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Consider the following infinite-horizon economy in continuous time. Production requires 1 firm - 1 worker. There is an exogenous population of workers normalized to 1, and free entry of firms. Each firm has the production function $f(k)$, where k is capital for worker chosen before the matching stage by the firm, with price normalized to 1. Assume that $f' > 0, \quad f'' < 0$. The capital stock of the firm is destroyed at the flow rate s, in which case the match between the worker and the firm is also dissolved. Search frictions are modeled as follows: if there are an average of q workers per vacancy of a certain type (distinguished by any observable characteristic), then the flow rate of match for workers is $\mu(q)$, which is assumed to be continuously differentiable with $\mu' < 0$. Similarly, the flow rate of matching for a vacancy of this type is $\eta(q) \equiv q\mu(q)$, with $\eta' > 0$. All agents maximize the discounted value of expected income, with a discount rate r. Wages are determined by Nash bargaining between a pair of worker-firm, with the bargaining power of the worker denoted by β.

1. Write the steady-state Bellman equations for a filled job with capital stock k, $J^F(k)$, a vacant job with capital stock k, $J^V(k)$, and an employed worker at a firm with capital stock k, $J^E(k)$.

2. Assume that workers observe no characteristic of firms (so search is undirected). Write down the steady state value of an unemployed worker, J^U. Explain why the free entry condition for firms should be written as $J^V(k) \leq k$ for all k and $J^V(k) = k$ if k is chosen by some firms in equilibrium. Determine the steady-state equilibrium with free entry (Hint: determine both the conditions for the equilibrium level of

investment and the equilibrium tightness in the labor market). Will there be more than one level of k chosen by firms in equilibrium?

3. Explain how this equilibrium differs from the constrained efficient allocation that a social planner would choose (you do not need to derive the constraint efficient allocation unless you have extra time).

4. Now suppose that workers can direct their search towards firms with different levels of capital stock (so while before we had q, now we should have $q(k)$ as workers per vacancy with capital stock k). Determine the free-entry equilibrium (Hint: you can simplify the derivation by assuming that all firms will choose the same level of k along the equilibrium path).

5. Compare the equilibria with directed and undirected search. Which one generates higher unemployment? Is the equilibrium necessarily inefficient now? Discuss with as much detail as you wish.

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Consider a world economy consisting of a Northern country and a large number of Southern countries. Normalize the total labor supply in the North and in the South to 1. There is free trading intermediates and no trade in final goods. All countries have access to the same production function for the final good

$$Y_t = \left(\int_0^n y_t(i)^\alpha di \right)^{1/\alpha},$$

where $y_t(i)$ is the amount of intermediate good i used in the production of the final good at time t and $\alpha \in (0, 1)$.

All countries are also inhabited by a representative consumer with preferences

$$\int_0^\infty \exp(-\rho t) \frac{C_t^{1-\theta} - 1}{1-\theta} dt.$$

The budget constraint of the economy is

$$X_t + C_t \leq Y_t,$$

where C is consumption and X R&D expenditure. The production function for each intermediate is

$$\tilde{y}_t(i) = l_t(i)$$

with market clearing requiring $\int_0^n l_t(i) \leq 1$. In this equation I distinguish between $\tilde{y}(i)$, which is production of intermediate good, i, different from the use of intermediate good i in the production of the final good because of international trade in intermediates.

Trade balance requires

$$\int_0^n p_t(i)\, y_t(i)\, di = \int_0^n p_t(i)\, \tilde{y}_t(i)\, di$$

where $p_t(i)$ is the world price of intermediate i at time t.

New goods are invented only in the North according to R&D technology

$$\dot{n}_t = \eta X_t.$$

The inventor of a new good in the North is the monopolist until the good is copied by the South, after which it is produced competitively (either in the South or in the North).

1. Characterize world prices and northern and southern wages assuming that there exist a total of n goods, n^N of which can only be produced in the North and the remaining n^S that can be produced both in the North and in the South. Will the North have necessarily higher wages than the South? If not, why not? (Hint: Notice that when the North does not produce any of the old, n^S, goods, all employers in the North are monopolists; whereas when some of the old goods are also produced

in the North, some producers face competitive markets).

2. Assume now that there is a constant rate of imitation, τ, of goods by Southern producers, so that

$$\dot{n}_t^S = \tau n_t^N.$$

Characterize the balanced growth path equilibrium where n^N and n^S grow at a constant (endogenous) rate g. What happens when τ increases? What happens when η increases?

3. Can an increase in τ hurt the North, the South? How would your answer be different if the growth rate of n^N were exogenous?