Lectures on Monetary Policy, Inflation and the Business Cycle
Monetary Policy Design: Extensions
[10/05 Preliminary and Incomplete/Do Not Circulate]
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1 Price Indexation and Inflation Inertia

1.1 Inflation Dynamics

Here we consider a variation of the baseline sticky price model in which firms index their prices to lagged inflation. In particular we assume that the price of goods sold in period $t+k$ by a firm that last re-optimized its price in period t is given by

$$P_{t+k|t} = P_{t+k-1|t} \ (\Pi_{t+k-1})^{\gamma} \tag{1}$$

for $k = 1, 2, ...$ with $P_{t|t} \equiv P_t^*$ and where $\gamma \in [0,1]$ is the degree of indexation. As in the baseline model firms reoptimize their prices only with probability $1-\theta$ in any given period. With indexation, a firm reoptimizing its price in period t will maximize choose a price:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ \left(P_{t+k|t} \ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the indexation rule (1) and the sequence of demand constraints

$$Y_{t+k|k} = (P_{t+k|t}/P_{t+k})^{-\epsilon_t} \ C_{t+k}$$

for $k = 0, 1, 2, ...$where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ is the stochastic discount factor for nominal payoffs, $\Psi_t(\cdot)$ is the cost function, and $Y_{t+k|k}$ denotes output in period $t+k$ for a firm that last reset its price in period t. Notice that, as in the last section of chapter 4, we are allowing for an exogenous, time varying price elasticity of demand (as in Steinsson (2003)). The first order condition for that problem takes the form

$$\sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ Y_{t+k|t} \ \left(P_t^* \ (\Pi_{t+k-1|k})^{\gamma} - \frac{\epsilon_t}{\epsilon_t - 1} \ P_{t+k} MC_{t+k|t} \right) \right\} = 0$$

where $\Pi_{t|k} \equiv P_t/P_{t-k}$. That optimality condition can be log-linearized around a zero inflation steady state to yield, after some manipulation:

$$\begin{aligned} p_t^* - p_{t-1} &= \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t \left\{ \frac{(1-\beta\theta)(1-\alpha)}{1+\alpha(\epsilon-1)} \; \widetilde{mc}_{t+k} + (1-\gamma\beta\theta) \; \pi_{t+k} \right\} \\ &= \beta\theta \; E_t\{p_{t+1}^* - p_t\} + \frac{(1-\beta\theta)(1-\alpha)}{1+\alpha(\epsilon-1)} \; \widetilde{mc}_t + (1-\gamma\beta\theta) \; \pi_t \quad (2) \end{aligned}$$

where $\widetilde{mc}_t \equiv mc_t + \overline{\mu}_t$ and $\overline{\mu}_t \equiv \log \frac{\epsilon_t}{\epsilon_t - 1}$.

Notice that in the presence of indexation, expected future inflation has a more limited impact on price setting, since firms realize they will be able to reduce its impact on their relative price through the automatic indexation mechanism (albeit partially and with a lag), until they have a change to re-optimize again.

On the other hand, the law of motion for the price level is given by

$$P_t \equiv \left[\theta \; (P_{t-1}\Pi_{t-1}^{\gamma})^{1-\epsilon} + (1-\theta) \; (P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

which can be log-linearized around the zero inflation steady state to yield

$$\pi_t = \gamma\theta \; \pi_{t-1} + (1-\theta) \; (p_t^* - p_{t-1}) \quad (3)$$

Combining (2) and (3) we obtain the following second order difference equation describing the dynamics of inflation:

$$\pi_t = \frac{\gamma}{1+\beta\gamma} \; \pi_{t-1} + \frac{\beta}{1+\beta\gamma} \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t + u_t$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)(1-\alpha)}{\theta(1+\alpha(\epsilon-1))}$ and $u_t \equiv -\lambda(\overline{\mu}_t - \mu)$. Using the simple relationship between marginal cost and the output gap derived in chapter 4, we can derive the following version of the new Keynesian Phillips curve in terms of quasi-differenced inflation $\widetilde{\pi}_t \equiv \pi_t - \gamma\pi_{t-1}$:

$$\widetilde{\pi}_t = \beta \; E_t\{\widetilde{\pi}_{t+1}\} + \kappa \; \widetilde{y}_t + u_t \quad (4)$$

where $\kappa \equiv \lambda(\sigma + \varphi)$.and $\widetilde{y}_t$ is the output gap, understood here as the log-deviation of output from its equilibrium level under flexible prices *and* a constant markup μ.

1.2 Welfare Approximation

As shown in chapter 5, a second order approximation to the representative consumer's utility losses takes the form

$$\mathbb{W} = -\frac{1}{2} \; E_0 \sum_{t=0}^{\infty} \beta^t \left((\sigma + \varphi) \; \widetilde{y}_t^2 + \epsilon \; var_i\{p_t(i)\} \right)$$

As shown in Woodford (2003, chap. 6)), in the presence of indexation of the sort assumed above the following relationship holds

$$\sum_{t=0}^{\infty} \beta^t var_i\{p_t(i)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t \; (\pi_t - \gamma \pi_{t-1})^2$$

Hence, we see how in the present environment relative price distortions arise as a result of deviations in inflation from the rate at which indexed prices are increasing, not as a consequence of inflation *in itself*. In particular, in the presence of full indexation ($\gamma = 1$) only changes in inflation, but not the level of inflation, have distortionary effects.

Combining the above results we can derive the following approximate welfare loss function for an economy with indexation:

$$\mathbb{W} = -\frac{1}{2} \; E_0 \sum_{t=0}^{\infty} \beta^t \left((\sigma + \varphi) \; \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \; \widetilde{\pi}_t^2 \right)$$

1.3 Optimal Monetary Policy with Inflation Inertia

We revisit the optimal policy under discretion and commitment, in the presence of inflation inertia resulting from backward-looking indexation by firms. For simplicity we assume the cost-push shock u_t follows a white noise process.

1.3.1 The Case of Discretion

The monetary authority minimizes the period loss function

$$\alpha_y \ \widetilde{y}_t^2 + \ \widetilde{\pi}_t^2$$

where $\alpha_y \equiv \frac{\kappa}{\epsilon}$, subject to the "tradeoff" equation

$$\widetilde{\pi}_t = \kappa \ \widetilde{y}_t + \nu_t$$

where $\nu_t \equiv \beta \ E_t\{\widetilde{\pi}_{t+1}\} + u_t$ is taken as given by the central bank.

The optimality condition for that problem is

$$\widetilde{y}_t = -\epsilon \ \widetilde{\pi}_t$$

Substituting this optimality condition into (4) and solving forward we obtain

$$\pi_t = \gamma \pi_{t-1} + \frac{1}{1+\kappa\epsilon} \ u_t$$

and

$$\widetilde{y}_t = -\frac{\epsilon}{1+\kappa\epsilon} \ u_t$$

Hence, and in contrast with the model with no indexation, the optimal policy with discretion implies persistent deviations of inflation form target, even in the limiting case of white noise disturbances. The intuition for that result is straighforward: due to backward-looking indexation the inflationary impact of the shock remains once the shock is gone; in order to minimize relative price distortions and the output gap the central bank should fully accommodate those "second round" inflationary effects of the shock.

1.3.2 The Case of Commitment

Now the monetary authority chooses a state-contingent policy $\{\widetilde{y}_t, \widetilde{\pi}_t\}_{t=0}^{\infty}$ that maximizes

$$-\frac{1}{2} \ E_0 \sum_{t=0}^{\infty} \beta^t \ [\alpha_y \ \widetilde{y}_t^2 + \ \widetilde{\pi}_t^2]$$

subject to a sequence of constraints

$$\widetilde{\pi}_t = \beta \ E_t\{\widetilde{\pi}_{t+1}\} + \kappa \ \widetilde{y}_t + u_t$$

First order conditions:

$$\alpha_y \ \widetilde{y}_t - \kappa \ \varphi_t = 0$$

$$\widetilde{\pi}_t + \varphi_t - \varphi_{t-1} = 0$$

for $t = 0, 1, 2, ...$and with $\varphi_{-1} = 0$.

Eliminating the multipliers and focusing on we have

$$\widetilde{y}_0 = -\epsilon \ \widetilde{\pi}_t$$

$$\widetilde{y}_t = \widetilde{y}_{t-1} - \epsilon \ \widetilde{\pi}_t$$

$t = 1, 2, ...$ which in turn can be written more compactly as

$$\widetilde{y}_t = -\epsilon \ (\widehat{p}_t - \gamma \widehat{p}_{t-1})$$

for $t = 0, 1, 2, ...$ where $\widehat{p}_t \equiv p_t - p_{-1}$. Substituting into the inflation equation and rearranging terms we obtain

$$\widetilde{p}_t = a \ \widetilde{p}_{t-1} + a\beta \ E_t\{\widetilde{p}_{t+1}\} + a \ u_t$$

where $\widetilde{p}_t \equiv \widehat{p}_t - \gamma \widehat{p}_{t-1}$ and $a \equiv \frac{1}{1+\kappa\epsilon+\beta}$. The stationary solution is given by:

$$\widetilde{p}_t = \delta \ \widetilde{p}_{t-1} + \delta \ u_t \tag{5}$$

where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$. Equivalently, in terms of deviations of the price level from the implicit target p_{-1} we have the following AR(2) process:

$$\widehat{p}_t = (\gamma + \delta) \ \widehat{p}_{t-1} - \delta\gamma \widehat{p}_{t-2} + \delta \ u_t$$

Notice also that, independently of γ, the output gap will evolve according to

$$\widetilde{y}_t = \delta \ \widetilde{y}_{t-1} - \delta\epsilon \ u_t$$

Hence, and to the extent that $\gamma + \delta > 1$, the optimal response of inflation to a transitory cost-push shock will display some positive serial correlation at short horizons. That "optimal inertia" arises from the desire to avoid the large relative price distortions associated with large deviations of inflation π_t from $\gamma \pi_{t-1}$.

See Fig 3.2 in Giannoni and Woodford (2005).

2 Transactions Frictions

In the section we restore an explicit role for money in our model by assuming that real balances genrate utility. Hence , as in chapter 2, preferences are now represented by a discounted sum of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

where M_t denotes monetary holdings in period t and P_t is the price index. The flow budget constraint now takes the form (once optimal allocation of expenditures is accounted for):

$$P_t \; C_t + R_t^{-1} B_t + M_t \leq B_{t-1} + M_{t-1} + W_t \; N_t - T_t$$

We specify the utility function to have the functional form

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \chi_m \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

As shown in Woodford (RES, 200x), in that environment a second order approximation to the utility of the representative household around a zero inflation, undistorted steady state takes the form:

$$\mathbb{W} = -\frac{1}{2} \; E_0 \sum_{t=0}^{\infty} \beta^t \; \left(\pi_t^2 + \alpha_y \; \widetilde{y}_t^2 + \alpha_r \; r_t^2\right) + t.i.p.$$

where, for simplicity, we ignore the zero lower bound on the nominal interest rate.

Intuition: the nominal rate acts as a tax on real balances, with associate deadweight loss convex in the “tax rate.”

2.1 Optimal Monetary Policy with Commitment

The monetary authority is assumed to choose a state-contingent policy $\{\widetilde{y}_t, \pi_t\}_{t=0}^{\infty}$ that maximizes

$$-\frac{1}{2}\ E_0 \sum_{t=0}^{\infty} \beta^t\ \left(\pi_t^2 + \alpha_y\ \widetilde{y}_t^2 + \alpha_r\ r_t^2\right)$$

subject to the sequence of constraints:

$$\pi_t = \kappa\ \widetilde{y}_t + \beta\ E_t\{\pi_{t+1}\} + u_t$$

$$\widetilde{y}_t = -\frac{1}{\sigma}\ (r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t) + E_t\{\widetilde{y}_{t+1}\}$$

The Lagrangean can be set up as follows:

$$\begin{aligned}\mathcal{L} \ &= \ \frac{1}{2}\ E_0 \sum_{t=0}^{\infty} \beta^t\ \Big[(\pi_t^2 + \alpha_y\ \widetilde{y}_t^2 + \alpha_r\ r_t^2) + \gamma_{1,t}\ (\pi_t - \kappa\ \widetilde{y}_t - \beta\ \pi_{t+1}) \\ &\quad + \gamma_{2,t}\ (\widetilde{y}_t + \frac{1}{\sigma}\ r_t - \frac{1}{\sigma}\ \pi_{t+1} - \widetilde{y}_{t+1})\Big]\end{aligned}$$

First order conditions:

$$\alpha_y\ \widetilde{y}_t - \frac{1}{2}\ (\kappa\ \gamma_{1,t} - \gamma_{2,t} + \beta^{-1}\gamma_{2,t-1}) = 0$$

$$\pi_t + \frac{1}{2}\ (\gamma_{1,t} - \gamma_{1,t-1} - \frac{1}{\sigma\beta}\gamma_{2,t-1}) = 0$$

$$\alpha_r\ r_t + \frac{1}{2}\ \frac{1}{\sigma}\ \gamma_{2,t} = 0$$

Substituting $\gamma_{2,t-1}$ we have (for $t = 1, 2, 3, ...$):

$$\alpha_y\ \widetilde{y}_t - \frac{1}{2}\ \kappa\ \gamma_{1,t} - \alpha_r\sigma\ r_t + \beta^{-1}\alpha_r\sigma\ r_{t-1} = 0$$

$$\pi_t + \frac{1}{2}\ (\gamma_{1,t} - \gamma_{1,t-1}) + \ \beta^{-1}\alpha_r\ r_{t-1} = 0$$

which in turn can be combined (for $t = 2, 3, 4, ...$)

$$\alpha_y\Delta\widetilde{y}_t + \kappa\pi_t + \kappa\beta^{-1}\alpha_r\ r_{t-1} - \alpha_r\sigma\ \Delta r_t + \beta^{-1}\alpha_r\sigma\ \Delta r_{t-1} = 0$$

Finally, rearranging terms, it yields the "super-inertial" Taylor rule:

$$r_t = (1 + \frac{\kappa}{\sigma\beta})\ r_{t-1} + \frac{1}{\beta}\ \Delta r_{t-1} + \frac{\kappa}{\alpha_r \sigma}\ \pi_t + \frac{\alpha_y}{\alpha_r \sigma}\ \Delta \widetilde{y}_t \tag{6}$$

which is independent of the statistical properties of the disturbances ("robustly optimal"). Together with the NKPC and IS, it can also be shown to have a locally unique solution.

See figure 3.3 in GW, with impulse responses to natural real rate shock.

Remarks:

- a policy tradeoff arises even in the absence of cost push shocks, resulting from the desire to avoid large fluctuations in interest rates.
- in the limiting case $\alpha_r = 0$, we recover:

$$\Delta \widetilde{y}_t = -\frac{\kappa}{\alpha_y}\ \pi_t$$

2.1.1

2.1.2 A Targeting Rule Representation

Let $q_t \equiv \frac{\kappa}{\alpha_r \sigma} \; \pi_t + \frac{\alpha_y}{\alpha_r \sigma} \; \Delta \widetilde{y}_t$. Then we can rewrite (6) as:

$$(1 - \lambda_1 L)(1 - \lambda_2 L) \; r_t = q_t$$

where $0 < \lambda_1 < 1 < \lambda_2$. Hence, at each period t interest rate must be set so that the condition:

$$(1 - \lambda_1 L) \; r_{t-1} = -\lambda_2^{-1} \sum_{k=0}^{\infty} \lambda_2^{-k} \; E_t\{q_{t+k}\}$$

is satisfied. Multiplying both sides by $-\frac{\alpha_r \sigma}{\kappa}(1 - \lambda_2^{-1})\lambda_2$ we obtain

$$F_t(\pi) + \phi \; F_t(\widetilde{y}) = \theta_y \; \widetilde{y}_{t-1} - \theta_r \; r_{t-1} - \theta_\Delta \; \Delta r_{t-1}$$

where $\phi = \theta_y \equiv \frac{\alpha_y}{\kappa}(1 - \lambda_2) < 0$, $\theta_r \equiv \frac{\alpha_r \sigma}{\kappa}(1 - \lambda_1)(1 - \lambda_2^{-1})\lambda_2 < 0$, and $\theta_\Delta \equiv \frac{\alpha_r \sigma}{\kappa} \lambda_1 (1 - \lambda_2^{-1})\lambda_2$, and $F_t(z) \equiv (1 - \lambda_2^{-1}) \sum_{k=0}^{\infty} \lambda_2^{-k} \; E_t\{z_{t+k}\}$ which is a weighted forecast with weights adding up to one.

Remarks:

- well defined rule, even though it does not determine r_t explicitly.
- interpretation: determines the target for the inflation forecast, as a function of output gap forecasts and past conditions.
- mean horizon for the relevan inflation projection is $(1-\lambda_2^{-1}) \sum_{k=0}^{\infty} \lambda_2^{-k} k = \frac{\lambda_2^{-1}}{1-\lambda_2^{-1}}$. Under Woodford's baseline calibration $\lambda_2^{-1} = 0.68$, implying a mean forecaset horizon of 2.1 quarters, much shorter than common practice by inflation targeting central banks.
- under a welfare-theoretic interpretation of the loss function $\alpha_y = \frac{\kappa}{\epsilon}$, thus implying $\phi = \frac{1}{\epsilon}(1 - \lambda_2)$, suggesting that forecasts of future output gaps should have little effect on the inflation forecast target.