Monetary Policy and Exchange Rate Volatility in a Small Open Economy

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Motivation

- The new Keynesian model for the closed economy

  - equilibrium dynamics: simple three-equation representation
  - optimal monetary policy design: optimality of inflation targeting

- How does the introduction of open economy elements affect that analysis and prescriptions?

- What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?

  $\Rightarrow$ main finding: equivalence result (for a benchmark model)

A New Keynesian Model of a Small Open Economy

Households

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \; \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$\int_0^1 P_{H,t}(j) C_{H,t}(j)\, dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j)\, dj\, di \; + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t$$

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \; C_{H,t}{}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \; C_{F,t}{}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

$$C_{H,t} = (\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} \; di \right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \; dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Firms

$$Y_t(i) = A_t \ N_t(i)$$

$$a_t = \rho \ a_{t-1} + u_t$$

Law of One Price

$$p_{i,t}(j) = e_{i,t} + p^i_{i,t}(j)$$

$$\Rightarrow \ p_{F,t} = e_t + p^*_t$$

Some Identities and Definitions

Terms of Trade

$$s_t \equiv p_{F,t} - p_{H,t}$$

CPI

$$\begin{aligned} p_t &\equiv (1-\alpha)\, p_{H,t} + \alpha\, p_{F,t} \\ &= p_{H,t} + \alpha\, s_t \end{aligned}$$

CPI Inflation vs. Domestic Inflation:

$$\pi_t = \pi_{H,t} + \alpha\, \Delta s_t$$

Real Exchange Rate and the Terms of Trade

$$\begin{aligned} q_t &\equiv (e_t + p_t^*) - p_t \\ &= s_t + (p_{H,t} - p_t) \\ &= (1-\alpha)\, s_t \end{aligned}$$

Optimal Intratemporal Allocation of Expenditures:

- within each category:

$$\begin{aligned} c_{H,t}(i) &= -\varepsilon \ (p_{H,t}(i) - p_{H,t}) + c_{H,t} \\ c_{F,t}(i) &= -\gamma \ (p_{F,t}(i) - p_{F,t}) + c_{F,t} \end{aligned}$$

- between categories:

$$\begin{aligned} c_{H,t} &= -\eta \ (p_{H,t} - p_t) + c_t \\ c_{F,t} &= -\eta \ (p_{F,t} - p_t) + c_t \end{aligned}$$

Other Optimality Conditions:

$$w_t - p_t = \sigma \ c_t + \varphi \ n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \ (r_t - E_t\{\pi_{t+1}\} - \rho)$$

International Risk Sharing (Complete Markets)

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}$$
$$\beta \left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma} \left(\frac{P^i_t}{P^i_{t+1}}\right) \left(\frac{\mathcal{E}^i_t}{\mathcal{E}^i_{t+1}}\right) = Q_{t,t+1}$$

Combining domestic and world foc's:

$$C_t = \vartheta_i \; C^i_t \; \mathcal{Q}_{i,t}{}^{\frac{1}{\sigma}}$$

Log-linearizing (after $\vartheta_i = 1$):

$$\begin{aligned} c_t &= c^*_t + \frac{1}{\sigma}\; q_t \\ &= c^*_t + \frac{1-\alpha}{\sigma}\; s_t \end{aligned}$$

Uncovered Interest Parity

Complete markets:

$$E_t\{Q_{t,t+1}\ [R_t - R_t^i\ (\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t})]\} = 0$$

Log-linearizing and aggregating over i:

$$r_t - r_t^* = E_t\{\Delta e_{t+1}\}$$

Combined with the definition of the terms of trade:

$$s_t = (r_t^* - E_t\{\pi_{t+1}^*\}) - (r_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\}$$

Integrating forward, and using $\lim_{T\to\infty} E_t\{s_T\} = 0$:

$$s_t = E_t\left\{\sum_{k=0}^{\infty} [(r_{t+k}^* - \pi_{t+k+1}^*) - (r_{t+k} - \pi_{H,t+k+1})]\right\}$$

Equilibrium Dynamics in the SOE: A Canonical Representation

$$\pi_{H,t} = \beta\ E_t\{\pi_{H,t+1}\} + \kappa_\alpha\ \widetilde{y}_t$$

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}\ (r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t)$$

where

$$\begin{aligned}
\widetilde{y}_t &= y_t - \overline{y}_t \\
\overline{y}_t &= \Omega + \Gamma\ a_t + \alpha\Psi\ y_t^* \\
\overline{rr}_t &\equiv \rho - \sigma_\alpha\Gamma(1-\rho_a)\ a_t + \alpha\sigma_\alpha(\Theta + \Psi)\ E_t\{\Delta y_{t+1}^*\}
\end{aligned}$$

$$\kappa_\alpha \equiv \lambda\,(\sigma_\alpha + \varphi) \quad ; \quad \sigma_\alpha \equiv \frac{\sigma}{(1-\alpha) + \alpha\omega} \quad ; \quad \Gamma \equiv \frac{1+\varphi}{\sigma_\alpha + \varphi} \quad ; \quad \Psi \equiv -\ \frac{\Theta\ \sigma_\alpha}{\sigma_\alpha + \varphi}$$

Aggregate Demand and Output Determination

World Market Clearing (WMC)

$$y_t^* = c_t^*$$

Domestic Market Clearing (DMC)

$$\begin{aligned} y_t &= c_t + \alpha\gamma\ s_t + \alpha\left(\eta - \frac{1}{\sigma}\right)\ q_t \\ &= c_t + \frac{\alpha\omega}{\sigma}\ s_t \end{aligned}$$

where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$.

Combining DMC and WMC with IRS we obtain:

$$y_t = y_t^* + \frac{1}{\sigma_\alpha}\ s_t \tag{1}$$

where $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} > 0$.

Finally, combining DMC with the Euler equation:

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha\omega}{\sigma}\,E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma}\,E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\,(r_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha\Theta\,E_t\{\Delta y^*_{t+1}\} \end{aligned}$$

where $\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$ and $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega}$.

Letting $\widetilde{y}_t = y_t - \overline{y}_t$,

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}\,(r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t)$$

where $\overline{rr}_t \equiv \rho + \sigma_\alpha(\Delta\overline{y}_{t+1} + \alpha\Theta\,E_t\{\Delta y^*_{t+1}\})$

The New Keynesian Phillips Curve in the Small Open Economy

Domestic Price Dynamics:

$$p_{H,t} \equiv \theta \; p_{H,t-1} + (1-\theta) \; \overline{p}_{H,t}$$

Optimal Price Setting:

$$\overline{p}_{H,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k} + p_{H,t}\}$$

Domestic Inflation Dynamics

$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \lambda \; \widehat{mc}_t \tag{2}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$

Marginal Cost and the Output Gap

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + \sigma\ c_t + \varphi\ n_t + \alpha\ s_t - a_t \\ &= -\nu + \sigma\ y_t^* + \varphi\ y_t + s_t - (1+\varphi)\ a_t \end{aligned}$$

Substituting for s_t using (1):

$$mc_t = -\nu + (\sigma_\alpha + \varphi)\ \ y_t + (\sigma - \sigma_\alpha)\ \ y_t^* - (1+\varphi)\ a_t$$

Under flexible prices,

$$-\mu = -\nu + (\sigma_\alpha + \varphi)\ \ \overline{y}_t + (\sigma - \sigma_\alpha)\ \ y_t^* - (1+\varphi)\ a_t$$

Thus,

$$\overline{y}_t = \Omega + \Gamma\ a_t + \alpha\Psi\ y_t^*$$

Also

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)\ \ \widetilde{y}_t$$

which combined with (2) yields:

$$\pi_{H,t} = \beta\ E_t\{\pi_{H,t+1}\} + \kappa_\alpha\ \widetilde{y}_t$$

Optimal Monetary Policy

Background and Strategy

A Special Case

$$\sigma = \eta = \gamma = 1$$

$$\Rightarrow \quad C_t = Y_t^{1-\alpha}(Y_t^*)^\alpha \tag{3}$$

Optimal Allocation:

$$\max \ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$$

subject to

$$\begin{aligned} C_t &= Y_t^{1-\alpha}(Y_t^*)^\alpha \\ &= (A_t N_t)^{1-\alpha}(Y_t^*)^\alpha \end{aligned}$$

Optimality condition:

$$N = (1-\alpha)^{\frac{1}{1+\varphi}}$$

Flexible Price Equilibrium

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= \overline{MC}_t \\ &= -\frac{(1-\tau)}{A_t}\, \overline{S}_t^{\alpha}\, \frac{U_N(\overline{C}_t, \overline{N}_t)}{U_C(\overline{C}_t, \overline{N}_t)} \\ &= \frac{(1-\tau)}{A_t}\, \frac{\overline{Y}_t}{\overline{C}_t}\, \overline{N}_t^{\varphi}\, \overline{C}_t \\ &= (1-\tau)\, \overline{N}_t^{1+\varphi} \end{aligned}$$

Optimality of Flexible Price Equilibrium:

$$(1-\tau)(1-\alpha) = 1 - \frac{1}{\varepsilon}$$

Implied Monetary Policy Objectives

$$y_t = \overline{y}$$

$$\pi_{H,t} = 0$$

for all t.

Implementation

$$r_t = \overline{rr}_t + \phi_\pi \; \pi_{H,t} + \phi_x \; x_t$$

where $\kappa_\alpha \; (\phi_\pi - 1) + (1 - \beta) \; \phi_x > 0$.

Other Macroeconomic Implications

Terms of Trade

$$\begin{aligned} \overline{s}_t &= \sigma_\alpha \; (\overline{y}_t - y_t^*) \\ &= \sigma_\alpha \Omega + \sigma_\alpha \Gamma \; a_t - \sigma_\alpha \Phi \; y_t^* \end{aligned}$$

where $\Phi \equiv \frac{\sigma + \varphi}{\sigma_\alpha + \varphi} > 0$.

Special case

$$\overline{s}_t = a_t - y_t^*$$

Exchange Rate

$$\overline{e}_t = \overline{s}_t - p_t^*$$

CPI

$$\begin{aligned} \overline{p}_t &= \alpha \; (\overline{e}_t + p_t^*) \\ &= \alpha \; \overline{s}_t \end{aligned}$$

Consequences of Suboptimal Policies

Welfare Losses (special case)

$$\mathbb{W} = -\frac{(1-\alpha)}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\,\pi_{H,t}^2 + (1+\varphi)\,x_t^2\right]$$

Taking unconditional expectations and letting $\beta \to 1$,

$$\mathbb{V} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}\,var(\pi_{H,t}) + (1+\varphi)\,var(x_t)\right]$$

Three Simple Rules

Domestic inflation-based Taylor rule (DITR)

$$r_t = \rho + \phi_\pi \, \pi_{H,t}$$

CPI inflation-based Taylor rule (CITR):

$$r_t = \rho + \phi_\pi \, \pi_t$$

Exchange rate peg (PEG)

$$e_t = 0$$

TABLE 1

Cyclical properties of alternative policy regimes

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0·95	0·68	0·72	0·86
Domestic inflation	0·00	0·27	0·27	0·36
CPI inflation	0·38	0·41	0·27	0·21
Nominal I. rate	0·32	0·41	0·41	0·21
Terms of trade	1·60	1·53	1·43	1·17
Nominal depr. rate	0·95	0·86	0·53	0·00

TABLE 2

Contribution to welfare losses

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1{\cdot}2, \varphi = 3$			
Var(domestic infl)	0·0157	0·0151	0·0268
Var(output gap)	0·0009	0·0019	0·0053
Total	0·0166	0·0170	0·0321
Low steady state mark-up $\mu = 1{\cdot}1, \varphi = 3$			
Var(Domestic infl)	0·0287	0·0277	0·0491
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0297	0·0296	0·0544
Low elasticity of labour supply $\mu = 1{\cdot}2, \varphi = 10$			
Var(Domestic infl)	0·0235	0·0240	0·0565
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0240	0·0261	0·0630
Low mark-up and elasticity of labour supply $\mu = 1{\cdot}1, \varphi = 10$			
Var(Domestic infl)	0·0431	0·0441	0·1036
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0436	0·0461	0·1101

Note: Entries are percentage units of steady state consumption.

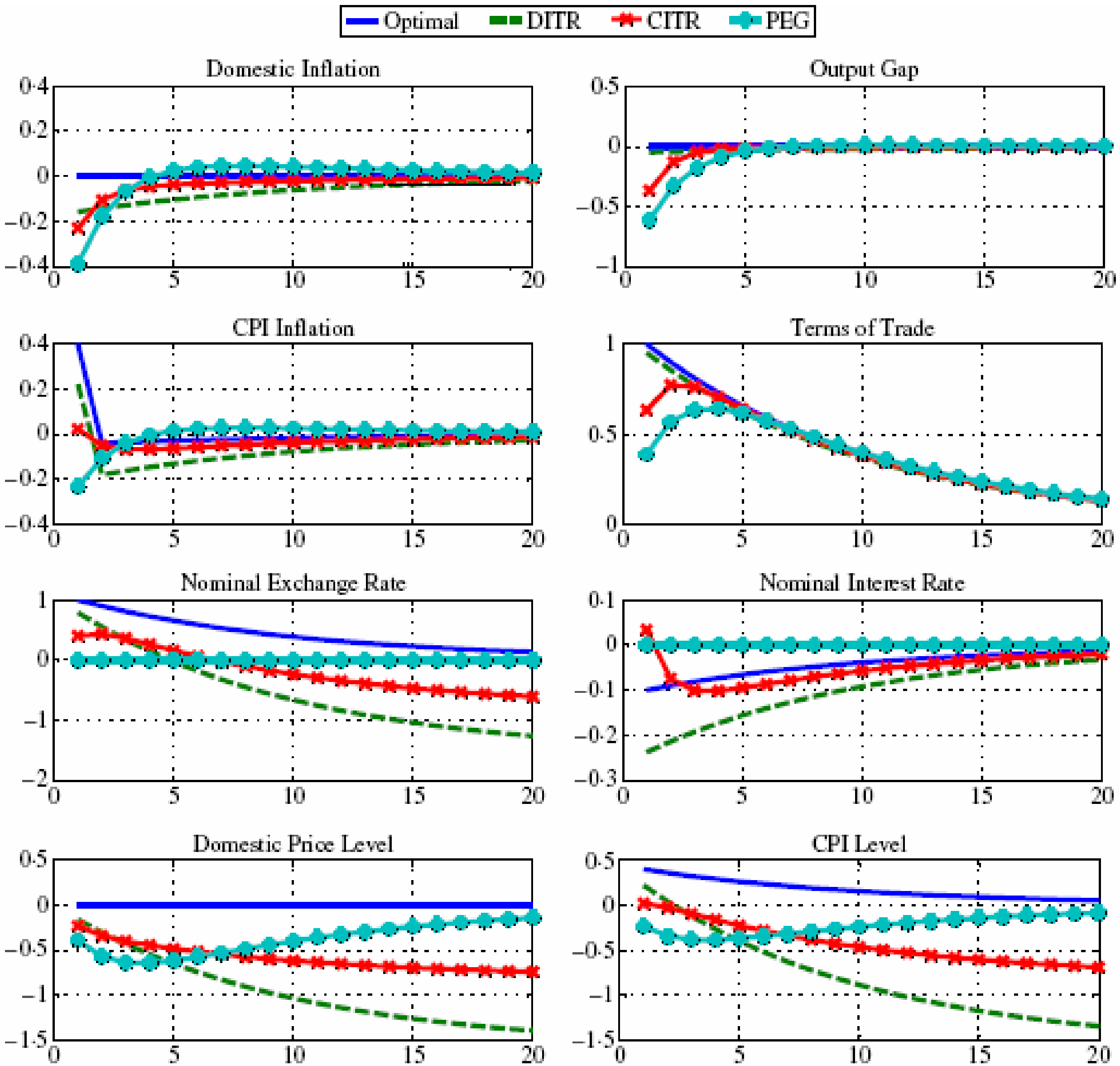

FIGURE 1

Impulse responses to a domestic productivity shock under alternative policy rules

Concluding Remarks

- small open economy version of the new Keynesian model
- under baseline assumptions (complete markets, full pass-through), equilibrium dynamics equivalent to the closed economy
- in a special (but not implausible) case: same optimal policy implications as in the closed economy (domestic inflation targeting).
- optimal policy associated with large fluctuations in nominal exchange rate.
- extensions:

    - sticky wages
    - limited pass-through
    - incomplete markets.
    - fiscal policy
    - optimal policy design in a monetary union