Monetary Policy and Exchange Rate Volatility
in a Small Open Economy

by

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March 2005
Motivation

- The new Keynesian model for the closed economy
  - equilibrium dynamics: simple three-equation representation
  - optimal monetary policy design: optimality of inflation targeting
- How does the introduction of open economy elements affect that analysis and prescriptions?
- What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?

⇒ main finding: equivalence result (for a benchmark model)
A New Keynesian Model of a Small Open Economy

Households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]
\]

subject to

\[
\int_0^1 P_{H,t}(j)C_{H,t}(j) \, dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j) \, dj \, di + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t
\]

\[
C_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\eta-1}
\]

\[
C_{H,t} = (\int_0^1 C_{H,t}(j)^{\frac{\eta-1}{\eta}} \, di)^{\frac{\eta}{\eta-1}}
\]

\[
C_{F,t} \equiv \left( \int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}} ; \quad C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}
\]
Firms

\[ Y_t(i) = A_t N_t(i) \]
\[ a_t = \rho a_{t-1} + u_t \]

Law of One Price

\[ p_{i,t}(j) = e_{i,t} + p_{i,t}^j(j) \]
\[ \Rightarrow p_{F,t} = e_t + p_t^* \]
Some Identities and Definitions

Terms of Trade

\[ s_t \equiv p_{F,t} - p_{H,t} \]

CPI

\[ p_t \equiv (1 - \alpha) p_{H,t} + \alpha p_{F,t} \]
\[ = p_{H,t} + \alpha s_t \]

CPI Inflation vs. Domestic Inflation:

\[ \pi_t = \pi_{H,t} + \alpha \Delta s_t \]

Real Exchange Rate and the Terms of Trade

\[ q_t \equiv (e_t + p_t^*) - p_t \]
\[ = s_t + (p_{H,t} - p_t) \]
\[ = (1 - \alpha) s_t \]
Optimal Intratemporal Allocation of Expenditures:

- within each category:

\[ c_{H,t}(i) = -\varepsilon (p_{H,t}(i) - p_{H,t}) + c_{H,t} \]
\[ c_{F,t}(i) = -\gamma (p_{F,t}(i) - p_{F,t}) + c_{F,t} \]

- between categories:

\[ c_{H,t} = -\eta (p_{H,t} - p_t) + c_t \]
\[ c_{F,t} = -\eta (p_{F,t} - p_t) + c_t \]

Other Optimality Conditions:

\[ w_t - p_t = \sigma c_t + \varphi n_t \]
\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{t+1}\} - \rho) \]
International Risk Sharing (Complete Markets)

\[
\begin{align*}
\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) &= Q_{t,t+1} \\
\beta \left( \frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \left( \frac{P^i_t}{P^i_{t+1}} \right) \left( \frac{E^i_t}{E^i_{t+1}} \right) &= Q_{t,t+1}
\end{align*}
\]

Combining domestic and world foc’s:

\[
C_t = \vartheta_i \ C^i_t \ Q_{i,t}^{\frac{1}{\sigma}}
\]

Log-linearizing (after \(\vartheta_i = 1\)):

\[
\begin{align*}
c_t &= c^*_t + \frac{\sigma}{\sigma} \ q_t \\
&= c^*_t + \frac{1 - \alpha}{\sigma} \ s_t
\end{align*}
\]
Uncovered Interest Parity

Complete markets:

\[ E_t\{Q_{t,t+1} \left[R_t - R_t^i (\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t})\right]\} = 0 \]

Log-linearizing and aggregating over \(i\):

\[ r_t - r_t^* = E_t\{\Delta e_{t+1}\} \]

Combined with the definition of the terms of trade:

\[ s_t = (r_t^* - E_t\{\pi_{t+1}^*\}) - (r_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\} \]

Integrating forward, and using \(\lim_{T \to \infty} E_t\{s_T\} = 0\):

\[ s_t = E_t \left\{ \sum_{k=0}^{\infty} \left[ (r_{t+k}^* - \pi_{t+k+1}^*) - (r_{t+k} - \pi_{H,t+k+1}) \right] \right\} \]
Equilibrium Dynamics in the SOE: A Canonical Representation

\[ \pi_{H,t} = \beta \ E_t\{\pi_{H,t+1}\} + \kappa_{\alpha} \ \tilde{y}_t \]

\[ \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_{\alpha}} \left( r_t - E_t\{\pi_{H,t+1}\} - \bar{r}_t \right) \]

where

\[ \tilde{y}_t = y_t - \bar{y}_t \]
\[ \bar{y}_t = \Omega + \Gamma \ a_t + \alpha \Psi \ y_t^* \]
\[ \bar{r}_t \equiv \rho - \sigma_{\alpha} \Gamma (1 - \rho_{\omega}) \ a_t + \alpha \sigma_{\alpha} (\Theta + \Psi) \ E_t\{\Delta y_{t+1}^*\} \]

\[ \kappa_{\alpha} \equiv \lambda \left( \sigma_{\alpha} + \varphi \right) \ ; \ \sigma_{\alpha} \equiv \frac{\sigma}{(1 - \alpha) + \alpha \omega} \ ; \ \Gamma \equiv \frac{1 + \varphi}{\sigma_{\alpha} + \varphi} \ ; \ \Psi \equiv - \frac{\Theta \sigma_{\alpha}}{\sigma_{\alpha} + \varphi} \]
Aggregate Demand and Output Determination

*World Market Clearing (WMC)*

\[ y_t^* = c_t^* \]

*Domestic Market Clearing (DMC)*

\[ y_t = c_t + \alpha \gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t \]

\[ = c_t + \frac{\alpha \omega}{\sigma} s_t \]

where \( \omega \equiv \sigma \gamma + (1 - \alpha) (\sigma \eta - 1) \).

Combining DMC and WMC with IRS we obtain:

\[ y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t \]

where \( \sigma_\alpha \equiv \frac{\sigma}{(1-\alpha) + \alpha \omega} > 0 \).
Finally, combining DMC with the Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha \omega}{\sigma} E_t\{\Delta s_{t+1}\}$$

$$= E_t\{y_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha \Theta}{\sigma} E_t\{\Delta s_{t+1}\}$$

$$= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha} (r_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha \Theta \ E_t\{\Delta \gamma^*_t\}$$

where $\Theta \equiv (\sigma \gamma - 1) + (1 - \alpha)(\sigma \eta - 1) = \omega - 1$ and $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha \omega}$.

Letting $\tilde{y}_t = y_t - \bar{y}_t$,

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha} (r_t - E_t\{\pi_{H,t+1}\} - \bar{rr}_t)$$

where $\bar{rr}_t \equiv \rho + \sigma_\alpha (\Delta \bar{y}_{t+1} + \alpha \Theta \ E_t\{\Delta \gamma^*_t\})$
The New Keynesian Phillips Curve in the Small Open Economy

Domestic Price Dynamics:

\[ p_{H,t} \equiv \theta \ p_{H,t-1} + (1 - \theta) \overline{p}_{H,t} \]

Optimal Price Setting:

\[ \overline{p}_{H,t} = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k \ E_t \{mc_{t+k} + p_{H,t}\} \]

Domestic Inflation Dynamics

\[ \pi_{H,t} = \beta \ E_t \{\pi_{H,t+1}\} + \lambda \ \widehat{mc}_t \]

where \( \lambda \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta} \)
Marginal Cost and the Output Gap

\[ mc_t = -\nu + (w_t - p_{H,t}) - a_t \]
\[ = -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \]
\[ = -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \]
\[ = -\nu + \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t \]

Substituting for \( s_t \) using (1):

\[ mc_t = -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t \]

Under flexible prices,

\[ -\mu = -\nu + (\sigma_\alpha + \varphi) \bar{y}_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t \]

Thus,

\[ \bar{y}_t = \Omega + \Gamma a_t + \alpha \Psi y_t^* \]

Also

\[ \hat{mc}_t = (\sigma_\alpha + \varphi) \bar{y}_t \]

which combined with (2) yields:

\[ \pi_{H,t} = \beta \ E_t\{\pi_{H,t+1}\} + \kappa_\alpha \bar{y}_t \]
Optimal Monetary Policy

Background and Strategy

A Special Case

\[ \sigma = \eta = \gamma = 1 \]

\[ \Rightarrow \quad C_t = Y_t^{1-\alpha}(Y^*_t)^\alpha \]

Optimal Allocation:

\[ \max \log C_t - \frac{N_t^{1+\varphi}}{1 + \varphi} \]

subject to

\[ C_t = Y_t^{1-\alpha}(Y^*_t)^\alpha \]
\[ = (A_t N_t)^{1-\alpha}(Y^*_t)^\alpha \]

Optimality condition:

\[ N = (1 - \alpha)^{\frac{1}{1+\varphi}} \]
Flexible Price Equilibrium

\[ 1 - \frac{1}{\varepsilon} = \bar{MC}_t \]

\[ = - \frac{(1 - \tau)}{A_t} S_t^\alpha \frac{U_N(C_t, \bar{N}_t)}{U_C(C_t, \bar{N}_t)} \]

\[ = (1 - \tau) \frac{\bar{Y}_t}{\bar{C}_t} \bar{N}_t^{\phi} \bar{C}_t \]

\[ = (1 - \tau) \bar{N}_t^{1+\phi} \]

Optimality of Flexible Price Equilibrium:

\[ (1 - \tau)(1 - \alpha) = 1 - \frac{1}{\varepsilon} \]

Implied Monetary Policy Objectives

\[ y_t = \bar{y} \]

\[ \pi_{H,t} = 0 \]

for all \( t \).
Implementation

\[ r_t = \overline{r}_t + \phi_\pi \pi_{H,t} + \phi_x x_t \]

where \( \kappa_\alpha (\phi_\pi - 1) + (1 - \beta) \phi_x > 0. \)

Other Macroeconomic Implications

Terms of Trade

\[ \overline{s}_t = \sigma_\alpha (\overline{y}_t - y_t^*) \]
\[ = \sigma_\alpha \Omega + \sigma_\alpha \Gamma a_t - \sigma_\alpha \Phi y_t^* \]

where \( \Phi = \frac{\sigma + \varphi}{\sigma_\alpha + \varphi} > 0. \)

Special case

\[ \overline{s}_t = a_t - y_t^* \]

Exchange Rate

\[ \overline{e}_t = \overline{s}_t - p_t^* \]

CPI

\[ \overline{p}_t = \alpha (\overline{e}_t + p_t^*) \]
\[ = \alpha \overline{s}_t \]
Consequences of Suboptimal Policies

Welfare Losses (special case)

\[ W = - \frac{(1 - \alpha)}{2} \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon}{\lambda} \pi_{H,t}^2 + (1 + \varphi) x_t^2 \right] \]

Taking unconditional expectations and letting \( \beta \to 1 \),

\[ V = - \frac{(1 - \alpha)}{2} \left[ \frac{\varepsilon}{\lambda} \text{var}(\pi_{H,t}) + (1 + \varphi) \text{var}(x_t) \right] \]
Three Simple Rules

*Domestic inflation-based Taylor rule (DITR)*

\[ r_t = \rho + \phi_\pi \pi_{H,t} \]

*CPI inflation-based Taylor rule (CITR):*

\[ r_t = \rho + \phi_\pi \pi_t \]

*Exchange rate peg (PEG)*

\[ e_t = 0 \]
TABLE 1

Cyclical properties of alternative policy regimes

<table>
<thead>
<tr>
<th></th>
<th>Optimal sd%</th>
<th>DI Taylor sd%</th>
<th>CPI Taylor sd%</th>
<th>Peg sd%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.95</td>
<td>0.68</td>
<td>0.72</td>
<td>0.86</td>
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<tr>
<td>Domestic inflation</td>
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<td>0.27</td>
<td>0.36</td>
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<td>CPI inflation</td>
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<td>0.41</td>
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<td>1.53</td>
<td>1.43</td>
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<tr>
<td>Nominal depr. rate</td>
<td>0.95</td>
<td>0.86</td>
<td>0.53</td>
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<tr>
<td></td>
<td>DI Taylor</td>
<td>CPI Taylor</td>
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<tr>
<td><strong>Benchmark ( \mu = 1.2, \phi = 3 )</strong></td>
<td></td>
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<tr>
<td>Var(domestic infl)</td>
<td>0.0157</td>
<td>0.0151</td>
<td>0.0268</td>
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<tr>
<td>Var(output gap)</td>
<td>0.0009</td>
<td>0.0019</td>
<td>0.0053</td>
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<tr>
<td>Total</td>
<td>0.0166</td>
<td>0.0170</td>
<td>0.0321</td>
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<tr>
<td><strong>Low steady state mark-up ( \mu = 1.1, \phi = 3 )</strong></td>
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<tr>
<td>Var(Domestic infl)</td>
<td>0.0287</td>
<td>0.0277</td>
<td>0.0491</td>
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<tr>
<td>Var(Output gap)</td>
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<td>0.0019</td>
<td>0.0053</td>
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<tr>
<td>Total</td>
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<td>0.0296</td>
<td>0.0544</td>
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<td><strong>Low elasticity of labour supply ( \mu = 1.2, \phi = 10 )</strong></td>
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<tr>
<td>Var(Domestic infl)</td>
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<tr>
<td>Var(Output gap)</td>
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<tr>
<td>Total</td>
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<td>0.0261</td>
<td>0.0630</td>
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<td><strong>Low mark-up and elasticity of labour supply ( \mu = 1.1, \phi = 10 )</strong></td>
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<tr>
<td>Var(Domestic infl)</td>
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<tr>
<td>Var(Output gap)</td>
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<td>0.0020</td>
<td>0.0064</td>
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<tr>
<td>Total</td>
<td>0.0436</td>
<td>0.0461</td>
<td>0.1101</td>
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</tbody>
</table>

*Note:* Entries are percentage units of steady state consumption.
FIGURE 1
Impulse responses to a domestic productivity shock under alternative policy rules
Concluding Remarks

- small open economy version of the new Keynesian model
- under baseline assumptions (complete markets, full pass-through), equilibrium dynamics equivalent to the closed economy
- in a special (but not implausible) case: same optimal policy implications as in the closed economy (domestic inflation targeting).
- optimal policy associated with large fluctuations in nominal exchange rate.
- extensions:
  - sticky wages
  - limited pass-through
  - incomplete markets.
  - fiscal policy
  - optimal policy design in a monetary union