14.461 Advanced Macroeconomics I: Part 2: Unemployment Differences, Fluctuations, Job Creation and Job Destruction

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1 Unemployment Facts

1.1 Introduction

A couple of introductory remarks are in order:

1. The simplest approach to unemployment is to ignore it, and lump it together with non-participation, under the heading of nonemployment. Then the supply and demand for work will determine wages and nonemployment. This is a very useful starting place. In particular, it emphasizes that often unemployment (nonemployment) is associated with wages above the market clearing wage levels.

2. However, the distinction between voluntary and involuntary unemployment is sometimes useful, especially since many workers claim to be

looking for work rather than being outside the labor force. Moreover, when there are very high levels of nonemployment among prime-age workers, the nonemployment framework may no longer be satisfactory. Therefore, three questions important in motivating the theories of unemployment are:

(a) Why do most capitalist economies have a significant fraction, typically at least around 5 percent, of their labor forces as "unemployed"?

(b) Why does unemployment increase during recessions? Why does it increase so much? (Or so little depending on what the benchmark is).

(c) Why has unemployment in continental Europe been extremely high over the past 25 years?

So far we have developed the search-theoretic framework for trading in labor (and other) markets. One distinctive feature of this framework is that it allows for unemployment and unfilled vacancies (jobs seeking workers) at the same time as an equilibrium phenomenon, because it recognizes that labor market matching is a time-consuming and costly process. We can now use this framework to understand both unemployment differences across countries (or regions) and unemployment fluctuations over time.

1.2 Some Basic Facts About Unemployment

Here is a very brief summary of some basic stylized facts about unemployment, useful to bear in mind when thinking about the search models.

1.2.1 Cyclical Patterns

1. The unemployment rate in the U.S. fluctuates around six percent, and is strongly countercyclical, sometimes with large fluctuations (see figure 1 below).

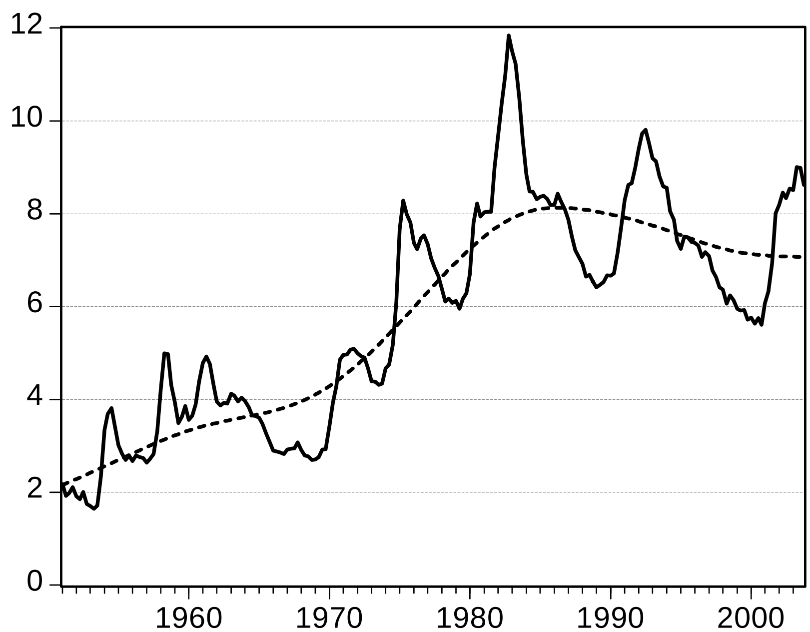

Figure 1

2. Vacancies (measured either as help-wanted ads in the United States, or as job openings in other countries) are even more strongly procyclical,

so that vacancy-unemployment ratio is procyclical (see figures 2 and 3 below).

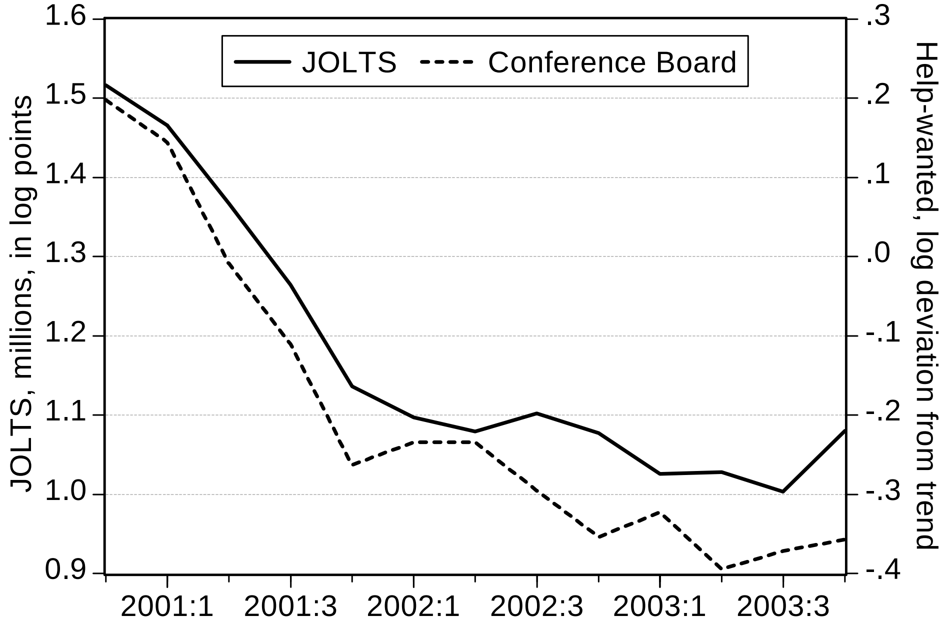

Figure 2

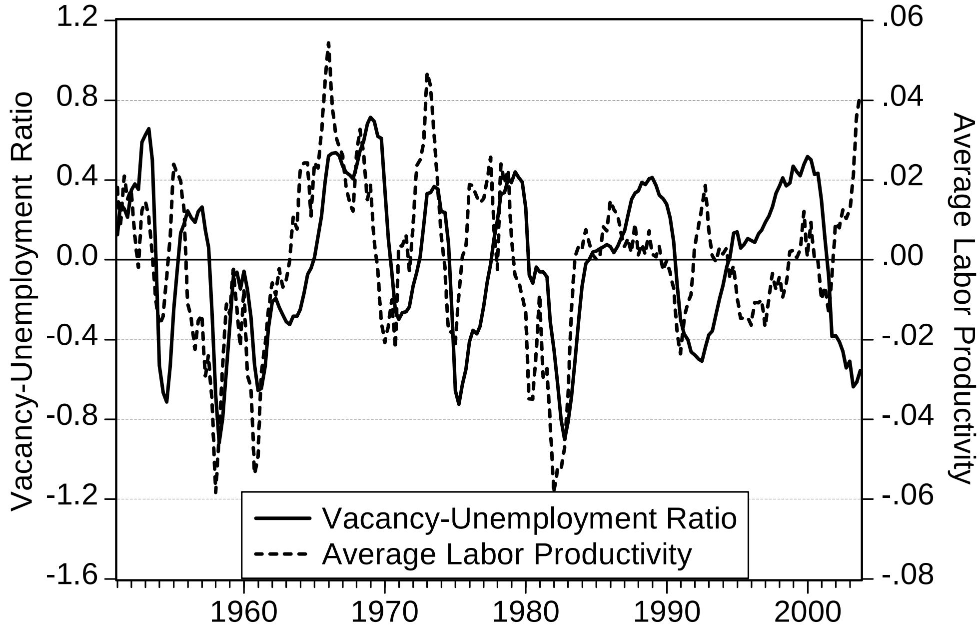

Figure 3

3. Short-run fluctuations in vacancies and unemployment correspond to a Beveridge curve, with a downward sloping relationship (see figure 4 below).

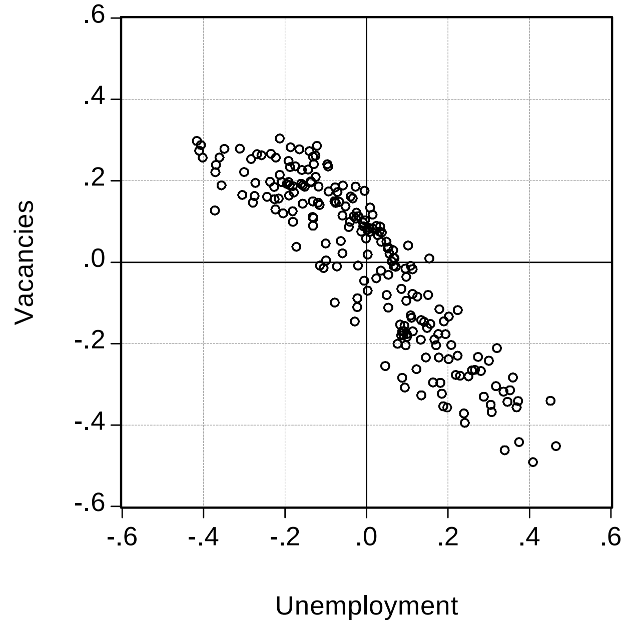

Figure 4

4. Although there is some debate on this point, wages are somewhat procyclical. Measuring the procyclicality of wages is difficult because of a number of reasons. First, what exact deflator, which time period and what frequency one looks at (i.e., the method of detrending) seems to matter a lot. More conceptually, there is the important issue of composition bias (workers who keep their jobs during recessions are different from those who lose their jobs). The survey by Abraham and Haltiwanger (JEL, 1995) presents a very comprehensive account of the issues. Figures 5 and 6 below give some pictures (depending on the trending etc.), which show slightly procyclical pattern, but again this is

quite sensitive to various different methods of detrending and deflating.

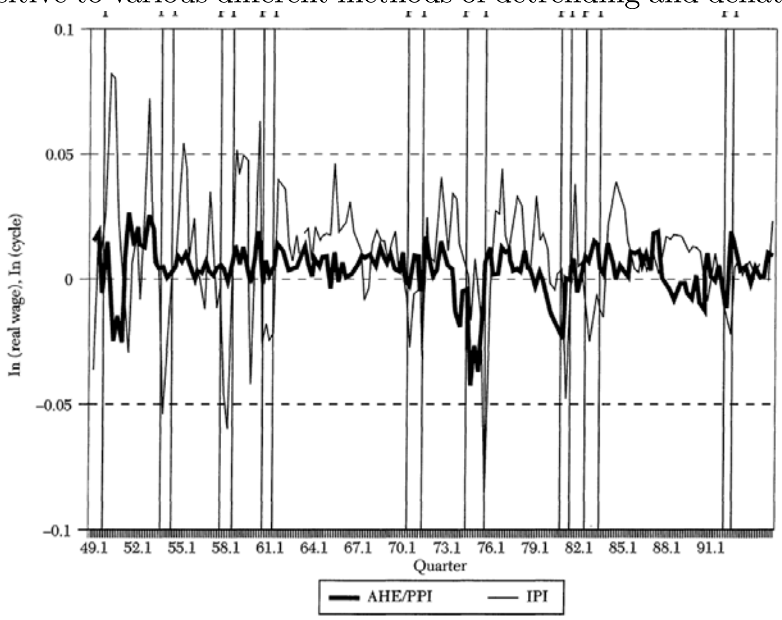

Figure 4b. Real Wages and IPI—Manufacturing, 1949–93 (First Differences)

Figure 5

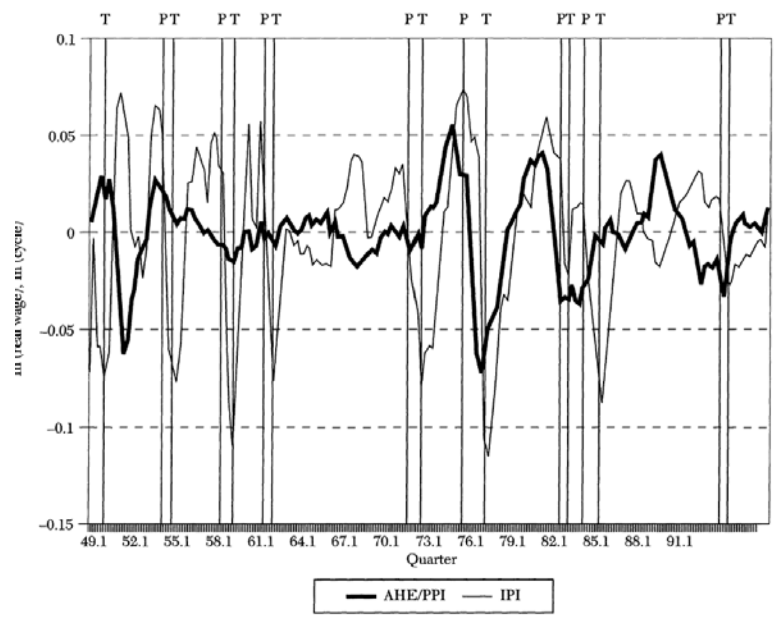

Figure 4c. Real Wages and IPI–Manufacturing, 1949–93 (HP Filter)

Figure 6

5. Davis and Haltiwanger show that job destruction in the manufacturing sector is strongly countercyclical, with significant plant closings or layoffs during recessions. New data from JOLTS survey, on the other hand, suggest that during the last recession, there has not been a significant increase in the entry of workers into unemployment. This may be because of differences between worker flows versus job flows, the cause manufacturing is different, or because the most recent recession is different. Hall (2005) based on JOLTS argues that we only need to look at job creation to understand unemployment dynamics.

1.2.2 Cross-Sectional or Cross-Country Patterns

1. There has been a large increase in unemployment among OECD countries, mostly driven by continental Europe.

Chart 1.1. **Unemployment in the OECD area, 1950-95**[1]
Millions

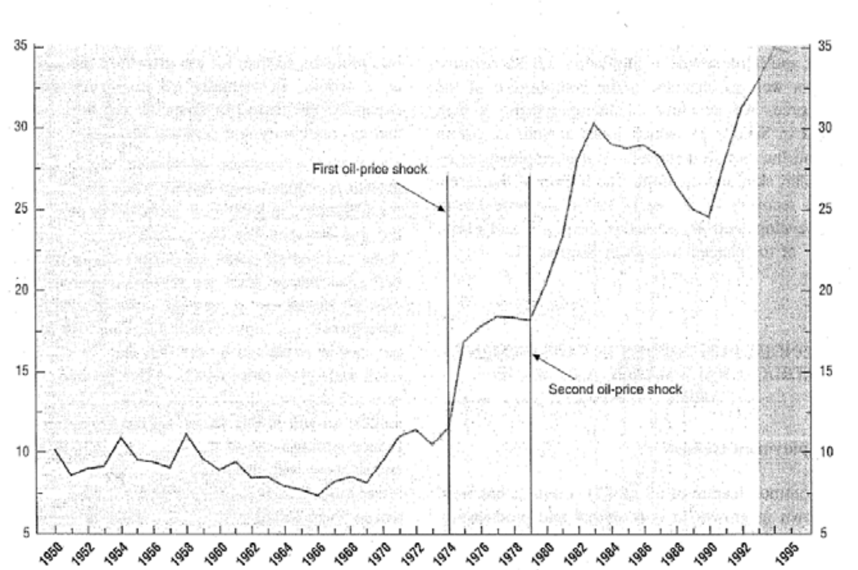

Figure 7

2. This is driven mostly by slow employment growth in continental Europe.

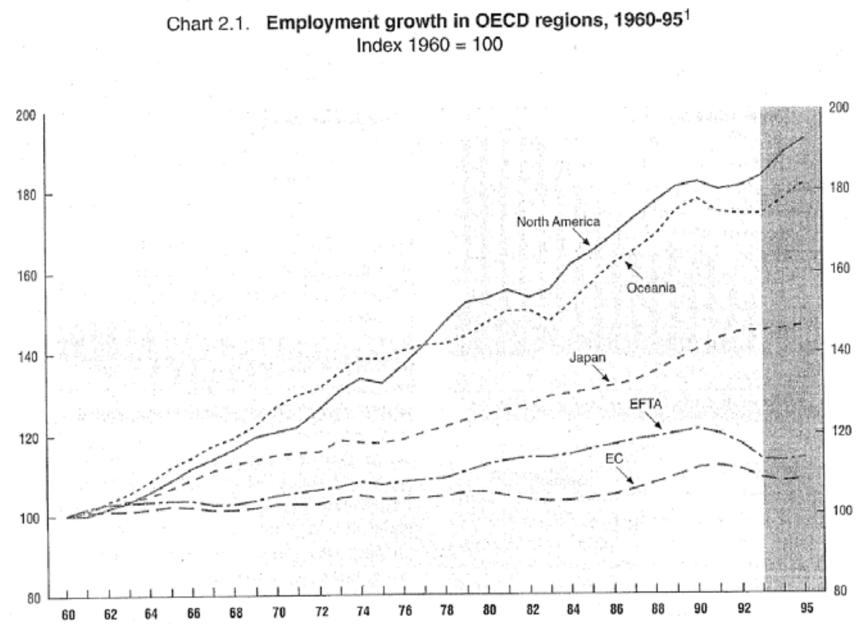

Figure 8

3. There is also some evidence that during this period, wages have grown faster in continental Europe than in North America. This pattern is also supported by the behavior of the labor share in GDP (see, for example, Blanchard "The Medium Run").

Chart 2.3. **Employment and real wages**
Index 1970 = 100

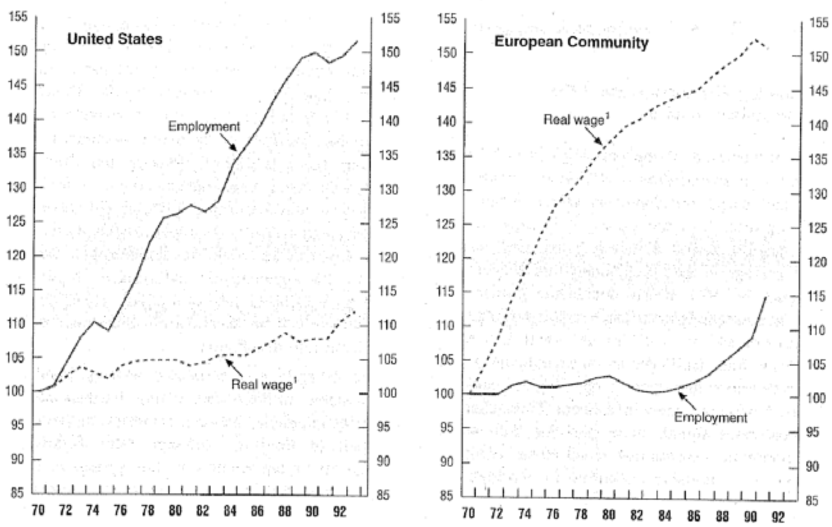

Figure 9

4. However, the unemployment rate in continental Europe was lower than in the U.S. throughout the postwar period, but rose above the U.S. level in the late 1970s or early 1980s, and has been consistently higher than the U.S. level.

5. The unemployment rate in some countries, notably Spain, has reached, and for a long while stayed at, 20 percent.

6. High unemployment in Europe reflects low rates of employment creation—that is, unemployed workers leave unemployment only slowly compared to the U.S.. Rates at which employed workers lose their jobs and be-

come unemployed are higher in the U.S. than in Europe. Unemployment in continental Europe increased especially among young workers, and nonemployment has affected prime-age males less, but there is still some effect. It is the young, the old or the women (in some countries) who do not work in Europe.

Chart 1.11. **Non-employment rates for the prime working-age population, 1972-92**[1]
Percentage of population aged 25-54 not in employment

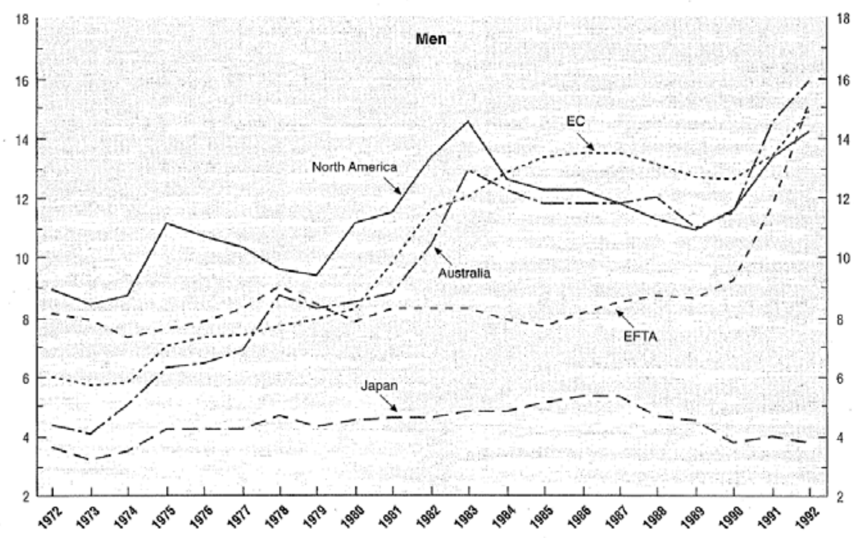

Figure 10

7. Unemployment in continental Europe increased both among the less and more educated workers (e.g., Nickell and Bell, 1994).

2 Review of the Basic Search Model

Now I discuss the application of the search-matching model developed before to thinking about unemployment. Given the material we have covered so far, this is simply a review of the basic Mortensen-Pissarides model, and an effort to replicate what we have done so far in their notation.

Matching Function: Matches $M = M(U, V)$

Let us now impose constant returns to scale from the beginning:

$$\begin{aligned} M &= xL = M(uL, vL) \\ &\Longrightarrow x = M(u, v) \end{aligned}$$

where

U =unemployment;

u =unemployment rate

V =vacancies;

v = vacancy rate (per worker in labor force)

L = labor force

x = match per labor force participant

Existing aggregate evidence suggests that the assumption of x exhibiting CRS is reasonable (e.g., Blanchard and Diamond, 1989)

Using the constant returns assumption, we can express everything as a function of the tightness of the labor market.

Therefore;

$$q(\theta) \equiv \frac{x}{v} \equiv \frac{M}{V} = M\left(\frac{u}{v}, 1\right),$$

where $\theta \equiv v/u$ is the tightness of the labor market

Since we are in continuous time, these things immediately map to flow rates. Namely

$q(\theta)$: Poisson arrival rate of match for a vacancy

$\theta q(\theta)$:Poisson arrival rate of match for an unemployed worker

What does Poisson mean?

Take a short period of time Δt, then the Poisson process is defined such that during this time interval, the probability that there will be one arrival, for example one arrival of a job for a worker, is

$$\Delta t \theta q(\theta)$$

The probability that there will be more than one arrivals is vanishingly small (formally, of order $o(\Delta t)$).

Therefore,

$1 - \Delta t \theta q(\theta)$: probability that a worker looking for a job will not find one during Δt

This probability depends on θ, thus leading to a potential externality— the search behavior of others affects my own job finding rate.

The search model is also sometimes called the flow approach to unemployment because it's all about job flows. That is about job creation and job destruction.

This is another dividing line between labor and macro. Many macroeconomists look at data on job creation and job destruction following Davis and Haltiwanger. Most labor economists do not look at these data. Presumably there is some information in them.

Job creation is equal to

$$\text{Job creation} = u\theta q(\theta)L$$

What about job destruction?

I will start with the simplest model of job destruction, which is basically to treat it as "exogenous".

Think of it as follows, firms are hit by adverse shocks, and then they decide whether to destroy or to continue.

$$\longrightarrow \text{Adverse Shock} \longrightarrow \text{destroy}$$
$$\longrightarrow \text{continue}$$

Exogenous job destruction: Adverse shock $= -\infty$ with "probability " s

Let us first start with steady states, which will replicate what we have done before, and then we will see the generalization to non-steady-state dynamics which is relevant for business cycle features.

As before, the key feature of a steady state is that

$$\text{flow into unemployment} = \text{flow out of unemployment}$$

Therefore, with exogenous job destruction:

$$s(1-u) = \theta q(\theta) u$$

This gives the steady-state unemployment rate as

$$u = \frac{s}{s + \theta q(\theta)} \tag{1}$$

This relationship is sometimes referred to as the Beveridge Curve, or the U-V curve, since it is similar to the empirical relationship shown in Figure 4 above. It draws a downward sloping locus of unemployment-vacancy combinations in the U-V space that are consistent with flow into unemployment being equal to the flow out of unemployment.

Some authors interpret shifts of this relationship as reflecting structural changes in the labor market, but we will see that there are many factors that might actually shift a generalized version of such a relationship.

Equation (1) is crucial even if you don't like the search model. It relates the unemployment rate to the rate at which people leave their jobs and and unemployment and the rate at which people leave the unemployment pool.

In a more realistic model, of course, we have to take into account the rate at which people go and come back from out-of-labor force status.

Let's turn next to the production side.

Let the output of each firm be given by neoclassical production function combining labor and capital:

$$Y = AF(K, N)$$

where the production function F is assumed to exhibit constant returns, K is the capital stock of the economy, and N is employment (different from labor force because of unemployment).

Defining $k \equiv K/N$ as the capital labor ratio, we have that output per worker is:

$$\frac{Y}{N} = Af(k) \equiv AF(\frac{K}{N}, 1)$$

because of constant returns.

Two interpretations $\longrightarrow$ each firm is a "job" and hires one worker

each firm can hire as many worker as it likes

For our purposes either interpretation is fine.

Hiring: Vacancy costs γ_0: fixed cost of hiring

r: cost of capital

δ: depreciation

The key assumption here is that capital is perfectly reversible.

Namely, let

J^V : PDV of a vacancy

J^F :PDV of a "job"

J^U :PDV of a searching worker

J^E :PDV of an employed worker

More generally, we have that worker utility is: $EU_0 = \int_0^\infty e^{-rt}U\left(c_t\right)$, but for what we care here, risk-neutrality is sufficient.

Utility $U(c) = c$, in other words, linear utility, so agents are risk-neutral.

Perfect capital market gives the asset value for a vacancy (in steady state) as

$$rJ^V = -\gamma_0 + q(\theta)(J^F - J^V)$$

Intuitively, there is a cost of vacancy equal to γ_0 at every instant, and the vacancy turns into a filled job at the flow rate $q\left(\theta\right)$.

Notice that in writing this expression, I have assumed that firms are risk neutral. Why?

$\longrightarrow$ workers risk neutral, or

$\longrightarrow$ complete markets

The question is how to model job creation (which is the equivalent of how to model labor demand in a competitive labor market).

Presumably, firms decide to create jobs when there are profit opportunities.

The simplest and perhaps the most extreme form of endogenous job creation is to assume that there will be a firm that creates a vacancy as soon as the value of a vacancy is positive (after all, unless there are scarce factors necessary for creating vacancies anybody should be able to create one).

This is sometimes referred to as the free-entry assumption, because it amounts to imposing that whenever there are potential profits they will be eroded by entry.

Free Entry $\Longrightarrow$

$$J^V \equiv 0$$

The most important implication of this assumption is that job creation can happen really "fast", except because of the frictions created by matching searching workers to searching vacancies.

Alternative would bethat the cost of opening vacancies is itself a function of the total number of vacancies or the tightness of the labor market, e.g.,

$$\gamma_0 = \Gamma_0(V) \text{ or } \Gamma_1(\theta)$$

If this were to case, there would be greater cost of creating vacancies when there are more vacancies created, adding additional sluggishness to dynamics.

Free entry implies that

$$J^F = \frac{\gamma_0}{q(\theta)}$$

Next, we can write another asset value equation for the value of a field job:

$$r(J^F + k) = Af(k) - \delta k - w - s(J^F - J^V)$$

Intuitively, the firm has two assets: the fact that it is matched with a worker, and its capital, k. So its asset value is $J^F + k$ (more generally, without the perfect reversability, we would have the more general $J^F(k)$). Its return is equal to production, $Af(k)$, and its costs are depreciation of capital and wages, δk and w. Finally, at the rate s, the relationship comes to an end and the firm loses J^F.

Perfect Reversability implies that w does not depend on the firm's choice of capital

$\Longrightarrow$ equilibrium capital utilization $f'(k) = r+\delta$— Modified Golden Rule

[...Digression: Suppose k is not perfectly reversible then suppose that the worker captures a fraction β all the output in bargaining. Then the wage depends on the capital stock of the firm, as in the holdup models discussed before.

$$\begin{aligned} w(k) &= \beta Af(k) \\ Af'(k) &= \frac{r+\delta}{1-\beta} \text{ ; capital accumulation is distorted} \end{aligned}$$

...]

Now, ignoring this digression

$$Af(k) - (r-\delta)k - w - \frac{(r+s)}{q(\theta)}\gamma_0 = 0$$

Now returning to the worker side, the risk neutrality of workers gives

$$rJ^U = z + \theta q(\theta)(J^E - J^U)$$

where z is unemployment benefits. The intuition for this equation is similar. We also have

$$rJ^E = w + s(J^U - J^E)$$

Solving these equations we obtain

$$\begin{aligned} rJ^U &= \frac{(r+s)z + \theta q(\theta)w}{r+s+\theta q(\theta)} \\ rJ^E &= \frac{sz + [r+\theta q(\theta)]\,w}{r+s+\theta q(\theta)} \end{aligned}$$

How are wages determined? Again Nash Bargaining, with the worker having bargaining power β.

Applying this formula, for pair i, we have

$$\begin{aligned} rJ_i^F &= Af(k) - (r+\delta)k - w_i - sJ_i^F \\ rJ_i^E &= w_i - s(J_i^E - J^U) \end{aligned}$$

thus the Nash solution will solve

$$\max(J_i^E - J^U)^\beta (J_i^F - J^V)^{1-\beta}$$

$$\beta \quad = \quad \text{bargaining power of the worker}$$

Since we have linear utility, thus " transferable utility", this implies

$$\Longrightarrow \quad J_i^E - J^U = \beta(J_i^F + J_i^E - J^V - J^U)$$

$$\Longrightarrow \quad w = (1-\beta)z + \beta\left[Af(k) - (r+\delta)k + \theta\gamma_0\right]$$

Here $[Af(k) - (r+\delta)k + \theta\gamma_0]$ is the quasi-rent created by a match that the firm and workers share. Why is the term $\theta\gamma_0$ there?

Now we are in this position to characterize the steady-state equilibrium.

Steady State Equilibrium is given by four equations

(1) The Beveridge curve:

$$u = \frac{s}{s + \theta q(\theta)}$$

(2) Job creation leads zero profits:

$$Af(k) - (r+\delta)k - w - \frac{(r+s)}{q(\theta)}\gamma_0 = 0$$

(3) Wage determination:

$$w = (1-\beta)z + \beta\left[Af(k) - (r+\delta)k + \theta\gamma_0\right]$$

(4) Modified golden rule:

$$Af'(k) = r + \delta$$

These four equations define a block recursive system

$(4) + r \longrightarrow k$

$k + r + (2) + (3) \longrightarrow \theta, w$

$\theta + (1) \longrightarrow u$

Alternatively, combining three of these equations we obtain the zero-profit locus, the VS curve, and combine it with the Beveridge curve. More specifically,

$(2), (3), (4) \Longrightarrow$ the VS curve

$$(1-\beta)\left[Af(k) - (r+\delta)k - z\right] - \frac{r + s + \beta\theta q(\theta)}{q(\theta)}\gamma_0 = 0 \tag{2}$$

Therefore, the equilibrium looks very similar to the intersection of "quasi-labor demand" and "quasi-labor supply".

Quasi-labor supply is given by the Beveridge curve, while labor demand is given by the zero profit conditions.

Given this equilibrium, comparative statics (for steady states) are straightforward.

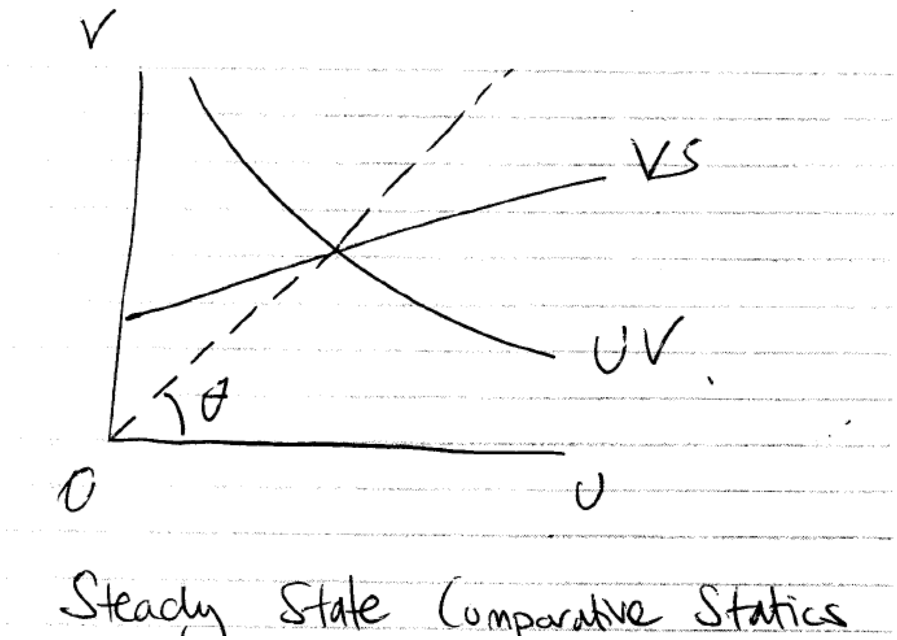

Figure 11

For example:

$s\uparrow$	$U\uparrow$	$V\uparrow$	$\theta\downarrow$	$w\downarrow$
$r\uparrow$	$U\uparrow$	$V\downarrow$	$\theta\downarrow$	$w\downarrow$
$\gamma_0\uparrow$	$U\uparrow$	$V\downarrow$	$\theta\downarrow$	$w\downarrow$
$\beta\uparrow$	$U\uparrow$	$V\downarrow$	$\theta\downarrow$	$w\uparrow$
$z\uparrow$	$U\uparrow$	$V\downarrow$	$\theta\downarrow$	$w\uparrow$
$A\uparrow$	$U\downarrow$	$V\uparrow$	$\theta\uparrow$	$w\uparrow$

Thus, a greater exogenous separation rate, higher discount rates, higher costs of creating vacancies, higher bargaining power of workers, higher unemployment benefits lead to higher unemployment. Greater productivity of jobs, leads to lower unemployment.

Interestingly some of those, notably the greater separation rate also increases the number of vacancies.

Can we think of any of these factors in explaining the rise in unemployment in Europe during the 1980s, or the lesser rise in unemployment in 1980s in in the United States?

3 Transitional Dynamics

A full analysis of business cycle dynamics requires a fully stochastic version of the above model. This can be done either in continuous time by introducing a continuous time stochastic process such as a Brownian motion or an Orenstein-Uhlenbeck process, or by specifying the model in discrete time and adding standard Markov-type disturbances.

Here, I only want to give a flavor of dynamics in this model, so I will continue to focus on the non-stochastic continuous time model we have developed so far, and think of a one-time (unanticipated or anticipated) shock hitting the economy.

This approach first requires us to write the dynamic programming equations allowing for non-steady state changes. Also to simplify, let us leave out capital choices, so that each job produces net output A. Then, we have

$$rJ^F - \dot{J}^F = A - w - s(J^F - J^V) \tag{3}$$

and

$$rJ^V - \dot{J}^V = -\gamma_0 + q(\theta)(J^F - J^V).$$

Free entry implies that

$$J^V \equiv 0,$$

i.e., the value of a vacant job is equal to zero at all times. Therefore,

$$\dot{J}^V = 0.$$

Consequently, we still have

$$J^F = \frac{\gamma_0}{q(\theta)}. \tag{4}$$

Now differentiating this equation with respect the time we have

$$\dot{J}^F = -\dot{\theta} q'(\theta) \frac{\gamma_0}{[q(\theta)]^2}$$

Now using (4), and defining the elasticity of the matching function as

$$\eta(\theta) \equiv -\frac{\theta q'(\theta)}{q(\theta)} > 0,$$

we have

$$\frac{\dot{J}^F}{J^F} = \frac{\dot{\theta}}{\theta} \eta(\theta)$$

Now using this with (3), (4) and $J^V \equiv 0$,

$$(r+s) - \frac{\dot{\theta}}{\theta} \eta(\theta) = \frac{(A-w) q(\theta)}{\gamma_0}.$$

The Nash bargaining solution is not affected (at least in the simplest version), so we still have

$$w = (1-\beta)z + \beta [A + \theta \gamma_0], \tag{5}$$

and combining this with the previous equation we have

$$\frac{\dot{\theta}}{\theta} = \frac{1}{\eta(\theta)} \left[(r+s) - \left(\frac{(1-\beta)(A-z) - \beta\theta\gamma_0}{\gamma_0} \right) q(\theta) \right]. \tag{6}$$

It is can be seen that (6), defines an unstable one dimensional differential equation (to see that it is unstable, it suffices to note that whenever $\dot{\theta} = 0$, the right hand side is increasing in θ), which follows from the fact that $q'(\theta) < 0$ (whether $\eta(\theta)$ is increasing or decreasing in θ does not matter since its derivative at $\dot{\theta} = 0$ multiplies an expression that's equal to zero).

The implication of equation (6) is therefore that there cannot be slow dynamics in θ. Starting from any non-steady-state value, i.e., $\theta_0 \neq \theta^*$, θ has to immediately jump to its steady-state value θ^* and after that initial instance, we will always have from then on having $\dot{\theta} = 0$. Moreover, it also implies that there will not been a dynamics in θ in response to an unanticipated permanent shock. Such a permanent shock would change the steady-state value θ^*, and there will be an immediate jump to this new steady-state value.

In fact, this is good, since θ is a control variable (not a pre-determined or state variable). It is determined by the number of vacancies that are opened at any given instant, which is a jump/control variable.

This therefore immediately establishes that adjustment of the vacancy to unemployment ratio in response to unanticipated permanent shocks will be instantaneous. For example, if A increases at some t^* in an unanticipated manner, θ^* at this point will jump up to a new steady-state level.

We have so far not talked about the dynamics of unemployment, which is our main focus. This is because of the *block recursive* structure of this model, which was noted earlier. We can always first find the equilibrium unemployment-vacancy ratio or the tightness, and then the unemployment rate.

More specifically, unemployment dynamics are given by our usual flow equation:

$$\dot{u} = s\,(1-u) - \theta q\,(\theta)\,u. \tag{7}$$

Therefore, in response to a permanent shock θ adjusts and stays at a constant level. However, unemployment being a state variable, will only adjust slowly following (7).

Instead of doing this in two steps, first looking at θ, and then at u, we could have naturally done this in one step. In that case the dynamical system would consist of the two equations, (6) and (7). It should be clear that this will give the same results. One easy way of seeing this is to note that in the neighborhood of the steady state, the qualitative behavior of this dynamical system could be represented as:

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} - & - \\ 0 & + \end{pmatrix} \begin{pmatrix} u \\ \theta \end{pmatrix}$$

Block recursiveness now corresponds to the fact that there is a "0" at the lower left-hand side corner. Given this pattern, it is obvious that this dynamical system, in the neighborhood of the steady state, can be represented by a linear system with one positive and one negative eigenvalue. Since there is one state and one control (jump) variable, this again amounts to the same thing, which is that θ will jump to its steady-state value immediately, and u will adjust slowly.

It is possible to get richer dynamics from this model in two different ways:

1. Consider transitory shocks. (Recall that there is also a debate on whether business cycles are best modeled as transitory shocks or permanent shocks).

2. Introduce some type of "wage rigidity" so that the wage equation (5) no longer applies. We will discuss this further below.

It turns out that neither of these are by themselves extremely useful, but this is to be discussed further below.

Question: how would you analyze the dynamic response of this system to an unanticipated transitory shock?

4 Endogenous Job Destruction

So far we treated the rate at which jobs get destroyed as a constant flow rate, s, giving us a simple unemployment-flow equation

$$\dot{u} = s(1-u) - \theta q\,(\theta)\,u$$

But presumably thinking of job destruction as exogenous is not satisfactory. Firms decide when to expand and contract, so it's a natural next step to endogenize s.

To do this, suppose that each firm consists of a single job (so we are now taking a position on firm size). Also assume that the productivity of each firm consists of two components, a common productivity and a firm-specific productivity.

In particular

$$\text{productivity for firm } i = \underbrace{p}_{\text{common productivity}} + \underbrace{\sigma \times \varepsilon_i}_{\text{firm-specific}}$$

where

$$\varepsilon_i \sim F(\cdot)$$

over support $\underline{\varepsilon}$ to $\bar{\varepsilon}$, and σ is a parameter capturing the importance of firm-specific shocks.

Moreover, suppose that each new job starts at $\varepsilon = \bar{\varepsilon}$, but does not necessarily stay there. In particular, there is a new draw from $F(\cdot)$ arriving at flow the rate λ.

[... How would you justify this assumption? Compar this with the assumption of idiosyncratic heterogeneity in match productivity in the baseline model covered earlier in the lectures...]

To simplify the discussion, let us ignore wage determination and set

$$w = b$$

This then gives the following value function (written in steady state) for a an active job with productivity shock ε (though this job may decide not to be active):

$$rJ^F(\varepsilon) = p + \sigma\varepsilon - b + \lambda \left[\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\{J^F(x), J^V\} dF(x) - J^F(\varepsilon) \right]$$

where J^V is the value of a vacant job, which is what the firm becomes if it decides to destroy. The max operator takes care of the fact that the firm has a choice after the realization of the new shock, x, whether to destroy or to continue.

Since with free entry $J^V = 0$, we have

$$rJ^F(\varepsilon) = p + \sigma\varepsilon - b + \lambda \left[E(J^F) - J^F(\varepsilon) \right] \tag{8}$$

where now I write $J^F(\varepsilon)$ to denote the fact that the value of employing a worker for a firm depends on firm-specific productivity.

$$E(J^F) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\left\{ J^F(x), 0 \right\} dF(x) \tag{9}$$

is the expected value of a job after a draw from the distribution $F(\varepsilon)$.

Given the Markov structure, the value conditional on a draw does not depend on history.

What is the intuition for this equation?

Differentiation of (8) immediately gives

$$\frac{dJ^F(\varepsilon)}{d\varepsilon} = \frac{\sigma}{r + \lambda} > 0 \tag{10}$$

Greater productivity gives a greater value to the firm.

When will job destruction take place?

Since (10) establishes that J^F is monotonic in ε, job destruction will be characterized by a cut-off rule, i.e.,

$$\exists\, \varepsilon_d : \varepsilon < \varepsilon_d \longrightarrow \text{ destroy}$$

Clearly, this cutoff threshold will be defined by

$$J^F(\varepsilon_d) = 0$$

But we also have $rJ^F(\varepsilon_d) = p + \sigma\varepsilon_d - b + \lambda\left[E(J^F) - J^F(\varepsilon_d)\right]$, which yields an equation for the value of a job after a new draw:

$$E(J^F) = -\frac{p + \sigma\varepsilon_d - b}{\lambda} > 0$$

This is an interesting result; it implies that since the expected value of continuation is positive (remember equation (9)), the flow profits of the marginal job, $p + \sigma\varepsilon_d - b$, must be negative!

Why is this? The answer is option value. Continuing as a productive unit means that the firm has the option of getting a better draw in the future, which is potentially profitable. For this reason it waits until current profits are sufficiently negative to destroy the job; in other words there is a natural form of *labor hoarding* in this economy.

Furthermore, we have a tractable equation for $J^F(\varepsilon)$:

$$J^F(\varepsilon) = \frac{\sigma}{r + \lambda}(\varepsilon - \varepsilon_d)$$

Let us now make more progress towards characterizing $E(J^F)$

By definition, we have

$$E(J^F) = \int_{\varepsilon_d}^{\bar{\varepsilon}} J^F(x) dF(x)$$

(where I have used the fact that when $\varepsilon < \varepsilon_d$, the job will be destroyed).

Now doing integration by parts, we have

$$\begin{aligned} E(J^F) &= \int_{\varepsilon_d}^{\bar{\varepsilon}} J^F(x) dF(x) = J^F(x)F(x)\Big|_{\varepsilon_d}^{\bar{\varepsilon}} - \int_{\varepsilon_d}^{\bar{\varepsilon}} F(x) \frac{dJ^F(x)}{dx} dx \\ &= J^F(\bar{\varepsilon}) - \frac{\sigma}{\lambda + r} \int_{\varepsilon_d}^{\bar{\varepsilon}} F(x) dx \\ &= \frac{\sigma}{\lambda + r} \int_{\varepsilon_d}^{\bar{\varepsilon}} [1 - F(x)] \, dx \end{aligned}$$

where the last line use the fact that $J^F(\varepsilon) = \frac{\sigma}{\lambda + r}(\varepsilon - \varepsilon_d)$, so incorporates $J^F(\bar{\varepsilon})$ into the integral.

Next, we have that

$$\begin{aligned} \underbrace{p + \sigma\varepsilon_d - b}_{\text{profit flow from marginal job}} &= -\frac{\lambda\sigma}{r + \lambda} \int_{\varepsilon_d}^{\bar{\varepsilon}} [1 - F(x)] \, dx \\ &< 0 \text{ due to option value} \end{aligned}$$

which again highlights the hoarding result. More importantly, we have

$$\frac{d\varepsilon_d}{d\sigma} = \frac{p - b}{\sigma} \left[\sigma\left(\frac{r + \lambda F(\varepsilon_d)}{r + \lambda}\right) \right]^{-1} > 0.$$

which implies that when there is more dispersion of firm-specific shocks, there will be more job destruction.

The job creation part of this economy is similar to before. In particular, since firms enter at the productivity $\bar{\varepsilon}$, we have

$$\begin{aligned} q(\theta)\, J^F(\bar{\varepsilon}) &= \gamma_0 \\ &\Longrightarrow \frac{\gamma_0(r+\lambda)}{\sigma(\bar{\varepsilon}-\varepsilon_d)} = q(\theta) \end{aligned}$$

Recall that as in the basic search model, job creation is "sluggish", in the sense that it is dictated by the matching function, so it cannot jump. Instead, it can only increase by investing more resources in matching.

On the other hand, job destruction is a jump variable so it has the potential to adjust much more rapidly (this feature was emphasized a lot when search models with endogenous job-destruction first came around, because at the time the general belief was that job destruction rates were more variable than job creation rates; now it's not clear whether this is true. It seems to be true in manufacturing, but not in the whole economy).

The Beveridge curve is also different now.

Flow into unemployment is also endogenous, so in steady-state we need to have

$$\lambda F(\varepsilon_d)(1-u) = \theta q(\theta) u$$

In other words:

$$u = \frac{\lambda F(\varepsilon_d)}{\lambda F(\varepsilon_d) + \theta q(\theta)},$$

which is very similar to our Beveridge curve above, except that $\lambda F(\varepsilon_d)$ replaces s.

The most important implication of this is that shocks (for example to aggregate productivity p) now also shift the Beveridge curve. For example, an increase in p will cause an inward shift of the Beveridge curve; so at a given level of creation, unemployment will be lower.

How do you think endogenous job destruction affects efficiency?

5 Magnitudes of Cross-Country Differences

An immediate application of these models would be to understand cross-country differences in unemployment rate. In this regard, the last model with endogenous job destruction, though extremely useful in explaining large volumes of job creation and destruction that simultaneously goes on in capitalist economies, seems to be second-order, since European economies don't seem to have higher levels of entry into unemployment. If anything, they have lower levels.

Therefore, an explanation for why unemployment is much higher (and has been much higher) in continental Europe has to turn to job creation.

We have already seen that comparative statics with respect to productivity (A), unemployment benefits (z) and bargaining power of workers (β) could all lead to differences in unemployment.

Here, there are reasons to suspect that perhaps differences in productivity are not essential. The reason is that roughly speaking, over the last century or so, while productivity has been growing, unemployment has been constant. This suggests that an appropriate model should have a structure that has "productivity-neutrality".

5.1 Balanced Growth

Motivated by this last observation, we may want to consider a balanced growth version of the search and matching model. This is easy to do. Here I sketch the simplest version. Suppose that both productivity and labor force grow. In particular assume that we have

n : population growth, i.e., $L(t) = L(0)\, e^{nt}$

g : growth of labor productivity, i.e., $A(t) = A(0)\, e^{gt}$

Steady state now implies

$$s(1-u)L + nL - q(\theta)\theta uL = unL$$

or re-expressing the steady state unemployment rate

$$u = \frac{s+n}{s+n+q(\theta)\theta}$$

This equation implies that a higher n will increase u for a given θ (or shift the Beveridge curve out). Why? Because new entrants arrive in the unemployment pool.

The rest of the analysis is similar to before. We need to change the first-order conditions for firms' capital choices along the same lines as we do in neoclassical growth models. In particular, for firm i

$$AF_k(K_i, e^{gt}N_i) - r - \delta = 0$$

so if we define effective capitallabor ratio as

$$k = \frac{K}{e^{gt}N},$$

and use the constant returns to scale properties, we have again the modified golden rule

$$Af'(k) = r + \delta.$$

But this is not sufficient to ensure balanced growth, because so far costs of opening vacancies in the unemployment benefit are constant. As productivity increases, firms will create more jobs and unemployment will fall.

To ensure "balanced growth," i.e., an equilibrium path with constant unemployment rate, these need to be scaled up with productivity as well. In particular, let us assume

$$\gamma_0 = \gamma w,$$

so that recruitment costs are in terms of labor, and

$$z = \lambda w,$$

so that there is an approximately constant "replacement rate" in the unemployment benefits system.

Then, going through the same steps as before, we have the following simple equilibrium conditions:

$$Af(k) - (r + \delta - g)k - w - \frac{(r + s - g)}{q(\theta)}\gamma w = 0,$$

which takes into account that now there is a capitalization affect from the fact that a job once created is becoming more productive over time at the rate g (and substitutes for γ_0 in terms of the wage).

Wage determination on the other hand implies

$$w = (1-\beta)\lambda w + \beta\left[Af(k) - (r+\delta-g)k + \theta\gamma w\right]$$

or

$$w = \frac{\beta\left[Af(k) - (r+\delta-g)k\right]}{1-(1-\beta)\lambda - \beta\theta\gamma}.$$

Now combining these equations we have a unique equation pinning down the equilibrium θ:

$$1-(1-\beta)\lambda - \beta\theta\gamma - \beta\left[1 + \frac{(r+s-g)}{q(\theta)}\gamma\right] = 0,$$

so that in balanced growth, the equilibrium unemployment rate is independent of both A and k. The second is not a desirable feature, since presumably unemployment should depend on capital-labor ratio as chosen by the firms, and we can consider modifications to incorporate this.

Note that introducing growth into the basic search model has led to a new and interesting implication, *the capitalization effect*: a higher growth rate implies that any job that opens today will become more productive in the future (because technological change is disembodied), so higher growth will be associated with more job creation and lower unemployment. Clearly, this is not inconsistent with lack of relationship between the secular increase in productivity and unemployment.

Although the capitalization effect is interesting, in the data there does not seem to be much evidence that faster growing countries have lower unemployment. This may be for three reasons:

1. This is a long-run prediction, difficult to find in the data.

2. This prediction is derived from the assumption that productivity growth is disembodied. With embodied productivity growth, this effect would not apply. In particular, if technological progress is "embedded" in new firms (and does not make existing firms more productive), the capitalization effect disappears.

3. An interesting argument by Aghion and Howitt (1994) combines the capitalization effect with a job destruction effect, based on the embodied technological progress idea. Faster growth creates both the capitalization effect, but also leads to faster destruction of existing jobs by new and more productive firms. Consequently, the relationship between unemployment and growth is ambiguous, and in their baseline specification is inverse U-shaped.

5.2 High Wages

The most popular explanation for high unemployment in Europe is that wages are high (relative to productivity). This has been pushed, for example, by The OECD Jobs Report (1994). This result is straightforward to obtain in the current context by considering high levels of z or β.

The problem is that the elasticity of unemployment in response to changes in unemployment benefit generosity, estimated from individual data in the United States for example, is not that high.

The bargaining parameter, β, on the other hand, could play a major role.

However, many economists do not see an obvious increase in the bargaining power of labor during the 1980s in Europe, which was the period of increasing unemployment.

This leads to the debate about whether the increase in unemployment in Europe was caused by

1. Demand Factors (e.g., Keynesian unemployment, or temporary contraction in A), as originally argued by Bruno and Sachs.

2. Institutional Factors (e.g., changes in z or β), as argued by Ed Lazear or Steve Nickell, for example.

3. Interaction Effects (e.g., interaction of demand/productivity factors with institutional factors). The importance of interaction effects was first suggested by Krugman (1994). It has been formalized many times, most famously by Ljungqvist and Sargent. In their model, unemployment benefits have a small effect until the economy is hit by a shock increasing churning, and following this shock, unemployment benefits prevent fast reallocation of workers from one sector to another (or from one set of firms to another set). Whether there has been such a churning shock or not is an open to question (not much evidence for greater

churning in job creation job destruction series or in tenure distributions).

An empirical version of this interaction story is documented by Blanchard and Wolfers, who show that an empirical model with interactions between time-invariant labor market institutions and time-varying TFP and labor demand type shocks does a good job of accounting for the increase and persistence of unemployment across countries.

Nevertheless, the existing evidence is mostly cross-country with few data points, so is not conclusive (see the recent paper by Nickell in the Economic Journal).

5.3 Firing Costs

Many economists emphasize other policy dimensions, for example firing costs. In particular, in the European context, there has been a lot of discussion of the "stifling role" of firing costs.

In particular, let us assume that when there is a separation, the firm has to pay the worker an amount f.

The usual value equations then become

$$rJ_i^F = A - w_i - s(J_i^F + f)$$

$$rJ^V = -\gamma_0 + q(\theta)\left(J^F - J^V\right)$$

$$rJ^U = b + \theta q(\theta)(J^E - J^U)$$

$$rJ_i^E = w_i + s(J^U + f - J_i^E)$$

Naturally, both employers and employees take into account that there will be payments for firing (separations). Moreover, they both are risk neutral and have the same discount factor.

Let us suppose that wages are again determined by Nash bargaining. In addition, let us first assume that at the beginning of the relationship, the worker and the firm engage in a **single bargain**, and write a **fully binding contract for the duration of the relationship**. The wage that will be agreed for the duration of the relationship will be the solution to

$$\max_{w_i} \left(J_i^E - J^U\right)^{\beta} \left(J_i^F - J^V\right)^{1-\beta}$$

This, combined with the free entry condition $J^V = 0$, implies that equilibrium wages are given by

$$w = \beta A + (1-\beta)\, b + \beta\gamma_0\theta - sf$$

The last term is the offset effect of firing costs. In other words, workers take a wage cut anticipating the future firing cost benefits. This wage cut is just sufficient to imply that firing costs will act as a simple transfer, and will have no effect on the equilibrium. Thus we have **neutrality** of firing costs.

What happens if workers and firms cannot write a fully binding contract? In that case, the effect of neutrality of firing costs will still apply as long as workers are not credit constrained. During the first instant of the relationship, workers will take a very large wage cut (i.e., they will be paid a negative wage) to undo the effect of firing costs in the future. Clearly, however, such a contract with a large payment from workers to firms is not realistic.

A different way of approaching the same problem, which avoids these problems, is to assume that firing costs become effective as soon as the bargaining process starts. This will change the Nash bargaining solution in the following way:

$$\max \left(J_i^E - J^U + sf\right)^{\beta} \left(J_i^F - J^U - sf\right)^{1-\beta}$$

Then wages are

$$w = \beta A + (1-\beta)\, b + \beta \gamma_0 \theta + rf$$

So now higher firing costs increase the bargaining power of the workers, thus wages. In other words: $f \uparrow$, $w \uparrow$, job creation $\downarrow$.

Which approach is a better approximation to the effect of firing costs in the data is an empirical matter, but it seems the previous approach presumes too much wage flexibility.

Moreover, part of the cost of firing cost regulations are not the transfers to the workers, but the difficulty and inflexibility they create for to firms. This is one of the arguments that those who believe firing costs have been important for the increase in European unemployment have used (e.g., Caballero and Hammour).

6 Magnitudes of Business Cycle Fluctuations

We have seen so far that the search approach to unemployment is quite flexible. It leads to easy equations with which we can work with and look at both cross-country differences and business cycle fluctuations. We have also seen that the extension with endogenous job destruction gives us the possibility of rapid changes in job destruction, for example, corresponding to be layoffs or plant closings, which is a feature that has sometimes been emphasized.

Nevertheless, an influential paper by Shimer (Shimer, AER 2005) has recently argued that this basic search-matching model does a poor job of matching the data. The basis of Shimer's critique is that even though the model generates qualitative features that are similar to those in the data, it can only generate significant movements in unemployment when shocks are implausibly large. In other words, to generate movements in u and θ similar to those in the data, we need much bigger changes in $Af(k) - (r + \delta)k$, A or p (labor productivity or TFP) than is in the data.

The reason for this is visible from the equations before. When A increases, so do the net present discounted value of wages, and this leaves less profits, and therefore there is less of a response from vacancies.

To see this point in greater detail, let us combine data and the basicc search model developed above. In particular, note that in the data the standard deviation of $\ln p$ (low productivity) is about 0.02, while the standard deviation of $\ln \theta$ is about 0.38. Therefore, to matched abroad facts with productivity-driven shocks, one needs an elasticity $d \ln \theta / d \ln p$ of approximately 20. Can the model generate this?

To investigate this, recall the equilibrium condition (2):

$$(1-\beta)\left[Af(k)-(r+\delta)k-z\right]-\frac{r+s+\beta\theta q(\theta)}{q(\theta)}\gamma_0=0.$$

This is very similar to the equilibrium condition that Shimer (AER 2005) derives. Rewrite this as

$$\frac{r+s+\beta\theta q(\theta)}{q(\theta)}\gamma_0=(1-\beta)\,p \tag{11}$$

where p is "net productivity", $p=Af(k)-(r+\delta)k-z$, the net profits that the firm makes over and above the outside option of the worker. This is not exactly the same as labor productivity, but as long as z is small, it is going to be very similar to labor productivity.

The quantitative predictions of the model will therefore depend on whether the elasticity implied by (11) comes close to a number like 20 or not. The basic point of Shimer (AER 2005) is that it does not.

To see this, let us differentiate (11) totally with respect to θ and p. We obtain:

$$-\frac{(r+s)}{q(\theta)}\gamma_0 q'(\theta)\,d\theta+\beta\gamma_0 d\theta=(1-\beta)\,dp$$

Now dividing the left-hand side by θ and the right hand side by p, and using the definition of the elasticity of the matching function $\eta(\theta)\equiv-\theta q'(\theta)/q(\theta)$

and the value of p from (11), we have

$$(r+s)\,\gamma_0\eta\,(\theta)\,\frac{d\theta}{\theta}+\beta\gamma_0\theta\frac{d\theta}{\theta}=(1-\beta)\,\frac{dp}{p}\frac{r+s+\beta\theta q(\theta)}{(1-\beta)\,q(\theta)}\gamma_0,$$

and since $dx/x=d\ln x$, we have the elasticity of the vacancy to unemployment ratio with respect to p as

$$\frac{d\ln\theta}{d\ln p}=\frac{r+s+\beta\theta q(\theta)}{(r+s)\,\eta\,(\theta)+\beta\theta q(\theta)}.$$

Therefore, this crucial elasticity depends on the interest rate, the separation rate, the bargaining power of workers, the elasticity of the matching function ($\eta\,(\theta)$), and the job finding rate of workers ($\theta q(\theta)$).

Let us take one period to correspond to a month. Then the numbers Shimer estimates imply that $\theta q(\theta)\simeq 0.45$, $s\simeq 0.34$ and $r\simeq 0.004$. Moreover, like other papers in the literature, Shimer estimates a constant returns to scale Cobb-Douglas matching function,

$$m\,(u,v)\propto u^{0.72}v^{0.28}.$$

As a first benchmark, suppose that we have efficiency, so that the Hosios condition holds. In this case, we would have $\beta=\eta\,(\theta)=0.72$. In that case, we have

$$\begin{aligned}\frac{d\ln\theta}{d\ln p} &\simeq \frac{0.034+0.004+0.72\times 0.45}{(0.034+0.004)\times 0.72+0.72\times 0.45}\\ &\simeq 1.03,\end{aligned}$$

which is substantially smaller than the 20-fold number that seems to be necessary.

One way to increase this elasticities to reduce the bargaining power of workers below the Hosios level. But this does not help that much. The upper bound on the elasticity is reached when $\beta = 0$ is

$$\begin{aligned}\frac{d\ln\theta}{d\ln p} &\simeq \frac{0.034+0.004}{(0.034+0.004)\times 0.72} \\ &\simeq 1.39,\end{aligned}$$

which is again not close to 20, which is the kind of number necessary to match the patterns in the data.

Yet another way would be to make p more variable than labor productivity, which is possible because p includes z, but it's unlikely that this will go very far.

In fact, what's happening is related to the cyclicality of wages. Recall that in the steady-state equilibrium,

$$w = (1-\beta)z + \beta\left[Af(k) - (r+\delta)k + \theta\gamma_0\right].$$

Therefore, the elasticity of the wage with respect to p is in fact greater than 1 (since it comes both from the direct affect and from the changes in θ); much of the productivity fluctuations are absorbed by the wage.

Naturally, in matching the data we may not want to limit our focus to just productivity shocks, especially since the correlation between productivity shocks and vacancy-unemployment rate is not that high (roughly about 0.4). So there must be other shocks, but what could those be?

One possibility is shocks to separation rates, for example because of plant closings. Hall (REStat 2005) also further argues that there is no way to resolve this puzzle by looking at the side of worker inflows into unemployment. He suggests that most of the action is in job creation. To a first approximation, job destruction or worker inflows into unemployment can be ignored. There is debate on this point, those like Steve Davis working on job destruction rates disagreeing, but it's an interesting perspective.

Shimer (AER 2005) suggests that one possible way of generating more fluctuations in unemployment in response to smaller changes in p is by introducing some type of wage rigidity. This is what Hall (AER 2005) does, assumes that wages are set by "social norms," and are therefore essentially constant. If wages are constant, changes in p will translate into bigger changes in J^F, and thus consequently to bigger changes in θ. Whether this is a satisfactory explanation remains to be seen. Certainly, assuming that wages are set by social norms is not very satisfactory, since where these social norms come from is left unanswered. A more promising avenue might be to model the wage process even more carefully, so that we can also have the "other shocks" correspond to wage shocks/supply shocks. This can certainly not be done by assuming that wages are set by black-box social norms.

This is an active area of research, and a number of papers at the moment try to understand how smaller changes in p could generate bigger fluctuations.