Lecture Notes on Monetary Economics

June 2005

1. Staggered Price Setting: the Calvo Model

1.1. Optimal Price Setting and Aggregate Price Dynamics

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(i) = A_t \; N_t(i) \tag{1.1}$$

and faces an isoelastic demand schedule.

Following the formalism introduced in Calvo (1983), we assume that each firm may reset its price only with probability $1-\theta$ each period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged.

Aggregate prices follow the law of motion:

$$p_t = \theta \; p_{t-1} + (1-\theta) \; p_t^*$$

implying

$$\pi_t = (1-\theta) \; (p_t^* - p_{t-1}) \tag{1.2}$$

Let $p_t^*(i)$ denote the (log) price set by a firm i adjusting its price in period t. If the were no constraints on the adjustment of prices the typical firm would set a price according to the rule $p_t^*(i) = \mu + mc_t^n(i)$, for all t.

Under the Calvo price-setting structure $p_{t+k}(i) = p_t^*(i)$ with probability θ^k for $k = 0, 1, 2, \ldots$ Hence when setting their price firms will have to be forward-looking. In a neighborhood of the zero inflation steady state, the optimal price seting rule can be shown to be given, up to a first order approximation, by:

$$p_t^*(i) = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k}^n(i)\} \tag{1.3}$$

Thus, firms will set a price equal to a markup μ over a weighted average of expected future nominal marginal costs, with the weights associated with each horizon k proportional to the probability that the chosen price remains effective k periods ahead.[1]

Using the fact that all firms resetting prices in period t will choose the same price p_t^* we can rewrite (1.3) as:

$$
\begin{aligned}
p_t^* - p_{t-1} &= \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{mc_{t+k} + (p_{t+k} - p_{t-1})\} \\
&= \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{mc_{t+k}\} + \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{\pi_{t+k}\} \\
&= (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{\widehat{mc}_{t+k}\} + \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{\pi_{t+k}\}
\end{aligned}
$$

where $\widehat{mc}_t \equiv mc_t - mc$, and $mc = -\mu$.

More compactly:

$$p_t^* - p_{t-1} = \beta\theta \ E_t\{p_{t+1}^* - p_t\} + (1-\beta\theta) \ \widehat{mc}_t + \pi_t$$

Combined with (1.2), yields the inflation equation:

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \lambda \ \widehat{mc}_t \tag{1.4}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

[1] A rigorous derivation of the optimal price-setting rule can be found in Yun (1996) or Woodford (1996), among others.

1.1.1. Extension with Decreasing Returns

Suppose that the individual firm's technology is given instead by

$$Y_t(i) = A_t \; N_t(i)^{1-\alpha}$$

The optimal price-setting rule takes into account that marginal cost is no longer common across firms:

$$p_t^*(i) = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t,t+k}^n\}$$

where $mc_{t,t+k}^n$ is the (log) marginal cost in period $t+k$ of a firm which last set its price in period t. Notice that $MC_{t,t+k} \equiv \frac{MC_{t,t+k}^n}{P_{t+k}}$ is given by

$$\begin{aligned} MC_{t,t+k} &= \frac{(W_{t+k}/P_{t+k})}{(1-\alpha)(Y_{t,t+k}/N_{t,t+k})} \\ &= MC_{t+k} \; \frac{(Y_{t+k}/N_{t+k})}{(Y_{t,t+k}/N_{t,t+k})} \\ &= MC_{t+k} \left(\frac{Y_{t,t+k}}{Y_{t+k}} \right)^{\frac{\alpha}{1-\alpha}} \\ &= MC_{t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{\frac{-\epsilon \; \alpha}{1-\alpha}} \end{aligned}$$

where the third equality uses the fact that $\frac{Y_t(i)}{N_t(i)} = A_t^{\frac{1}{1-\alpha}} \; Y_t(i)^{-\frac{\alpha}{1-\alpha}}$. Thus, in logs, we have

$$mc_{t,t+k} = mc_{t+k} - \frac{\epsilon\alpha}{1-\alpha} \; (p_t^* - p_{t+k})$$

Combining the latter expression with the optimal setting rule, and after some tedious algebra, we can derive the inflation equation with the average real marginal cost as a driving force:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda_\alpha \; \widehat{mc}_t$$

where $\lambda_\alpha \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \; \frac{1-\alpha}{1+\alpha(\epsilon-1)} < \lambda$.

1.2. Equilibrium

The equilibrium (log) real marginal cost is given by

$$mc_t = (\sigma + \varphi)\ y_t - (1 + \varphi)\ a_t$$

Under *flexible prices*, $mc_t = -\mu \equiv mc$, all t. We define the natural level of output $\overline{y}_t$ implicitly by:

$$mc = (\sigma + \varphi)\ \overline{y}_t - (1 + \varphi)\ a_t$$

thus implying

$$\begin{aligned} \overline{y}_t &= -\left(\frac{\mu}{\sigma + \varphi}\right) + \left(\frac{1 + \varphi}{\sigma + \varphi}\right)\ a_t \\ &\equiv y + \psi_a\ a_t \end{aligned}$$

Thus, it follows that

$$\widehat{mc}_t = (\sigma + \varphi)\ (y_t - \overline{y}_t) \tag{1.5}$$

where $y_t - \overline{y}_t \equiv \widetilde{y}_t$ is referred to in the literature as the *output gap.*

Combining (1.5) with (1.4) we obtain the so called *New Keynesian Phillips Curve*

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t \tag{1.6}$$

where $\kappa \equiv \lambda(\sigma + \varphi)$

Notice that we can also rewrite now the IS equation in terms of the output gap:

$$\widetilde{y}_t = -\frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t) + E_t\{\widetilde{y}_{t+1}\} \tag{1.7}$$

where

$$\begin{aligned} \overline{rr}_t &\equiv \rho + \sigma\ E_t\{\Delta \overline{y}_{t+1}\} \\ &= \rho - \sigma\psi_a(1 - \rho_a)\ a_t \end{aligned}$$

is the *natural rate of interest* (i.e., the one that would obtain under flexible prices).

1.2.1. Extension

Derive the NKPC, the NIS equation and the money market condition in terms of the output gap when the goods market clearing condition is given by $y_t = c_t + g_t$.

1.3. Equilibrium under a Simple Interest Rate Rule

Consider the simple interest rate rule:

$$r_t = v_t + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t \tag{1.8}$$

where v_t is an exogenous (possibly stochastic) component with mean ρ.

Combining (1.6), (1.7), and (1.8) we can represent the equilibrium conditions by means of the following system of difference equations.

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\overline{rr}_t - v_t) \tag{1.9}$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa+\beta(\sigma+\phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}$.

The solution to (1.9) is locally unique if $\mathbf{A}_T$ has both eigenvalues within the unit circle. If we restrict ourselves to non-negative values for (ϕ_π, ϕ_y) a necessary and sufficient condition is given by:

$$\kappa \, (\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

which we assume to hold for the time being.

1.3.1. The Effects of an Exogenous Monetary Policy Shock

Let v_t follow an AR(1) process

$$v_t = \rho_v \, v_{t-1} + \varepsilon_t^v$$

Calibration (Walsh (2003)): $\rho_v = 0.5$, $\phi_\pi = 1.5$, $\phi_y = 0$, $\beta = 0.99$, $\sigma = \varphi = 1$, $\theta = 0.8$.

Dynamic effects of an exogenous increase in the nominal rate (Figure 5.3).

1.3.2. The Effects of an Exogenous Non-Monetary Shock

Need to determine implied process for the natural rate and simulate effects of shock (set $v_t = \rho$)

Example: technology shock, with AR(1) process for technology parameter. Implied process for natural rate:

$$\overline{rr}_t - \rho = \rho_a(\overline{rr}_{t-1} - \rho) - \sigma\psi_a(1 - \rho_a)\varepsilon_t^a$$

1.4. Equilibrium under an Exogenous Money Supply

We assume an exogenous path for the growth rate of the money supply

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{1.10}$$

where $\rho_m \in [0,1)$ and $\{\varepsilon_t^m\}$ is white noise. In addition we assume

$$\Delta a_t = \rho_a \, \Delta a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0,1)$ and $\{\varepsilon_t^a\}$ is white noise. Hence, $\overline{rr}_t = \rho + \sigma \psi_a \rho_a \Delta a_t$.

We rewrite the money market equilibrium condition in terms of the output gap, as follows:

$$\widetilde{y}_t - \eta \, r_t = m_t - p_t - \overline{y}_t \equiv mpy_t \tag{1.11}$$

Combining (1.11) into (1.7) we obtain:

$$\left(1 + \frac{1}{\sigma\eta}\right) \widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} + \frac{1}{\sigma\eta} \, mpy_t + \frac{1}{\sigma} \, E_t\{\pi_{t+1}\} + \psi_a \rho_a \Delta a_t \tag{1.12}$$

Furthermore, we have

$$mpy_{t-1} = mpy_t + \pi_t - \Delta m_t + \psi_a \Delta a_t \tag{1.13}$$

Hence, the equilibrium dynamics are described by the dynamical system made up of equations (1.6), (1.12), and (1.13), which can be written as follows

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ mpy_{t-1} \end{bmatrix} = \mathbf{A_M} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ mpy_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \Delta m_t \\ \Delta a_t \end{bmatrix} \tag{1.14}$$

where

$$\mathbf{A_M} \equiv \begin{bmatrix} 1 + \frac{1}{\sigma\eta} & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \frac{1}{\sigma\eta} & \frac{1}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} 1 + \frac{1}{\sigma\eta} & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & \psi_a \rho_a \\ 0 & 0 \\ -1 & \psi_a \end{bmatrix}$$

The system above has one predetermined variable and two nonpredetermined variables. Accordingly, the solution will be unique if only if $\mathbf{A_M}$ has two eigenvalues inside the unit circle and one outside.

1.4.1. Quantitative Analysis of a Calibrated Model (Gali (2002))

- Calibration: $\rho_m = 0.5$, $\rho_a = 0$, $\sigma = \varphi = 1$, $\eta = 4$, $\theta = 0.75$
- Effects of Money Supply Shocks
- Effects of Technology Shock

1.5. Appendix: Derivation of Optimal Price Setting Rule

1.5.1. Aggregate Price Level Dynamics

The aggregate price level evolves according to

$$P_t = \left[\theta \ (P_{t-1})^{1-\epsilon} + (1-\theta) \ (P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{1.15}$$

or, alternatively, dividing by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta \ + (1-\theta) \ \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{1.16}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $\frac{P_t^*}{P_{t-1}} = 1$.

Log-linearization around a zero inflation ($\Pi = 1$) steady state implies:

$$\pi_t = (1-\theta) \ (p_t^* - p_{t-1}) \tag{1.17}$$

1.5.2. Optimal Price Setting

Let P_t^* denote the price set by a firm that adjusts its price in period t.[2] Under the Calvo price-setting structure P_t^* will still be effective in period t with probability θ^k for $k = 0, 1, 2, ...$ Let $\Psi_t(\cdot)$ represent period t cost function, in nominal terms. Thus, P_t^* will be chosen in order to maximize the current value of the expected stream of profits generated during the life of the price:

$$\sum_{k=0}^{\infty} \theta^k \ E_t \left\{Q_{t,t+k} \ \left(P_t^* \ Y_{t+k}(j) - \Psi_{t+k}(Y_{t+k}(j))\right)\right\}$$

subject to the sequence of demand constraints.

$$Y_{t+k}(j) = (P_t^*/P_{t+k})^{-\epsilon} \ C_{t+k} \equiv Y_{t+k}^d(P_t^*) \tag{1.18}$$

Notice that in the problem above the expectation is conditional on P_t^* remaining effective. $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor.

[2]Notice that the problem above will be the same for all firms resetting prices in period t , and so will their choice of price P_t^*, which explains the absence of a firm index for the latter variable.

The optimal choice P_t^* must satisfy the first order condition:

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; Y_{t+k}^d(P_t^*) \; \left(P_t^* - \frac{\epsilon}{\epsilon - 1} \; MC_{t+k}^n \right) \right\} = 0$$

where $MC_{t+k}^n = \frac{W_{t+k}}{A_{t+k}}$ is the nominal marginal cost and $\frac{\epsilon}{\epsilon-1}$ is the frictionless optimal gross markup. More compactly,

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} E_t \left\{ \omega_{t,t+k} \; MC_{t+k}^n \right\}$$

where $\omega_{t,t+k} \equiv \frac{\theta^k \; Q_{t,t+k} \; Y_{t+k}^d(P_t^*)}{\sum_{h=0}^{\infty} \theta^h \; E_t[Q_{t,t+h} \; Y_{t+h}^d(P_t^*)]}$.

Letting $\Pi_{t,t+k} \equiv (P_{t+k}/P_t)$, we can rewrite

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; Y_{t+k}^d(P_t^*) \; \left(\frac{P_t^*}{P_{t-1}} - \frac{\epsilon}{\epsilon - 1} \; \Pi_{t-1,t+k} \; MC_{t+k} \right) \right\} = 0 \qquad (1.19)$$

Letting $\mu \equiv \log(\frac{\epsilon}{\epsilon-1})$ and using the fact that $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \Pi_{t,t+k}^{-1}$ and $\frac{\epsilon}{\epsilon-1} MC_{t+k} = \frac{MC_{t+k}}{MC}$ log-linearization of (1.19) around a zero inflation steady state yields:

$$\begin{aligned} p_t^* - p_{t-1} &= (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{(p_{t+k} - p_{t-1}) + \widehat{mc}_{t+k}\} \\ &= \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\pi_{t+k}\} + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\widehat{mc}_{t+k}\} \end{aligned}$$

which can be interpreted as the sum of an "inflation catch-up" and a "markup catch-up" terms.

More compactly:

$$p_t^* - p_{t-1} = \beta\theta \; E_t\{(p_{t+1}^* - p_t)\} + \pi_t + (1 - \beta\theta) \; \widehat{mc}_t \qquad (1.20)$$

Finally, using the fact that $\widehat{mc}_t = mc_t^n - p_t + \mu$, we can rewrite (1.20) as:

$$p_t^* = \beta\theta \; E_t\{p_{t+1}^*\} + (1 - \beta\theta) \; (mc_t^n + \mu)$$

which in turn yields:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k}^n\}$$

1.5.3. Inflation Dynamics

Combining (1.20) and (1.17), and rearranging terms yields the inflation dynamics equation:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t \tag{1.21}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

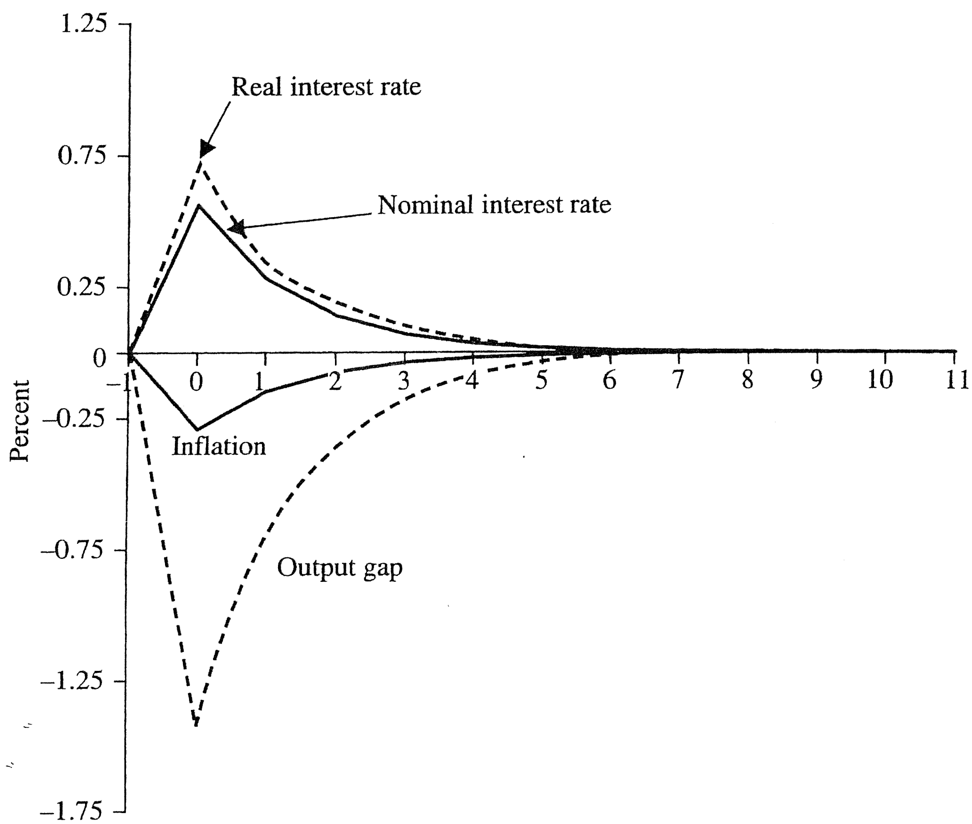


Figure 5.3
Output, Inflation, and Real Interest Rate Responses to a Policy Shock in the New Keynesian Model

Figure 3: Dynamic Responses to a Monetary Shock

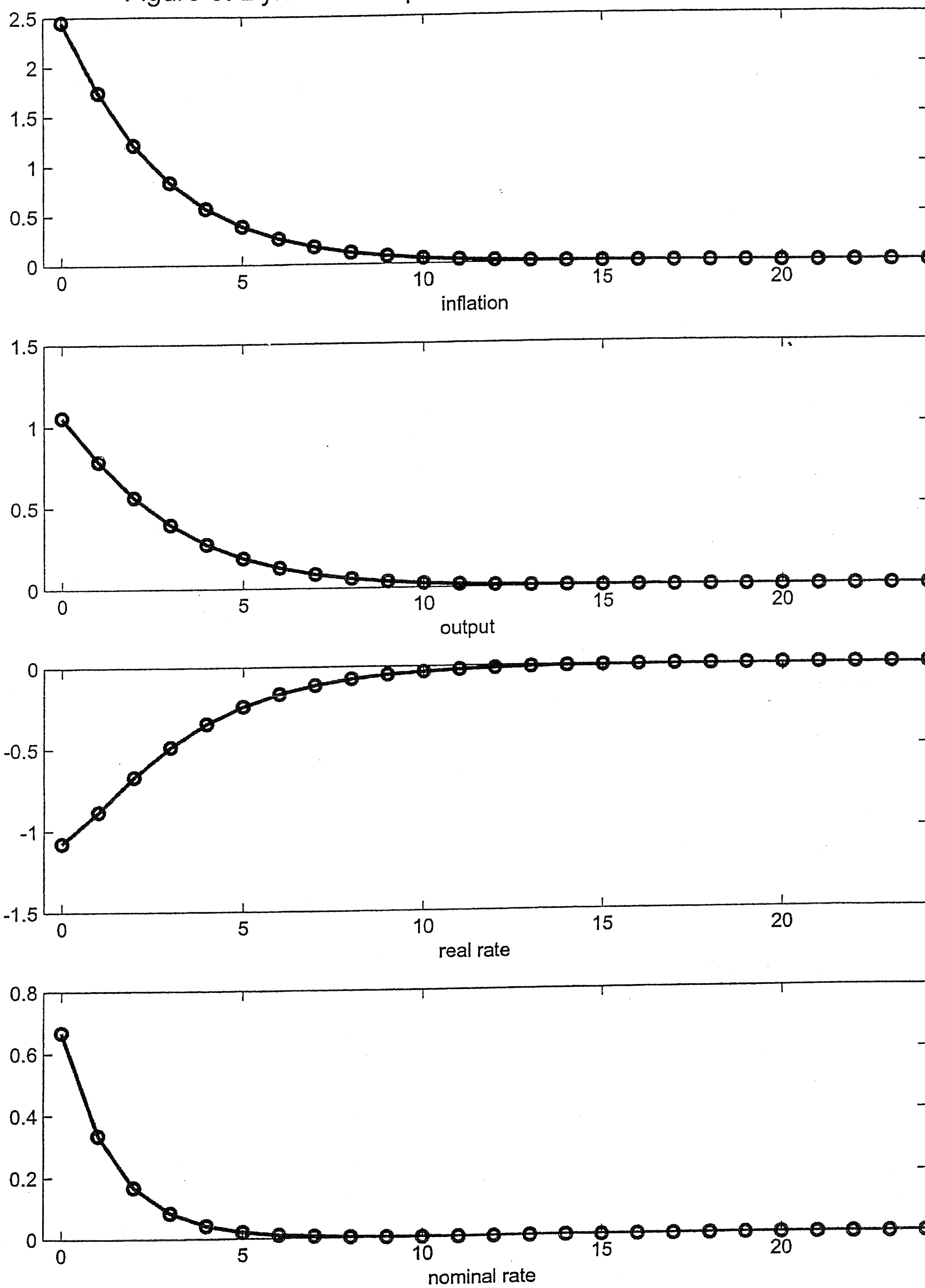
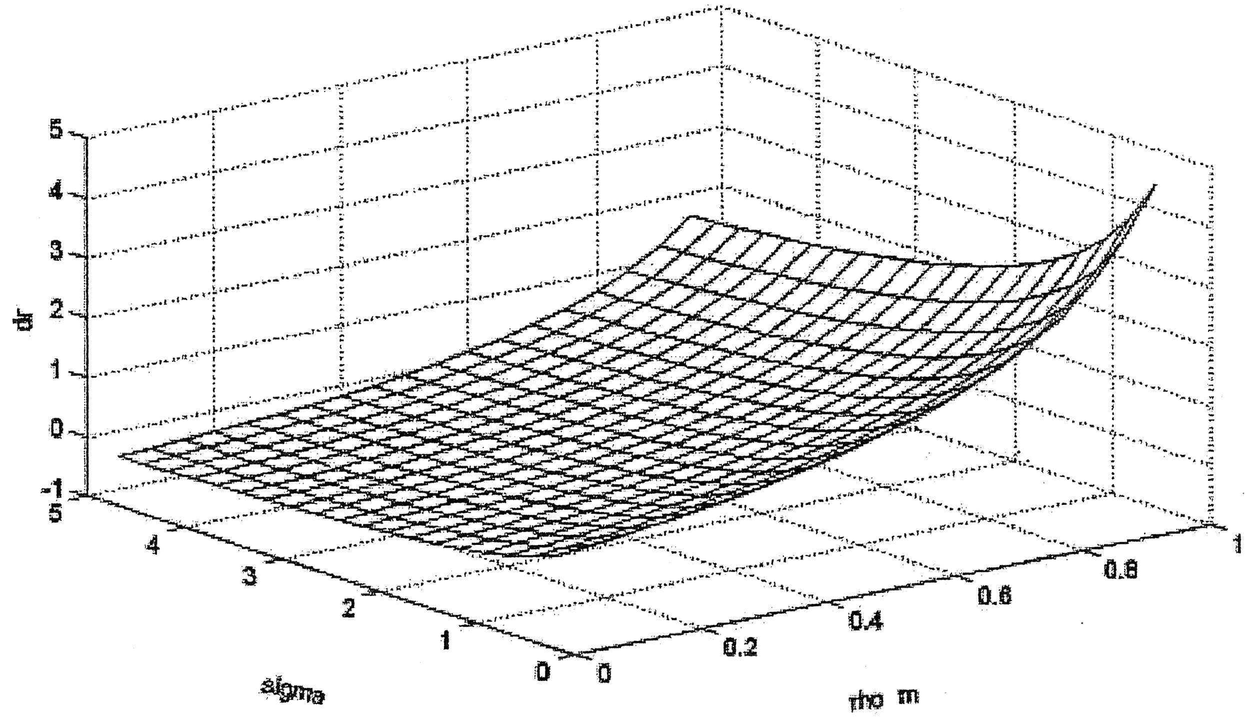


Figure 4: Monetary Shocks and the Liquidity Effect



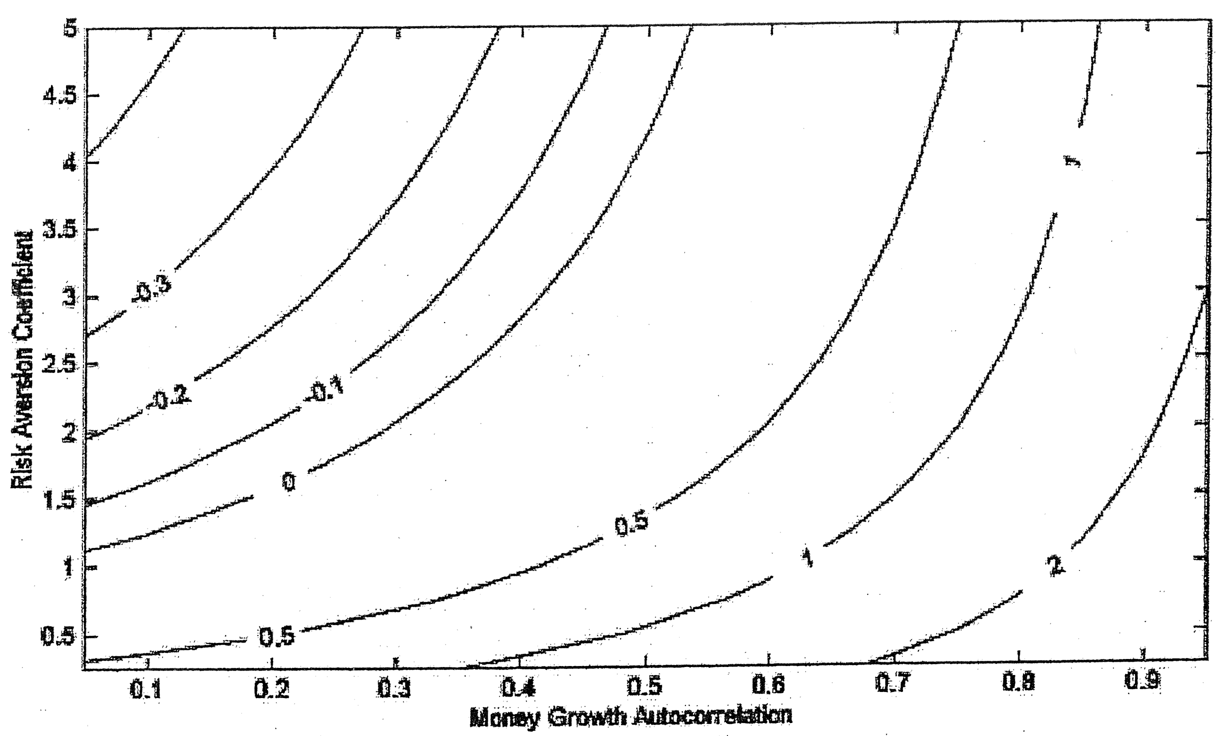
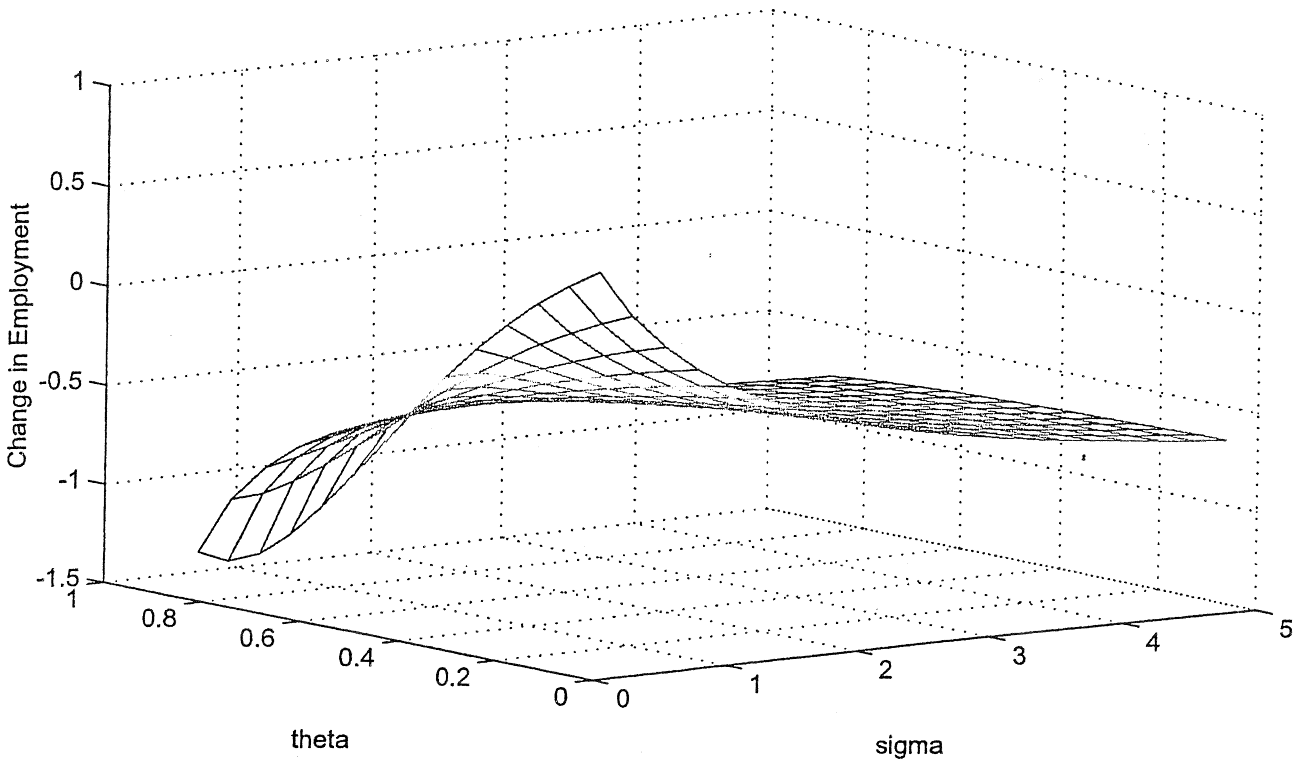


Figure 6: Technology Shocks and Employment



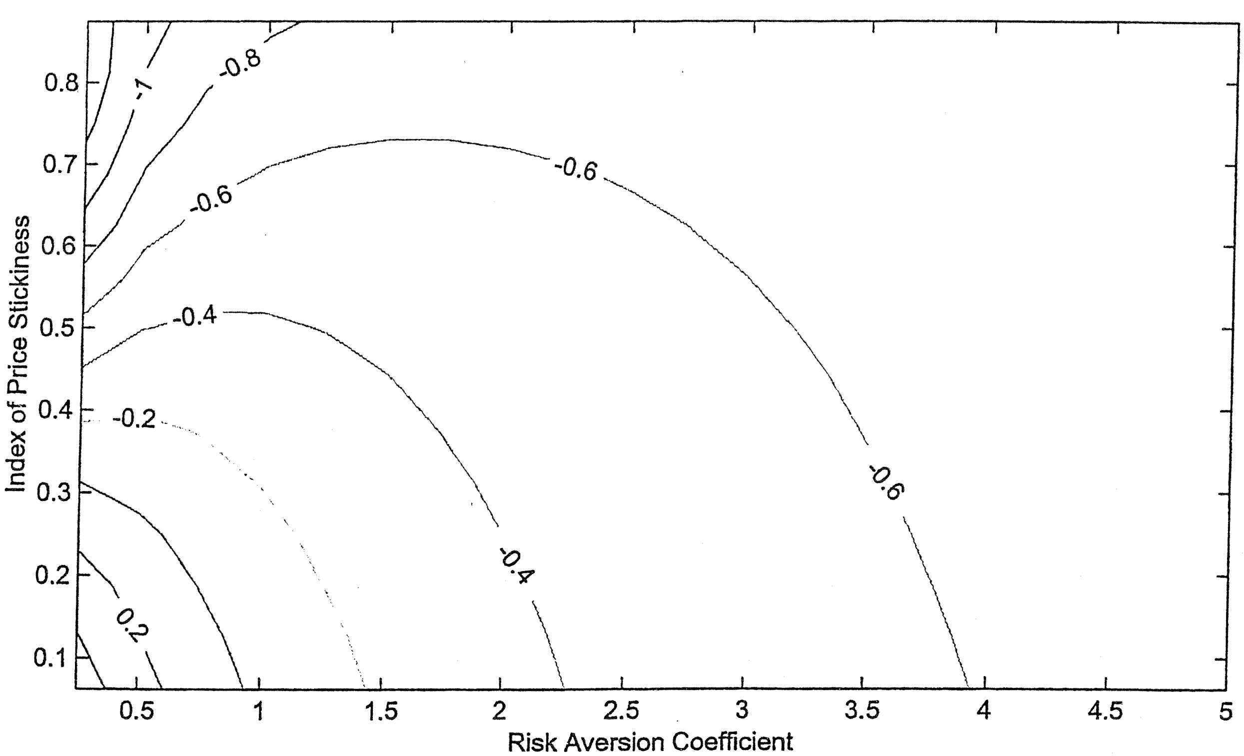


Figure 5: Dynamic Responses Responses to a Technology Shock