Lectures on Monetary Policy, Inflation, and the Business Cycle

Chapter 3:
Flexible Prices

preliminary and incomplete
do not circulate

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1 Optimal Price Setting with Flexible Prices

Consider, as in chapter 2, a monopolistically competitive firm producing a differentiated good with a technology

$$Y_t(i) = A_t \ N_t(i)$$

and facing a demand schedule

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \ C_t \tag{1}$$

Under the assumption that the firm can readjust the price anytime at no cost, each period the firm will maximize current profits

$$Y_t(i) \ \left(P_t(i) - \frac{W_t}{A_t}\right)$$

subject to current demand (1). The corresponding optimality condition requires setting the price according to the constant markup rule:

$$\begin{aligned} P_t(i) &= \frac{\epsilon}{\epsilon - 1} \frac{W_t}{A_t} \\ &= \frac{\epsilon}{\epsilon - 1} \ MC_t(i) \ P_t \end{aligned}$$

for all $i \in [0,1]$ and $t = 0, 1, 2, ...$We can rewrite the previous price-setting rule in log-linear terms as:

$$p_t(i) = \mu + (mc_t(i) + p_t)$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon - 1}$.

Under our assumptions on technology, marginal cost is common across firms, so we can write $mc_t(i) = mc_t$. Furthermore, in a symmetric equilibrium all firms will follow an identical rule, implying $p_t(i) = p_t$, for all $i \in [0,1]$. It follows that

$$mc_t = -\mu \tag{2}$$

for all t. Next we combine the previous equilibrium condition with the remaining conditions derived in the previous chapter.

2 Equilibrium under Flexible Prices

Recall the "common" equilibrium conditions derived in chapter 2:

$$y_t = n_t + a_t \tag{3}$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(\widehat{r}_t - E_t\{\pi_{t+1}\}) \tag{4}$$

$$mc_t = (\sigma + \varphi)\; y_t - (1 + \varphi)\; a_t \tag{5}$$

When needed we will also invoke the money market clearing condition

$$m_t - p_t = y_t - \eta\; \widehat{r}_t \tag{6}$$

and which was derived in the previous chapter.

Combining the previous conditions with condition (5) derived above we obtain the following expressions for equilibrium output, employment and the (ex-ante) real interest rate

$$\overline{y}_t = \gamma + \psi_a\; a_t \tag{7}$$

$$\overline{n}_t = \gamma + (\psi_a - 1)\; a_t \tag{8}$$

$$\begin{aligned} \overline{rr}_t &= \rho + \sigma\; E_t\{\Delta y_{t+1}\} \\ &= \rho + \sigma\psi_a\; E_t\{\Delta a_{t+1}\} \end{aligned} \tag{9}$$

where $\gamma \equiv -\frac{\mu}{\sigma+\varphi} < 0$ and $\psi_a \equiv \frac{1+\varphi}{\sigma+\varphi} > 0$, and where the upper-bar is meant to stress the fact that these are equilibrium values under the assumption of flexible prices.

Notice that the equilibrium dynamics of employment, output, and the real interest rate are determined *independently of monetary policy.* In other words, in monetary policy is *neutral* with respect to those real variables. As discussed below, the previous equilibrium values are an important benchmark when we depart from the assumption of flexible prices and are referred as *natural* levels of output, employment and the real rate.

On the other hand, determination of the equilibrium behavior of nominal variables requires that we specify how monetary policy is conducted. Next we consider several monetary policy rules and their implied outcomes.

3 Price Level Determination under Flexible Prices

We start by examining the implications of some interest rate rules. Later we introduce rules that involve monetary aggregates.

3.0.1 An Exogenous Path for the Nominal Interest Rate

Assume that the nominal interest rate follows an *exogenous* stationary process $\{r_t\}$ with mean ρ (thus consistent with zero inflation in the steady state). Notice that a particular case of this rule corresponds to a constant interest rate $r_t = r$ all t.

Equilibrium condition (9) can be rewritten as

$$E_t\{\pi_{t+1}\} = r_t - \overline{rr}_t$$

where, as discussed above, $\{\overline{rr}_t\}$ is determined independently of the policy rule.

Hence, we see that expected inflation is pinned down by the previous equation, but actual inflation is not. In fact, any path for the price level which satisfies

$$p_{t+1} = p_t + r_t - \overline{rr}_t + \xi_{t+1}$$

where $E_t\{\xi_{t+1}\} = 0$ for all t is consistent with equilibrium. In other words, under an exogenous nominal rate rule the economy displays *price level indeterminacy*. Notice that when (6) is operative the equilibrium path for the money supply (which is endogenous under the present rule) is given by

$$m_t = p_t + \overline{y}_t - \eta\ \widehat{r}_t$$

and hence it inherits the indeterminacy of p_t.

3.0.2 A Simple Taylor Rule

Suppose that the central bank adjusts the nominal interest rate according to the rule

$$r_t = \rho + \phi_\pi\ \pi_t$$

Combining the previous rule with (4) we obtain

$$\phi_\pi \; \pi_t = E_t\{\pi_{t+1}\} + \sigma E_t\{\Delta \overline{y}_{t+1}\} \tag{10}$$

If $\phi_\pi > 1$, there is only one stationary solution to the previous difference equation (i.e. a solution that remains in a neighborhood of the steady state). That solution can be obtained by solving (10) forward, which yields

$$\pi_t = \sigma \sum_{k=1}^{\infty} \left(\frac{1}{\phi_\pi}\right)^k \; E_t\{\Delta \overline{y}_{t+k}\}$$

Hence, if $\{a_t\}$ follows a stationary AR(1) process

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a$$

with $\rho_a \in [0,1]$ we will have $E_t\{\Delta \overline{y}_{t+k}\} = -(1-\rho_a)\rho_a^{k-1} \; \psi_a \; a_t$ and hence

$$\pi_t = -\frac{(1-\rho_a) \; \psi_a}{\phi_\pi - \rho_a} \; a_t$$

On the other hand, if $\phi_\pi < 1$, any process π_t satisfying

$$\pi_{t+1} = \phi_\pi \; \pi_t - \sigma E_t\{\Delta \overline{y}_{t+1}\} + \xi_{t+1}$$

where $E_t\{\xi_{t+1}\} = 0$ for all t is consistent with equilibrium, while remaining in a neighborhood of the steady state (*price level indeterminacy*).

Notice that the condition for a determinate price level, $\phi_\pi > 1$, requires that the central bank adjust nominal interest rates more than one-for-one in response to any change in inflation. The previous result can be viewed as a particular instance of an interest rate rule being required to satisfy the so-called *Taylor principle* in order to guarantee a determinate equilibrium.

3.0.3 An Exogenous Path for Money Growth

Suppose that the central bank sets an exogenous path for the money supply $\{m_t\}$. We can combine (4) and (6) in order to eliminate the nominate, and obtain the following difference equation for the price level:

$$p_t = \left(\frac{\eta}{1+\eta}\right) \; E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) \; m_t + u_t$$

where $u_t \equiv -\frac{1}{1+\eta}\ \overline{y}_t + \frac{\sigma\eta}{1+\eta}\ E_t\{\Delta\overline{y}_{t+1}\}$ evolves independently of $\{m_t\}$.

Assuming $\eta > 0$ and solving forward we obtain:

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \ E_t\left\{m_{t+k}\right\} + u_t'$$

where $u_t' \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \ E_t\left\{u_{t+k}\right\}$ is, again, independent of monetary policy.

Equivalently, we can be rewrite the previous expression in terms of expected future growth rate of money:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \ E_t\left\{\Delta m_{t+k}\right\} + u_t' \tag{11}$$

Hence, we see how an arbitrary exogenous path for the money supply always determines the price level uniquely. Given the price level, as determined above, we can then use (6) to solve for the nominal interest rate:

$$\begin{aligned} \widehat{r}_t &= \eta^{-1}\ [\overline{y}_t - (m_t - p_t)] \\ &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \ E_t\left\{\Delta m_{t+k}\right\} + u_t^{"} \end{aligned}$$

where $u_t^{"} \equiv \eta^{-1}(u_t' + \overline{y}_t)$.

Example. Consider the case in which money growth follows an AR(1) process.

$$\Delta m_t = \rho_m\ \Delta m_{t-1} + \varepsilon_t^m$$

For simplicity let's assume a constant technology, implying a constant $\overline{y}_t$ which we can normalize to zero, without loss of generality. Then it follows from (11) that

$$p_t = m_t + \frac{\eta\rho_m}{1+\eta(1-\rho_m)}\ \Delta m_t$$

Hence, in response to an exogenous shock to the money supply, and as long as $\rho_m > 0$ (as seems empirically relevant), the price level should respond more then one-for-one with the increase in the money supply, a prediction which contrasts starkly with the sluggish response of the price level observed in empirical estimates of the effects of monetary policy shocks.

The nominal interest rate will in turn be given by

$$\widehat{r}_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \, \Delta m_t$$

i.e. in response to an expansion of the money supply, an as long as $\rho_m > 0$, the nominal interest rate is predicted to go up. In other words, the model implies the absence of a *liquidity effect*, in contrast with much of the evidence.