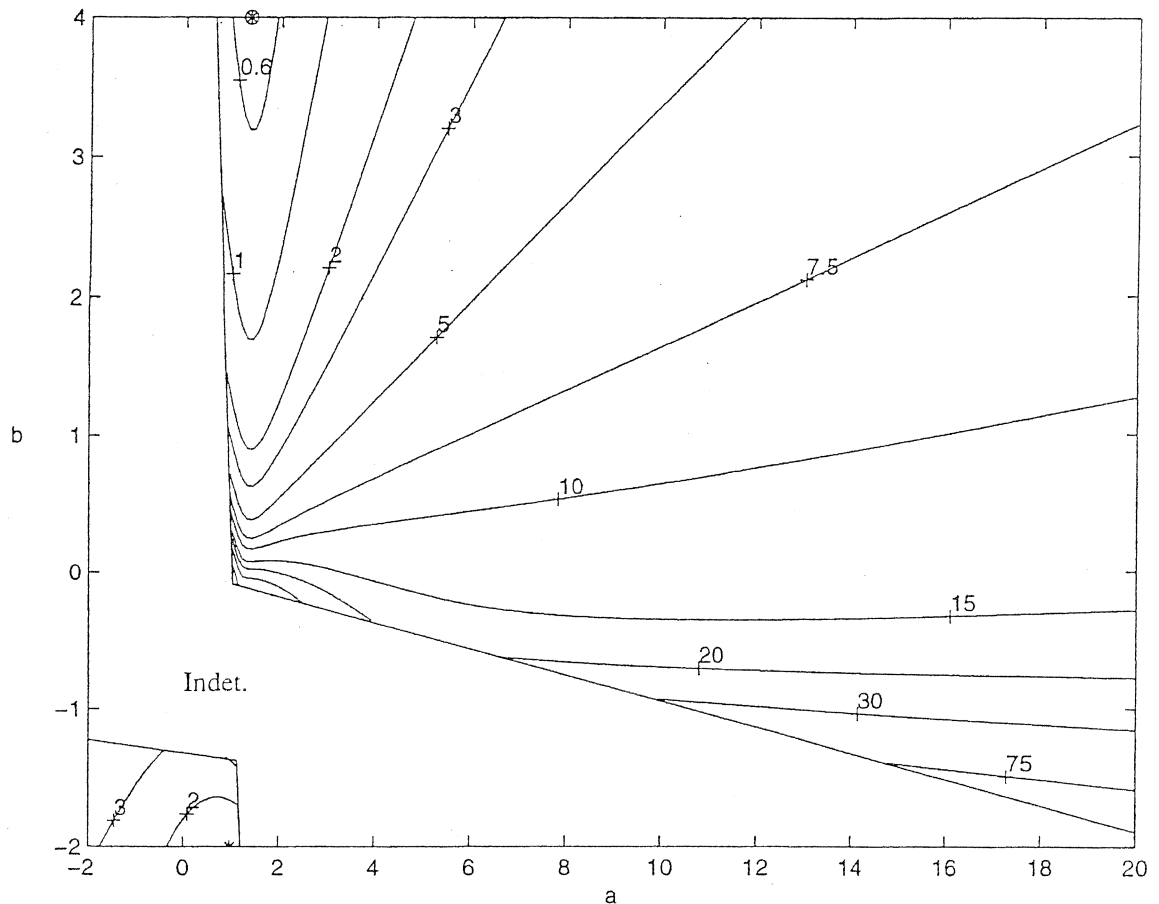
Fig. 2.8   Simple Taylor rules: $\text{var}\{\hat{Y}\}$ as a function of a and b

these two interpretations may be difficult to disentangle because we identify $\hat{Y}^s$ by measuring shifts in the empirically estimated AS equation given by (22). Unfortunately, changes in desired markups will shift this equation just as much as changes in technology or other changes in the welfare-maximizing level of output.

2.2.3   Rules That Involve a Lagged Interest Rate

We achieve improvements in household welfare if we generalize the family of simple Taylor rules to allow the funds rate to respond also to lagged values of itself. We thus consider generalized Taylor rules of the form

$$(41) \qquad \hat{r}_t = a\hat{\pi}_t + b\hat{Y}_t + c\hat{r}_{t-1},$$

where we now allow c to be greater than zero. This allows for interest rate smoothing, so that sustained changes in output and inflation lead to only gradual changes in interest rates. Actual policy in the United States and elsewhere seems to involve some degree of interest rate smoothing, though academic commentators have often questioned why this should be so.[25] Nor is there any reason to restrict attention to the case $0 \leq c \leq 1$, though only in that case can
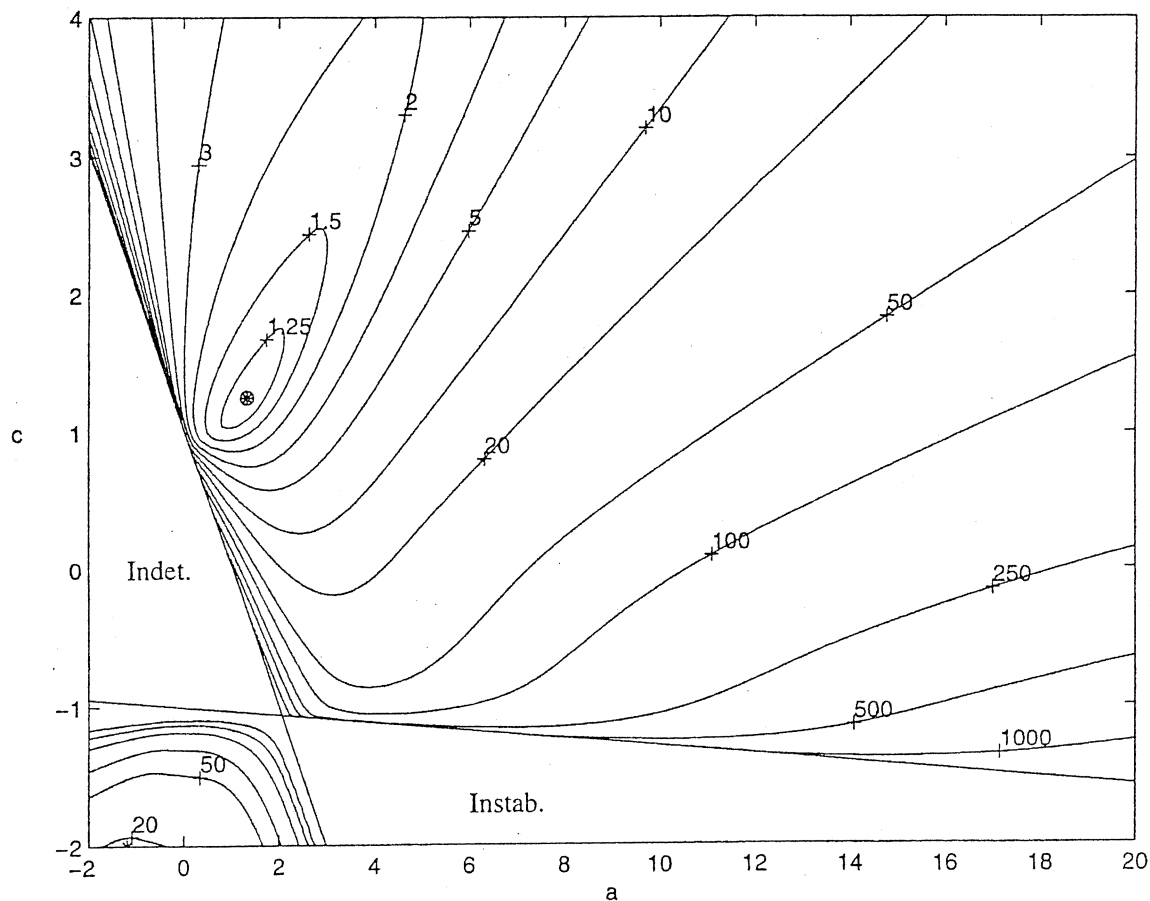
Fig. 2.14   Generalized Taylor rules: $L + \pi^{*2}$ as a function of a and c

omy as a whole can be distilled. A further difficulty with responding to con-temporaneous variables may be that, even if these are observable immediately, the political process of responding to them takes time.

None of this denies that the central bank continually updates its estimate of the current state of the economy. And it should be recalled that our model of the delays in the response of output and inflation implies that the relevant data exist in principle in the quarter *prior* to the one in which the data must be used under rules (40) and (41). However, it is reasonable to suppose that the central bank's estimate of the state of the economy generally differs from the econo-my's actual state. In this case, responding to the current estimate of the current state differs from rules (40) and (41). If rules of the form (40) and (41) are applied to the error-ridden current estimates, the interest rate is affected by the measurement error, and a thorough evaluation of these rules would require an analysis of these effects.

Thus we now suppose instead that the Federal Reserve does not respond to output and inflation variations except with a one-quarter lag. In this class of rules,
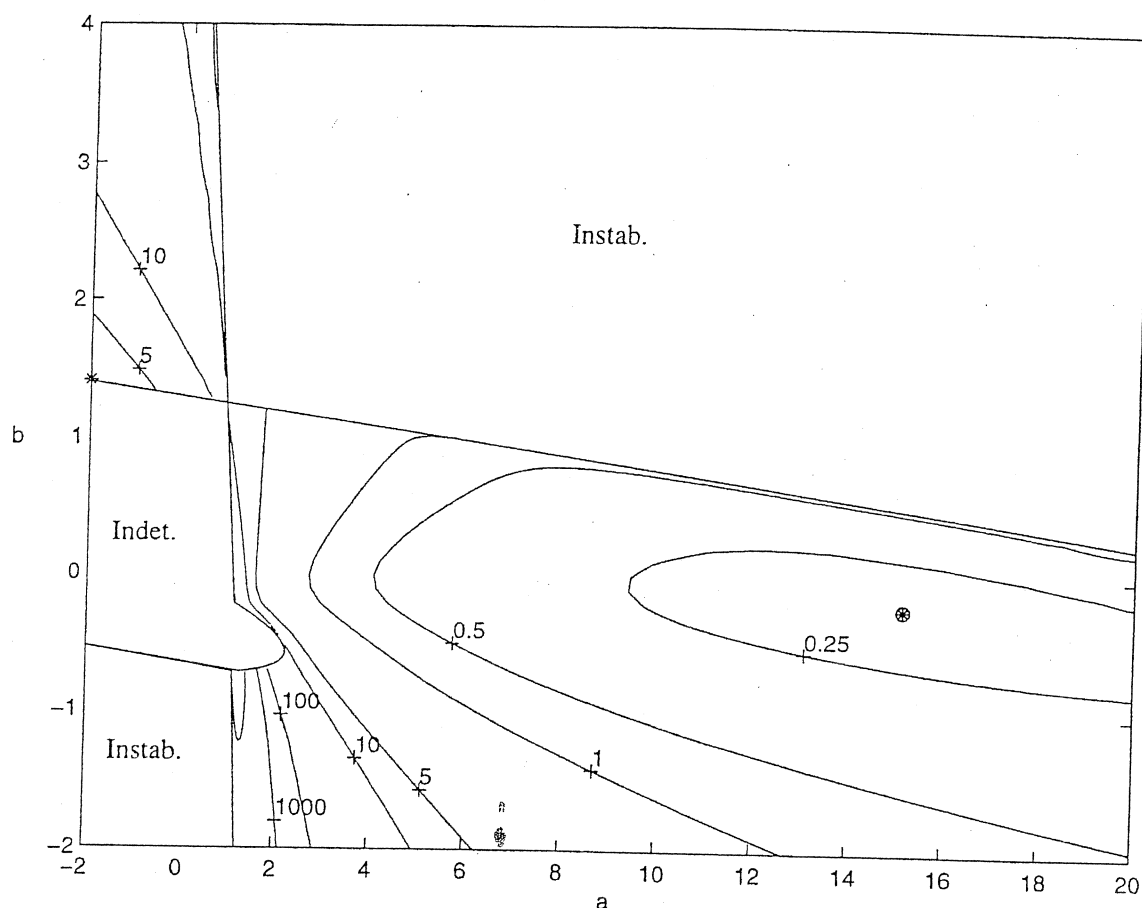
Fig. 2.15   Lagged response rules: var$\{\hat{\pi}\}$ as a function of a and b

Considering the effect of such a lag also allows us to compare our results with other papers in this volume since some of these also include the rules we label A_1 through D_1 in table 2.1.

Even if the Fed had a reasonably accurate estimate of the current state of the economy, there would be good reasons to be interested in lagged-data rules of this form. In particular, the use of such rules would make Fed operations more transparent to the public at large if the public only had this lagged information. By avoiding the use of information that the public does not have, it becomes both easier to describe Fed operations and easier for people to detect when the Fed has departed from the rule. An alternative, of course, might be to respond to internal estimates and publish these estimates of the state of the economy as they become available. The study of this alternative, and its effects on transparency given that this estimate will at least sometimes be wrong, is clearly beyond the scope of this paper.

We start in figure 2.15 by displaying how the variance of inflation varies with a and b when c is set equal to zero. This figure is quite different from figure 2.5, which involves the same parameters and performance criterion in

Cumulated Coefficients on Inflation
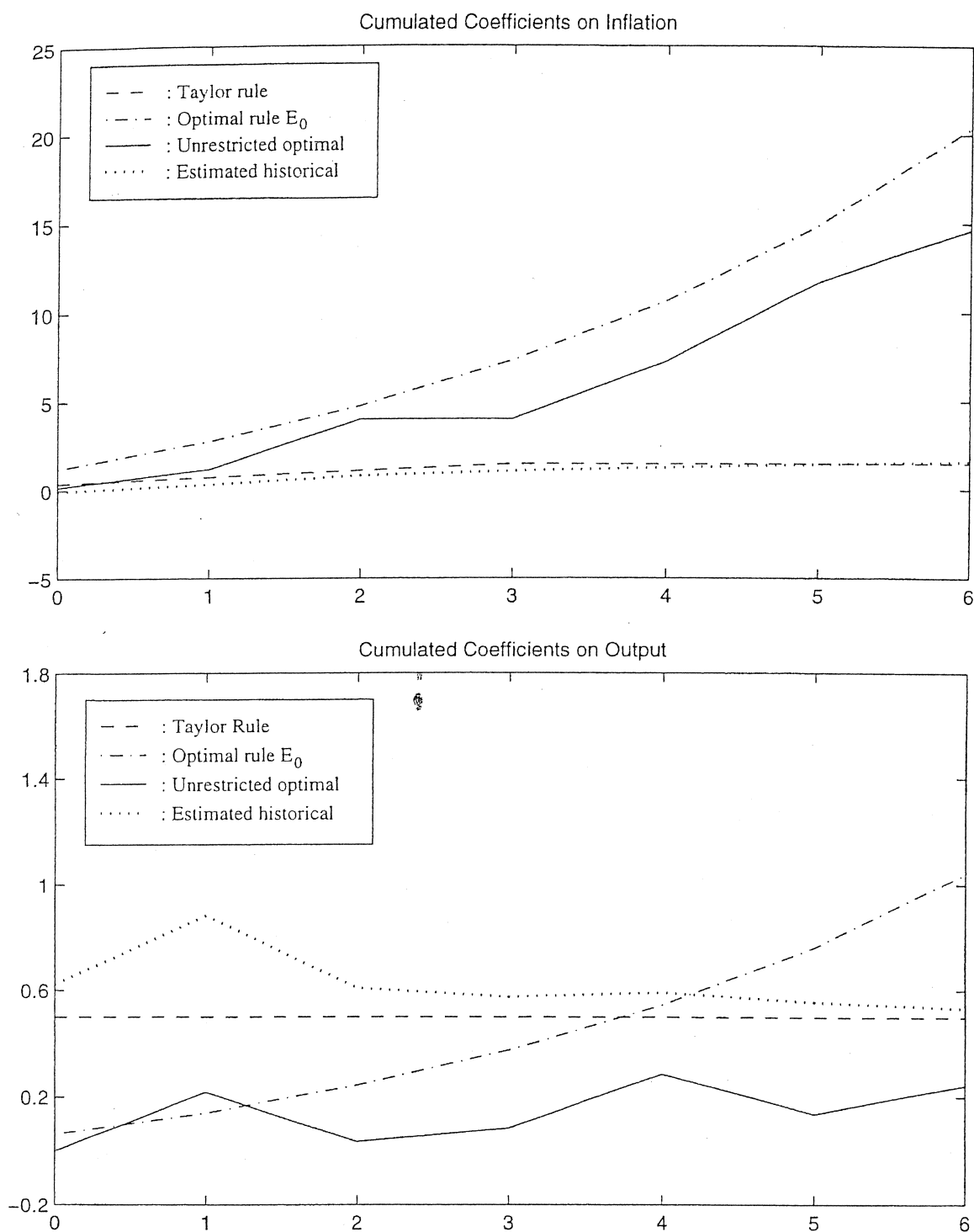
Cumulated Coefficients on Output

Fig. 2.19   Cumulative interest rate response under alternative policies

of the optimal rules. For the unrestricted optimal rule, the reaction remains more muted for the entire six-quarter horizon displayed here. This indicates an important difference between actual policy, at least as either Taylor or we have characterized it, and optimal policy according to our model: our model suggests that interest rate responses to output above trend should be much weaker, at least in the first few quarters, than they actually are. On the other hand, this does not mean that optimal policy would not involve interest rates eventually