Lectures on Monetary Policy, Inflation and the Business Cycle
Chapter 7: Sticky Wages and Prices
[10/05 Preliminary and Incomplete/Do Not Circulate]
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1 A Model with Staggered Wage Setting

In this chapter we lay out a model of an economy in which nominal wages, as well as prices, are sticky. Following Erceg et al. (2000), wage stickiness is introduced in a way analogous to price stickiness. In particular we assume a continuum of differentiated labor services, all of which are used by each firm. Each household is specialized in one type of labor, which it supplies monopolistically.[1] Each period only a (constant) fraction of households/labor types, drawn randomly from the population, can adjust their posted nominal wage. As a result, the aggregate nominal wage responds sluggishly to shocks, and wage inflation brings about relative wage distortions and an inefficient allocation of labor. Next we describe the problem facing firms and households in this environment.

1.1 Firms

The technology available to the typical firm producing a differentiated good, say good i, is assumed to be given by

$$Y_t(i) = A_t \ N_t(i)^{1-\alpha}$$

where $N_t(i)$ is now a composite index defined by

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} \ dj \right]^{\frac{\epsilon_w}{\epsilon_w-1}}$$

where $N_t(i,j)$ denotes the quantity of type-j labor employed by firm i in period t. Notice that parameter ϵ_w represents the elasticity of substitution

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[1]More realistically, one can think of all households/workers specialized in a given labor service being represented by a trade union, with the latter setting the wage on behalf of its members

among labor types. Notice that we assume a continuum of labor types, indexed by $j \in [0, 1]$.

Let $W_t(j)$ denote the nominal wage for type j labor. As discussed below, wages are set by workers of each type (or a union representing them) and taken as given by all firms. Cost minimization yields a set of demand schedules for each firm i and labor type j, given the firm's total employment $N_t(i)$:

$$N_t(i, j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t(i) \tag{1}$$

for all $i, j \in [0, 1]$, where

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} \, dj\right]^{\frac{1}{1-\epsilon_w}}$$

is the aggregate wage index. Furthermore, substituting (1) into the definition of $N_t(i)$, we obtain $\int_0^1 W_t(j) N_t(i, j) \, dj = W_t N_t(i)$.

Hence, and conditional on the optimal allocation of the wage bill among the different types of labor implied by (1), a firm adjusting its price in period t will solve the following problem, which is identical to the one analyzed in chapter 4:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k \, E_t \left\{ Q_{t,t+k} \, \left(P_t^* \, Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = (P_t^*/P_{t+k})^{-\epsilon_p} \, C_{t+k}$$

for $k = 0, 1, 2, \ldots$ where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor for nominal payoffs, $\Psi_{t+k}(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t + k$ for a firm that last reset its price in period t. Notice that we have added a subscript p to parameters θ and ϵ, for symmetry with their labor market counterparts.

As shown in chapter 4, the aggregation of the resulting price setting rules yields, to a first order approximation and in a neighborhood of the zero inflation steady state, the following difference equation for price inflation π_t^p:

$$\pi_t^p = \beta \, E_t\{\pi_{t+1}^p\} - \lambda_p \, \widehat{\mu}_t^p \tag{2}$$

where $\widehat{\mu}_t^p \equiv \mu_t^p - \mu^p = -\widehat{mc}_t$ and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p \, (1+\alpha(\epsilon-1))}$.

1.2 Households

Each household is assumed to specialize in the supply of different type of labor. Households post the (nominal) wage at which they are willing to supply their labor services to firms demanding them. Many households may specialize in the same type of labor (with their joint mass remaining infinitesimal), in which case they are assumed to collude by delegating the wage decision to a union. In a way analogous to firms' price setting, each period only a fraction $1 - \theta_w$ of households/unions, drawn randomly from the population, reoptimize their posted wage. Under the assumption of full consumption risk sharing (resulting from complete markets), all households/unions resetting their wage in any given period will choose the same wage, since they will face a identical problem. Next we lay out and solve that problem.

1.2.1 Optimal Wage Setting

Let W_t^* denote the wage newly set in period t. The choice of W_t^* must solve:

$$\max_{W_t^*} \sum_{k=0}^{\infty} (\beta\theta_w)^k \ E_t\{U(C_{t+k|t}, N_{t+k|t})\}$$

subject to the sequence of labor demand schedules and flow budget constraints that are effective while remains W_t^* in place, i.e.

$$N_{t+k|t} = (W_t^*/W_{t+k})^{-\epsilon_w} \ N_{t+k}$$

$$P_{t+k} \ C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k+1}D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^* \ N_{t+k|t} - T_{t+k}$$

for $k = 0, 1, 2, ...$where $N_{t+k|t}$ is the quantity of labor services provided in period $t + k$ by a household that last reset its wage in period t, and where $N_t \equiv \int_0^1 N_t(i)di$ is an index of aggregate employment. $D_{t+k|t}$ is the market value in period $t+k$ of the securities portfolio held at the end of the previous period by the household the wage of whose labor type is reset in period t.

The first order condition associated with the problem above is given by:

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k \ E_t \left\{ U_c(C_{t+k|t}, N_{t+k|t})\frac{W_t^*}{P_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} \ U_n(C_{t+k|t}, N_{t+k|t}) \right\} = 0$$

Notice also that the assumption complete markets, combined with separability of consumption in the utility function (and identical initial conditions),

guarantee that $C_{t+k|t} = C_{t+k}$ for all $k = 0, 1, 2, ...$ a result that we invoke in what follows.

Letting $MRS_{t+k|t} \equiv -\frac{U_n(C_{t+k}, N_{t+k|t})}{U_c(C_{t+k}, N_{t+k|t})}$ we can rewrite the optimality condition above as

$$\sum_{k=0}^{\infty} \theta_w^k \, E_t \left\{ Q_{t,t+k} \left[\frac{W_t^*}{P_t \, \Pi_{t,t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} \, MRS_{t+k|t} \right] \right\} = 0 \qquad (3)$$

where $\Pi_{t,t+k} \equiv P_{t+k}/P_t$.

In a perfect foresight, zero inflation steady state we have

$$\frac{W^*}{P} = \frac{\epsilon_w}{\epsilon_w - 1} \, MRS$$

We log-linearize (3) around that steady state, which yields, after some manipulation:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \qquad (4)$$

where $\mu^w \equiv \log \frac{\epsilon_w}{\epsilon_w - 1}$ is the log of the desired gross wage markup, i.e. the one that would prevail in the absence of wage rigidities.

The intuition behind wage setting rule (4) is straightforward. First, w_t^* is increasing in expected future prices, since households want to preserve the purchasing power of its nominal wage, given a desired average real wage. Second, w_t^* is increasing in the expected average marginal disutilities of labor (in terms of goods) over the life of the wage, since households want to adjust their expected average real wage accordingly, given expected future prices.

Notice that

$$
\begin{aligned}
mrs_{t+k|t} &= mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}) \\
&= mrs_{t+k} - \epsilon_w \varphi(w_t^* - w_{t+k})
\end{aligned}
$$

Hence we can rewrite (4) as

$$
\begin{aligned}
w_t^* &= \frac{1 - \beta\theta_w}{1 + \epsilon_w\varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left\{ \mu_w + mrs_{t+k} + \epsilon_w\varphi \, w_{t+k} + p_{t+k} \right\} \\
&= \frac{1 - \beta\theta_w}{1 + \epsilon_w\varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left\{ (1 + \epsilon_w\varphi) \, w_{t+k} - \widehat{\mu}_{t+k}^w \right\} \\
&= \beta\theta_w \, E_t \{w_{t+1}^*\} + (1 - \beta\theta_w) \left(w_t - (1 + \epsilon_w\varphi)^{-1} \, \widehat{\mu}_t^w \right) \qquad (5)
\end{aligned}
$$

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where $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denotes the deviations of the economy's average wage markup $\mu_t^w \equiv (w_t - p_t) - mrs_t$ from its steady state (as well as frictionless) level μ^w.

1.2.2 Wage Inflation Dynamics

On the other hand, the evolution of the aggregate wage index is given by

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1 - \theta_w) W_t^{*1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}}$$

Log-linearization of this equation around the zero (wage) inflation steady state yields:

$$w_t = \theta_w \ w_{t-1} + (1 - \theta_w) \ w_t^* \tag{6}$$

Combining (5) and (6), and letting $\pi_t^w = w_t - w_{t-1}$, we can obtain, after some manipulation, our baseline wage inflation equation:

$$\pi_t^w = \beta E_t \{ \pi_{t+1}^w \} - \lambda_w \ \widehat{\mu}_t^w \tag{7}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Notice that this wage inflation equation has a form analogous to (2), the equation describing the price inflation dynamics.

1.2.3 Other Optimality Conditions

In the presence of sticky wages and wage setting by monopolistic workers/unions, the wage inflation equation (7) replaces the condition $\omega_t = mrs_t$ (i.e. the competitive labor supply schedule) in the list of optimality conditions associated with the household's problem. Nevertheless, the latter still yields a conventional Euler equation as an additional optimality condition, as derived in chapter 2, and which must hold independently from the assumptions on wage setting. In its log-linearized version, we recall that the Euler equation takes the form:

$$c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} (\widehat{r}_t - E_t \{ \pi_{t+1}^p \})$$

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2   Equilibrium

We start our analysis of the equilibrium by deriving a version of the equations for price and wage inflation in terms of the output gap $\widetilde{y}_t \equiv y_t - \overline{y}_t$. Importantly, the concept of natural output used in the present chapter is to be understood as referring to the equilibrium level of output in the absence of *both* price and wage rigidities. We also introduce a new variable, the *real wage gap,* denoted by $\widetilde{\omega}_t$, and formally defined as

$$\widetilde{\omega}_t \equiv \omega_t - \overline{\omega}_t$$

where $\omega_t \equiv w_t - p_t$ denotes the real wage, and $\overline{\omega}_t = \log(1-\alpha) + (\overline{y}_t - \overline{n}_t) - \mu^p$ is the *natural real wage*, i.e. the real wage prevailing in the absence of nominal rigidities.

First, we relate the average price markup fluctuations to the output and real wage gaps:

$$
\begin{aligned}
\widehat{\mu}_t^p &= (mpn_t - \omega_t) - \mu^p \\
&= (\widetilde{y}_t - \widetilde{n}_t) - \widetilde{\omega}_t \\
&= -\frac{\alpha}{1-\alpha}\,\widetilde{y}_t - \widetilde{\omega}_t
\end{aligned}
\tag{8}
$$

Hence, combining (2) and (8) we obtain the following equation for price inflation as a function of the output and real wage gaps:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \frac{\lambda_p \alpha}{1-\alpha}\,\widetilde{y}_t + \lambda_p\,\widetilde{\omega}_t \tag{9}$$

Similarly, we have

$$
\begin{aligned}
\widehat{\mu}_t^w &= \omega_t - mrs_t - \mu^w \\
&= \widetilde{\omega}_t - (\widetilde{y}_t + \varphi\widetilde{n}_t) \\
&= \widetilde{\omega}_t - \left(1 + \frac{\varphi}{1-\alpha}\right)\,\widetilde{y}_t
\end{aligned}
\tag{10}
$$

Combining (7) and (10) we obtain an analogous version of the wage inflation equation in terms of the two gaps:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \lambda_w\left(1 + \frac{\varphi}{1-\alpha}\right)\,\widetilde{y}_t - \lambda_w\,\widetilde{\omega}_t \tag{11}$$

In addition, we have the identity relating the changes in the wage gap to price inflation, wage inflation and the natural wage:

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta\overline{\omega}_t \tag{12}$$

The previous equations can be combined with a dynamic IS equation familiar from earlier chapters:

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}^p\} - \overline{rr}_t\right) + E_t\{\widetilde{y}_{t+1}\} \tag{13}$$

where the natural interest rate $\overline{rr}_t$ should now be understood as the one prevailing in an equilibrium with flexible wages and prices.

Finally, and in order to close the model we need to specify how interest rates are determined. We can do so, e.g. by postulating an interest rate rule of the form

$$r_t = v_t + \phi_p\,\pi_t^p + \phi_w\,\pi_t^w + \phi_y\,\widetilde{y}_t \tag{14}$$

where v_t is an exogenous component with mean ρ, possibly a function of $\overline{rr}_t$ and $\Delta\overline{\omega}_t$.

Plugging (14) into (13) to eliminate the interest rate, and collecting the remaining conditions (9), (11), (12), (13) and (14) we can represent the equilibrium dynamics by means of a system of the form

$$\mathbf{x}_t = \mathbf{A}_w\,E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_w\,\mathbf{z}_t \tag{15}$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t,\ \pi_t^p,\ \pi_t^w,\ \widetilde{\omega}_{t-1}]'$, $\mathbf{z}_t \equiv [\overline{rr}_t - v_t,\ \Delta\overline{\omega}_t]'$, and where the elements of $\mathbf{A}_w$ and $\mathbf{B}_w$ are functions of the model's parameters. Given an exogenous process for $\{\mathbf{z}_t\}$, any solution to (15) is an equilibrium, and viceversa.

Remark: notice that even under the assumption that $v_t = \overline{rr}_t$ the dynamical system does *not* have a solution satisfying $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ unless $\overline{\omega}_t$ is constant.

3 Monetary Policy Design with Sticky Wages and Prices

Second Order Approximation to Welfare Losses (EHL 00)

$$(\sigma + \varphi) \ var(\widetilde{y}_t) + \frac{\epsilon_p}{\lambda_p} \ var(\pi_t^p) + \frac{\epsilon_w}{\lambda_w} \ var(\pi_t^w)$$

Key policy issues

- replicating the frictionless equilibrium allocation is no longer feasible (as long as it requires real wage changes).

- price inflation targeting is no longer optimal

- evaluation of alternative simple rules (EHL Table)

3.1 Policy Tradeoffs and a Nearly Optimal Monetary Policy

We start by deriving a key relationship between the output gap and the wage and price markups.[2] Notice that:

$$\mu_t^p + \mu_t^w = (mpn_t - \omega_t) + (\omega_t - mrs_t)$$
$$= mpn_t - mrs_t$$

Given our assumptions on preferences and technology (together with the market clearing condition $y_t = c_t$), we can rewrite the above condition in terms of deviations from the flexible price and wage equilibrium as follows:

$$\widehat{\mu}_t^p + \widehat{\mu}_t^w = (\widetilde{y}_t - \widetilde{n}_t) - (\sigma\widetilde{y}_t + \varphi\widetilde{n}_t)$$
$$= -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\widetilde{y}_t \qquad (16)$$

Combining (2) and (16) we obtain a version of the new Keynesian Phillips curve which allows for wage markup variations resulting from sticky wages:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa_p\,\widetilde{y}_t + \lambda_p\,\widehat{\mu}_t^w \qquad (17)$$

where $\kappa_p \equiv \lambda_p\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)$.

Analogously, combining (7) and (16) yields a wage inflation equation in terms of the output gap:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w\widetilde{y}_t + \lambda_w\,\widehat{\mu}_{p,t} \qquad (18)$$

where $\kappa_w \equiv \lambda_w\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)$.

The previous representations make clear that the presence of nominal wage rigidities (with the consequent variations in the wage markups $\widehat{\mu}_t^w$) generate a tradeoff between stabilization of price inflation and stabilization of the output gap. Analogously, sticky prices (and the resulting fluctuations in $\widehat{\mu}_{p,t}$) imply a tradeoff between stabilization of wage inflation and stabilization of the output gap.

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[2]See Galí, Gertler and López-Salido (2005) for an empirical analysis of the "gap" between the marginal rate of substitution and the the marginal product, and a decomposition of that gap in terms of the two markups.

Notice that we can combine equations (17) and (18) to obtain the following simple relationship between the output gap and a composite inflation measure:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \, \widetilde{y}_t$$

where

$$\pi_t \equiv \vartheta \, \pi_t^p + (1 - \vartheta) \, \pi_t^w$$

with coefficients given by $\vartheta \equiv \frac{\lambda_w}{\lambda_p + \lambda_w} \in (0,1)$ and $\kappa \equiv \vartheta \kappa_p = (1 - \vartheta)\kappa_w$.

Notice that π_t is a weighted average of price and wage inflation, with the respective weights being a function of the relative stickiness of prices versus wages. Hence, the weight on price inflation is increasing in the degree of price stickiness (θ_p), the steepness of the marginal cost schedule (α), and the price elasticity of demand (ϵ_p). By contrast, a larger degree of wage stickiness (θ_w), a steeper labor supply (φ), and higher wage elasticity of labor demand (ϵ_w) tend to increase the weight of wage inflation in the composite measure. Finally, notice that the coefficient on the output gap is smaller that the respective coefficients in the separate wage and price inflation equations.

The quantitative analysis in EHL and Woodford (2003. ch. 6) implies that a monetary policy that seeks to stabilize the output gap (or, equivalently, the composite inflation index π_t) is nearly optimal.

Table 2
Welfare costs of alternative policy rules[a]

		Strict targeting			Hybrid targeting	
	Optimal policy	Price inflation	Wage inflation	Output gap	Price inflation output gap	Price inflation wage inflation
Baseline parameters	2.4	18.6	3.5	2.4	2.4	2.4
Wage contract mean duration $(1-\xi_w)^{-1}$						
1	0	0	3.5	0	0	0
2	1.6	6.5	3.5	1.6	1.6	1.6
4	2.4	18.6	3.5	2.4	2.4	2.4
8	2.7	36.8	3.5	2.7	2.7	2.7
Labor elasticity of output (α)						
0	2.2	702.5	2.6	2.2	2.2	2.2
0.1	2.3	77.8	2.8	2.3	2.3	2.3
0.2	2.4	33.5	3.1	2.4	2.4	2.4
0.3	2.4	18.6	3.5	2.4	2.4	2.4
0.4	2.4	11.3	4.0	2.5	2.5	2.4
0.5	2.3	7.22	4.7	2.5	2.5	2.3
Price markup rate (θ_p)						
0.05	7.3	18.6	16.9	10.9	8.3	7.6
0.10	5.0	18.6	9.0	5.9	5.6	5.1
0.25	2.9	18.6	4.3	2.9	2.9	2.9
0.50	1.9	18.6	2.7	1.9	1.9	1.9

[a]Each welfare loss is expressed as a fraction of Pareto-optimal consumption, divided by the productivity innovation variance.

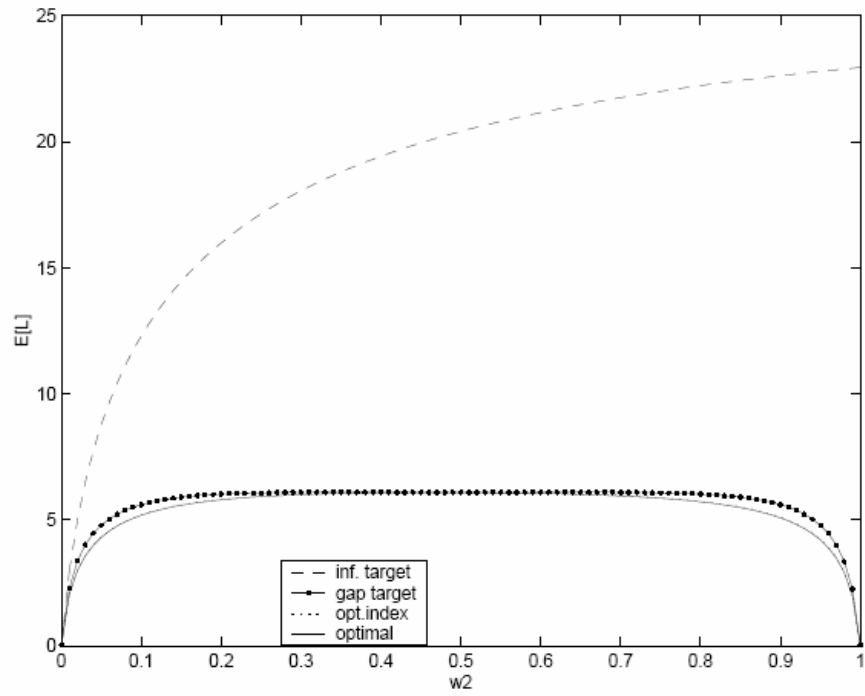Source: Erceg, Henderson, Levin (2000)

Figure 6.4: Welfare losses under alternative policies with sticky wages and prices.

Source: Michael Woodford (2003), chapter 6.