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Non Separable Utility: An Example

Households

Preferences

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X(C_t, M_t/P_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$\begin{aligned} X(C_t, M_t/P_t) &\equiv \left[(1-\vartheta)C_t^{1-\nu} + \vartheta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \qquad for\ \nu \neq 1 \\ &\equiv C_t^{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{\vartheta} \qquad for\ \nu = 1 \end{aligned}$$

Optimality conditions

$$\frac{W_t}{P_t} = N_t^{\varphi}\ X_t^{\sigma-\nu}\ C_t^{\nu}\ (1-\vartheta)^{-1}$$

$$1 = \beta R_t\ E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\nu}\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\sigma}\ \Pi_{t+1}^{-1}\right\}$$

$$\frac{M_t}{P_t} = C_t\ (1-R_t^{-1})^{-\frac{1}{\nu}}\left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{1}{\nu}}$$

Log-linearized optimality conditions:

- Money Demand:

$$m_t - p_t = c_t - \eta \ \widehat{r}_t$$

where $\eta \equiv [\nu(R-1)]^{-1}$.

- Labor Supply:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \\ &= \sigma c_t + \varphi n_t + \chi(\nu - \sigma) \ [c_t - (m_t - p_t)] \\ &= \sigma c_t + \varphi n_t + \chi\eta(\nu - \sigma) \ \widehat{r}_t \end{aligned}$$

where $\chi \equiv \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}+\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}} \in [0,1)$.

Define $S_m \equiv \frac{M/P}{C}$. Using the money demand equation we have

$$S_m = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)} \right)^{\frac{1}{\nu}}$$

Hence, we can write

$$w_t - p_t = \sigma c_t + \varphi n_t + \omega \ \widehat{r}_t$$

where $\omega \equiv \frac{S_m \ \beta}{1+S_m(1-\beta)}(1 - \frac{\sigma}{\nu})$.

- Consumption/Saving:

$$\begin{aligned} c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}[\widehat{r}_t - E_t\{\pi_{t+1}\} - (\nu - \sigma) \ E_t\{(c_{t+1} - x_{t+1}) - (c_t - x_t)\}] \\ &= E_t\{c_{t+1}\} - \frac{1}{\sigma}[\widehat{r}_t - E_t\{\pi_{t+1}\} - \chi(\nu - \sigma) \ E_t\{\Delta c_{t+1} - \Delta(m_{t+1} - p_{t+1})\}] \\ &= E_t\{c_{t+1}\} - \frac{1}{\sigma}[\widehat{r}_t - E_t\{\pi_{t+1}\} - \omega \ E_t\{\Delta \widehat{r}_{t+1}\}] \end{aligned}$$

Flexible Price Equilibrium

$$\begin{aligned} -\mu &= mc_t \\ &= (\sigma + \varphi)\; y_t - (1+\varphi)\; a_t + \omega\; \widehat{r}_t \end{aligned}$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}[\widehat{r}_t - E_t\{\pi_{t+1}\} - \omega\; E_t\{\Delta\widehat{r}_{t+1}\}]$$

$$m_t - p_t = y_t - \eta\; \widehat{r}_t$$

Notice that:

$$y_t = \gamma + \psi_a\; a_t - \psi_r\; \widehat{r}_t$$

where $\psi_r \equiv \frac{\omega}{\sigma+\varphi}$.

$\Longrightarrow$ monetary non-neutrality

$\Longrightarrow$ determination of real variables requires specification of monetary policy

Quantitative Analysis (Walsh, Yun)

Exogenous process for money

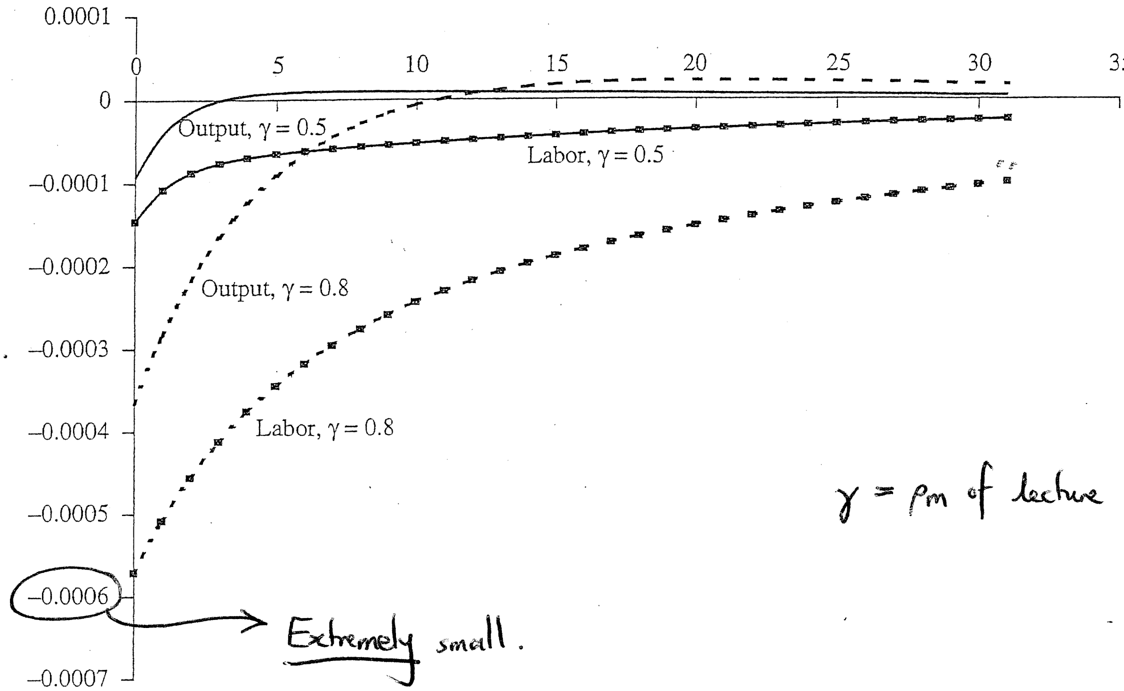
$$\Delta m_t = (1-\rho_m)\gamma_m + \rho_m\; \Delta m_{t-1} + \varepsilon_t^m$$

Impulse responses

From: Carl E. Walsh (2003), Monetary Theory and Policy, Second Edition
Chapter 2, pgs 78, 79.

Money growth
⇒ Nom int rate ↑
(Fisher effect)
⇒ Cons, output, empl. ↓.

NOT like in empirics.



γ = ρm of lecture notes.

Figure 2.3
Output and Labor Responses to a Money Growth Shock

SMALL
+ OPP. DIR.
+ FAILS TO GENERATE
LIQUIDITY EFFECT.

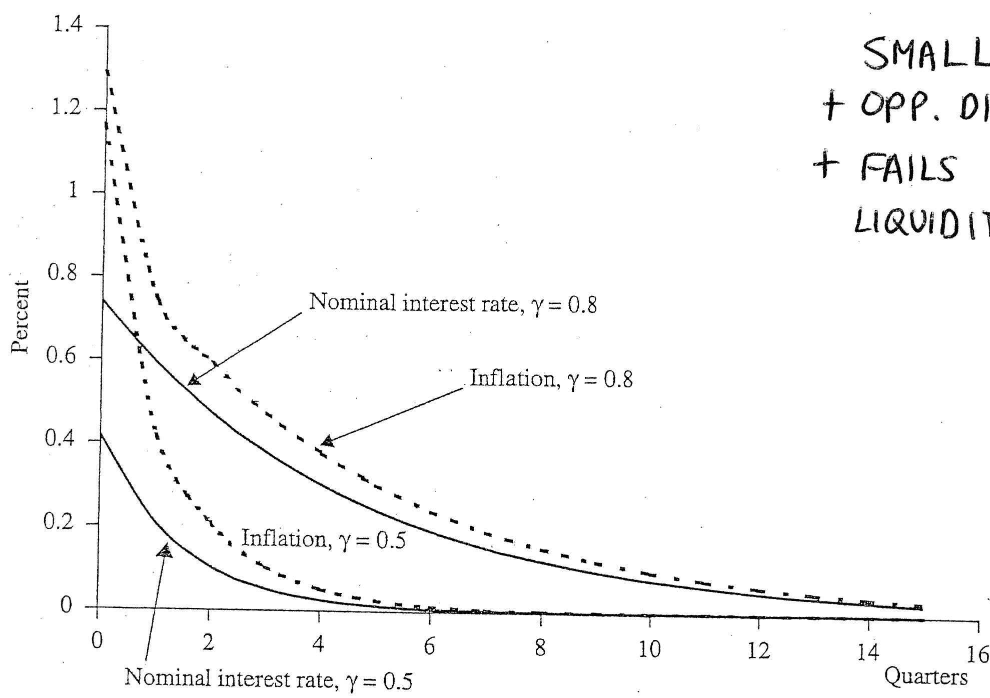


Figure 2.4
Nominal Interest Rate and Inflation Response to a Money Growth Shock (solid lines, nominal interest rate response; dashed lines, inflation response)