### Recitation 7: Empirics and Theory on Monetary Policy Design

Over the last couple of weeks in lectures, we have considered optimal monetary policy in the baseline model and in models with extensions. Professor Gali provided an empirical overview of monetary policy rules in different periods, for example the US experience pre- and post-Volcker. In the context of cost-push shocks, we discussed monetary policy with and without commitment.

This recitation handout covers two papers by Clarida, Gali and Gertler (QJE 2000 and JEL 1999), and we examine the topics mentioned above in more detail.

## 1. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory (CGG, QJE 2000)

Clarida, Gali and Gertler (2000) begin by using GMM in order to estimate the coefficients on monetary policy rules for the US Federal Reserve during pre- and post-Volcker periods. Then a theoretical model is used in order to predict two real-world consequences of the change in the monetary policy rule: (i) sunspot fluctuations and (ii) volatility with respect to fundamental shocks. The authors propose that this accounts for the differential performance of inflation and output during the pre- and post-Volcker periods.

#### 1.1 GMM Estimation of a Forward-Looking Rule

The authors postulate the following linear equation for the monetary policy rule:

$$r_t^* = r^* + \beta \left( E[\pi_{t,k} | \Omega_t] - \pi^* \right) + \gamma E[x_{t,q} | \Omega_t]$$
(1)

where  $\pi_{t,k}$  denotes the average value of inflation between periods t and k, and  $x_{t,q}$  denotes the equivalent term for the deviation of output from target between periods t and q. Such a rule is optimal for a central bank with a loss function that is quadratic in the deviations of inflation and output from their respective targets. Empirically, such rules have been fairly successful in describing monetary policy as practised by the US Federal Reserve.

 $r^*$  is the *target* nominal interest rate. To make the model more consistent with actual US monetary policy, it is assumed that the *actual* nominal interest rate follows a partial adjustment process:

$$r_t = \rho(L)r_{t-1} + (1-\rho)r_t^* \tag{2}$$

Each period the Federal Reserve adjusts the nominal rate to eliminate  $(1 - \rho)$  of the gap between its current target level and some linear combination of its past values. Combining (1) and (2):

$$r_{t} = (1 - \rho) \left\{ rr^{*} - (\beta - 1)\pi^{*} + \beta\pi_{t,k} + \gamma x_{t,q} \right\} + \rho(L)r_{t-1} + \varepsilon_{t}$$
(3)

where  $\varepsilon_t \equiv -(1-\rho) \left\{ \beta(\pi_{t,k} - E[\pi_{t,k}|\Omega_t]) + \gamma(x_{t,q} - E[x_{t,q}|\Omega_t]) \right\}$ and  $rr^* = r^* - \pi^*$  is the long run equilibrium real rate.

Notice that the error term is a linear combination of forecast errors and is therefore orthogonal to any variable in the information set  $\Omega_t$ . Let  $\mathbf{z}_t$  denote a vector of instruments known when  $r_t$  is set. Equation (3) then implies the following set of orthogonality restrictions:

$$E\{(r_t - (1 - \rho)\{rr^* - (\beta - 1)\pi^* + \beta\pi_{t,k} + \gamma x_{t,q}\} - \rho(L)r_{t-1})\mathbf{z}_t\} = 0$$
(4)

This is the restriction used in the GMM estimation.

In the absence of further assumptions we can estimate  $rr^* - (\beta - 1)\pi^*$ , but not  $rr^*$  or  $\pi^*$  separately. Imposing the assumption that the observed sample average value of the real interest rate is  $rr^*$ . This enables us to estimate  $\pi^*$ .

#### 1.2 Evidence on Policy Reaction Functions Pre- and Post-Volcker

TABLE I Aggregate Volatility Indicators							
	Standard Deviation of:						
	Inflation		Output				
	Level	hp	Gap	hp			
Pre-Volcker	2.77	1.48	2.71	1.83			
Volcker-Greenspan post–82	$\begin{array}{c} 2.18 \\ 1.00 \end{array}$	$0.96 \\ 0.79$	$2.36 \\ 2.06$	1.49 1.34			

The historical evidence on economic performance is summarized as follows:

Baseline estimates for the key parameters of the monetary policy rule  $(\pi^*, \beta, \gamma, \rho)$  are presented for the case k = q = 1. CBO estimates of the output gap as used for  $x_t$ .

TABLE II Baseline Estimates							
π*	β	γ	ρ	р			
4.24	0.83	0.27	0.68	0.834			
3.58	(0.07) 2.15	0.93	0.79	0.316			
-	$   \begin{array}{r} & \\ \pi^{*} \\ \hline \\ 4.24 \\ (1.09) \\ 3.58 \\ (0.50) \end{array} $	TABLE II           BASELINE ESTIMA $\pi^*$ $\beta$ 4.24         0.83           (1.09)         (0.07)           3.58         2.15           (0.50)         (0.40)	TABLE II BASELINE ESTIMATES $\pi^*$ $\beta$ $\gamma$ 4.24         0.83         0.27           (1.09)         (0.07)         (0.08)           3.58         2.15         0.93           (0.50)         (0.40)         (0.42)	TABLE II         BASELINE ESTIMATES $\pi^*$ $\beta$ $\gamma$ $\rho$ 4.24       0.83       0.27       0.68         (1.09)       (0.07)       (0.08)       (0.05)         3.58       2.15       0.93       0.79         (0.50)       (0.40)       (0.42)       (0.04)			

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

The evidence points to substantial differences in the policy reaction function between periods. Most importantly, the coefficient for  $\beta$  is far below 1 during the pre-Volcker period and far above 1 during the post-Volcker period. The estimate for  $\gamma$  is only marginally significant for the post-Volcker period. The estimate for the smoothing parameter  $\rho$  is high in both periods, suggesting considerable interest rate inertia: only between 10 and 30 percent of a change in the interest rate target is reflected in the nominal interest rate in the quarter of the change.

The paper conducts further robustness analysis.

#### 1.3 Model of US Economy

The authors use a baseline model that is a version of those found in King and Wolman (1996), Woodford (1996, 1998) and Yun (1996), among others. The equilibrium conditions are log-linearized around a zero inflation steady state to yield:

$$\pi_t = \delta E[\pi_{t+1}|\Omega_t] + \lambda(y_t - z_t) \tag{5}$$

$$y_t = E[y_{t+1}|\Omega_t] - \frac{1}{\sigma}(r_t - E[\pi_{t+1}|\Omega_t]) + g_t$$
(6)

$$r_t^* = \beta E[\pi_{t+1}|\Omega_t] + \gamma x_t \tag{7}$$

$$r_t = \rho r_{t-1} + (1-\rho)r_t^* \tag{8}$$

 $y_t - z_t$  is the deviation of output from the natural value. We restrict ourselves to k = q = 1.

#### **1.4 Sunspot Fluctuations**

If the coefficient  $\beta$  is far below unity, the equilibrium will be indeterminate. In this sense, the policy feedback rule itself may be a source of macroeconomic instability. Under indeterminacy there may be sunspot fluctuations, i.e. macroeconomic fluctuations that occur because they are expected to occur and not due to changes in fundamentals. The intuition is as follows. If  $\beta$  is less than 1, an increase in expected inflation induces a rise in the nominal interest rate from (7), but not of a sufficient magnitude to prevent the real interest rate from falling. The fall in the real interest rate stimulates the economy and generates higher inflation, from equation (5). Therefore the expectations of higher inflation are self-fulfilling.

What do these sunspot fluctuations look like? The authors draw the "sunspot shocks" from a standard Normal distribution, as detailed in their 1997 working paper. The impulse response to a sunspot shock are as follows:





These are not too dissimilar to the actual behaviour of the US economy.



#### **1.5 Near-Indeterminacy and Fundamental Shocks**

In some of the specifications in the paper, the standard errors for the estimate of  $\beta$  are too large to rule out that its true value is unity. In this case, it is possible that the economy is just outside the region of indeterminacy.

However, this does not mean that there will be no difference in the performance of the economy between the pre- and post-Volcker regimes. Even though sunspot fluctuations are no longer feasible in the economy, in an economy buffetted by shocks inflation will be more unstable in the regime where the estimate of  $\beta$  is lower, i.e. in the pre-Volcker regime.

TABLE VII Fundamental Shocks								
		Supply shocks		Demano	Demand shocks			
β	$\sigma(\pi)$	$\sigma(x)$	$\sigma(y)$	$\sigma(\pi)$	$\sigma(y)$			
2.0	1.00	1.00	1.00	1.00	1.00			
1.5	1.48	1.36	1.29	1.61	1.67			
1.1	2.57	2.16	2.26	3.04	1.96			
1.0	3.20	2.61	2.88	3.88	4.25			

The cyclical response of the economy to fundamental shocks is quite sensitive to  $\beta$ . The increase in  $\beta$  from 1 to 2 causes the volatility of both output and inflation to fall by more than a half, for *both* supply and demand shocks. If  $\beta$  is low, the central bank comes close to fully accommodating the inflationary shock, and the absence of a stabilizing movement from the real rate of interest means that the economy fluctuates more in response to the shock.

Look at the impulse response functions below for the response to a supply shock. The key point is that the shock produces a persistent effect on inflation only for values of  $\beta$  near 1.





Again, this may explain the difference in economic performance between pre- and post-Volcker periods.

# 2. The Science of Monetary Policy: A New Keynesian Perspective (CGG, JEL 1999)

Clarida, Gali and Gertler (1999) survey many aspects of the monetary policy literature. This handout focuses upon the determination of optimal policy under various assumptions of the degree of credible commitment by the central bank. We will use the notation from the lecture notes rather than the paper.

The problem of the central bank is to minimize its loss function subject to the Phillips Curve:

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \alpha \widetilde{y}_t^2 + \pi_t^2 \right] \right\}$$
(9)

s.t. 
$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \widetilde{y}_t + u_t$$
(10)

And the interest rate is given by substituting the solution to the above problem into the IS equation:

$$\widetilde{y}_{t} = E_{t} \left\{ \widetilde{y}_{t+1} \right\} - \frac{1}{\sigma} \left( r_{t} - E_{t} \left\{ \pi_{t+1} \right\} - \overline{r} \overline{r}_{t} \right)$$

$$\tag{11}$$

#### 2.1 Optimal Policy with Discretion

This has been covered in the lecture notes "Optimal Policy with a Cost-Push Shock," Section 1.2. The central bank has no ability to influence expectations about the future, so it takes the private sector's expectations as given in each period.

$$\min \left[ \alpha \widetilde{y}_t^2 + \pi_t^2 \right]$$
  
s.t.  $\pi_t = \kappa \widetilde{y}_t + \theta_t$ 

where  $\theta_t \equiv \beta E_t \{\pi_{t+1}\} + u_t$  is taken as given.

As in the lecture notes, the optimality condition for this problem is:

$$\widetilde{y}_t = -\frac{\kappa}{\alpha} \pi_t \tag{12}$$

Let  $q \equiv \frac{1}{\kappa^2 + \alpha(1 - \beta \rho_u)}$ . Then under the optimal time consistent policy:

$$\widetilde{y}_t = -\kappa q u_t \tag{13}$$

$$\pi_t = \alpha q u_t \tag{14}$$

#### 2.2 Classical Inflation Bias

The classical inflation bias problem was formulated by Kydland and Prescott (1979) and Barro and Gordon (1983). It considers the possibility that the central bank's target for the output gap is not 0 but k > 0. The central bank solves:

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \alpha \left( \widetilde{y}_t - k \right)^2 + \pi_t^2 \right] \right\}$$
  
s.t.  $\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa \widetilde{y}_t + u_t$ 

Then the optimality condition under discretion is:

$$\widetilde{y}_t^k = -\frac{\kappa}{\alpha} \pi_t^k + k \tag{15}$$

where the superscript k highlights the fact that the objective function has been amended to allow for k > 0. This condition (15) can be used to characterize the optimal output gap and inflation, then we compare to the solution without a positive target for the output gap.

$$\widetilde{y}_t^k = \widetilde{y}_t \tag{16}$$

$$\pi_t^k = \pi_t + \frac{\alpha}{\kappa}k \tag{17}$$

The key point is as follows. A central bank with a positive output gap target does not in fact succeed in raising the output gap in equilibrium. Rather, the inflation simply rises, by the amount  $\frac{\alpha}{\kappa}k$ . Note that we have assumed  $\beta = 1$ .

A central banker with a higher weight on inflation relative to output will have a lower  $\alpha$ , and hence a lower inflation bias term. That appointing a conservative central banker can lead to welfare gains for society was proposed by Rogoff (1985). There are gains from increasing credibility. If commitment by the central bank is possible, then it will enhance welfare by allowing the central bank to commit to a response akin to the weight conservative central bank.

#### 2.3 Optimal Policy with Commitment

But the ability to commit leads to welfare gains even if k = 0. This was illustrated in the lecture notes in class. I briefly summarize the main results.

The monetary authority chooses a state-contingent policy  $\left\{ \tilde{y}_t, \pi_t \right\}_{t=0}^{\infty}$  to solve:

$$\max -\frac{1}{2}E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \alpha \widetilde{y}_t^2 + \pi_t^2 \right] \right\}$$
  
i.t.  $\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa \widetilde{y}_t + u_t \text{ for } t = 0, 1, 2, .$ 

The Lagrangean can be set up:

$$\max L = -\frac{1}{2}E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \alpha \widetilde{y}_t^2 + \pi_t^2 + \gamma_t \left( \pi_t - \kappa \widetilde{y}_t - \beta \pi_{t+1} \right) \right] \right\}$$

The solution obtained can be represented:

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$$\widetilde{y}_0 = -\frac{\kappa}{\alpha} \pi_0 \tag{19}$$

$$\widetilde{y}_t = \widetilde{y}_{t-1} - \frac{\kappa}{\alpha} \pi_t \text{ for } t = 1, 2, 3, \dots$$
(20)

This is discussed in the lecture notes. Intuitively, there is history dependence because by committing to respond to the current shock partly today and partly in the future, the tradeoff between inflation and output is more favourable today.

#### 2.4 Optimal Policy within a Simple Family of Policy Rules

Now we allow an intermediate degree of commitment, something that is not covered in the lecture notes. To be specific, the central bank can commit to follow a policy where the output gap depends *linearly* upon the *contemporaneous* shock term:

$$\widetilde{y}_t^c = -\omega u_t \tag{21}$$

Notice that such a rule includes the optimum under discretion as a special case. (Therefore welfare must be weakly higher than in the discretion case.) Combining equation (21) with the Phillips Curve implies that inflation under this rule is also linear in the cost-push shock:

$$\begin{aligned} \pi_t^c &= \beta E_t \left\{ \pi_{t+1}^c \right\} + \kappa \widetilde{y}_t^c + u_t \\ &= E_t \left\{ \sum_{i=0}^\infty \beta^i \left[ \kappa \widetilde{y}_{t+i}^c + u_{t+i} \right] \right\} \\ &= E_t \left\{ \sum_{i=0}^\infty \beta^i \left[ -\kappa \omega u_{t+i} + u_{t+i} \right] \right\} \\ &= \frac{1 - \kappa \omega}{1 - \beta \rho_u} u_t \end{aligned}$$

It is possible to express this as:

$$\pi_t^c = \frac{\kappa}{1 - \beta \rho_u} \widetilde{y}_t^c + \frac{1}{1 - \beta \rho_u} u_t \tag{22}$$

We now find the optimal value of  $\omega$  by solving the following problem:

$$\max -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \alpha \left( \widetilde{y}_{t+i}^c \right)^2 + \left( \pi_{t+i}^c \right)^2 \right] \right\}$$
$$= -\frac{1}{2} \left[ \alpha \left( \widetilde{y}_t^c \right)^2 + \left( \pi_t^c \right)^2 \right] L_t$$
s.t. Equation (22)

where  $L_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ u_{t+i}/u_t \right]^2 \right\} > 0.$ The optimality condition is:

$$\widetilde{y}_t^c = -\frac{\kappa}{\alpha^c} \pi_t^c \tag{23}$$

where  $\alpha^c \equiv \alpha(1 - \beta \rho) < \alpha$ .

Since  $\alpha^c < \alpha$ , commitment takes the form of promising to engineer a greater contraction in output in response to inflationary shocks. There is a more aggressive response to prevent inflation from changing much from its target value. From a policy standpoint, Rogoff's (1985) point about a weight conservative central banker carries over to this case. The central bank can secure a higher level of welfare if it promises to respond more vigorously to an inflationary shock than the economy's loss function would warrant. One way in which such a commitment could be made is to appoint a central banker with preferences different from the general population, in particular one that assigns more relative weight on controlling inflation rather than output.