1 14.461 Advanced Macro; Problem Set 1

Question 1 (Sequential Search with Separations): Consider the McCall search model with a mass 1 of risk neutral individuals with discount factor equal to β and an exogenously given stationary distribution of wages $F(w)$. Assume that the unemployment benefit is equal to b. Once a worker finds and accepts a job, he will be employed in this job until the job is destroyed exogenously, which happens with independent probability equal to s in every period. Once the job is destroyed, the individual returns to the unemployment pool. Suppose that at $t = 0$ all workers start out as unemployed.

1. Find the reservation wage of the individual. Is it constant over time?
2. Find the the law of motion of unemployment. Where does it converge to?
3. What happens to reservation wages and the unemployment process when b increases?
4. What happens to reservation wages and the unemployment process when s increases?
5. What happens when $F(w)$ shifts to a new distribution $\tilde{F}(w)$, which is a mean preserving spread of F? What happens if $\tilde{F}$ second-order stochastically dominates F?

Question 2 (Modified Diamond Coconut Model): Consider a variant of the Diamond's coconut model in continuous time. Agents with a coconut run into each other at the rate $b(e)$ where e is the fraction of agents with a coconut looking for a partner, and those without a coconut find a tree at the rate a. Collecting a coconut costs a constant amount, c, but different trees have coconuts of different sizes. Let the size distribution of coconuts be denoted by the distribution function $G(q)$, where q is also the utility that an agent obtains from the consumption of a coconut of size q. On this island, agents can consume their own coconut, but they have to meet another agent with the coconut before doing so. Thus, in steady state the value of an agent with the coconut of size q is given by

$$rV^E = b(e)\left[q + V^U - V^E\right]$$

Characterize which coconuts an agent will collect as a function of the fraction of agents with a coconut, e. Determine the relationship between e and collection decision consistent with steady state. Can there exist more than one steady state equilibrium? How are the externalities in this economy different from those in Diamond's original model?

Question 3 (Leftovers From Class):

1. Prove Theorem for Homework in Section 3 of the class notes.

2. In the search model analyzed in Section 5 of the class notes, impose $J^V = 0$. Assume that the matching function $M(U, V)$ is constant returns to scale. Characterize the equilibrium, and determine the comparative statics with respect to b, γ, β and r. Why are some of these different from the model in which the number of firms was kept constant? Provide intuition.

   Hint: Note that p and q will change in response to changes in the above variables.

3. In this same model, analyze the constrained efficient allocation in this model and compare to the decentralized equilibrium by setting up the planner's objective function as

$$\int_0^\infty e^{-rt} \left[\int_{\tilde{x}^*}^\infty x n\left(x\right) dF(x) + bU - \gamma V \right]$$

   where I again suppress time dependence, and $n\left(x\right)$ is the number of jobs with productivity x, with the additional constraints that

$$\int_{\tilde{x}^*}^\infty n\left(x\right) dF(x) + V = N$$

   and

$$\dot{n}\left(x\right) = a\left(x\right) f\left(x\right) M\left(U, V\right) - sn\left(x\right)$$

   where $a\left(x\right)$ denotes the fraction of matches of productivity x that the planner turns into jobs.

   Explain these constraints, derive the solution and compare it to decentralized allocation.

Question 4 (Limits of Search Economies): Consider the search with bargained prices model with a constant number of firms and workers discussed in the lecture (recall the productivity distribution conditional a matching is $F\left(x\right)$). Assume that the matching technology is given by

$$M = \phi m\left(U, V\right)$$

where m exhibits constant returns to scale and $\phi \geq 0$ is that parameter.

Characterize the equilibrium and analyze it as $\phi \to \infty$. What happens to the productivity distribution of jobs? What happens to wages?

Now solve the same model with free entry, such that $J^V = 0$ at all times. How does this change the conclusions?

Question 5 (Search Effort): Consider the following continuous-time search model. There is continuum of workers normalized to 1. Each filled vacancy produces y and gets destroyed at the exogenous rate s. Keeping a vacancy open involves instantaneous cost 0 and the number of vacancies is determined by the free entry condition. Wages are exogenously set at $w = y/2$ and there is no unemployment benefit. The matching function of this economy that determines the flow of new jobs as a function of unemployment and the number of vacancies is

$$m(U, V) = U^{\theta} V^{\eta - \theta}$$

where $1 + \theta > \eta$ and $0 < \theta < 1$.

(i) Derive the Beveridge curve and show that it is downward sloping in the U-V space.

(ii) Derive the zero profit condition in terms of U and V and show that it is always upward sloping in the U-V space so that we have a unique steady state. What is the intuition for this uniqueness?

Now consider a modified model where the matching function is

$$m(eU, V) = (eU)^{\theta} V^{\eta - \theta}$$

where e is average search effort. Worker i's flow rate of matching is $e_i m(eU, V)/eU$.

The cost of search effort in money terms, $c(e_i)$, is increasing and strictly convex with $c\,(0)$ and $c'\,(0)$.

(iii) Show that the individually optimal search effort in steady state is given by

$$c'(e_i) = (eU)^{\theta - 1} V^{\eta - \theta} (J_E - J_U)$$

where J_E the value of being employed for a worker and J_U is the value of being employed and they are evaluated at the equilibrium search effort and e is the average search effort. Next, show that in a symmetric equilibrium, search effort is given by

$$c'(e) = (eU)^{\theta - 1} V^{\eta - \theta} \frac{\frac{y}{2} c\,(e)}{r + s + e^{\theta} U^{\theta - 1} V^{\eta - \theta}}$$

Show that e is increasing in V (holding U constant).

(iv) Write down the zero profit condition and holding U constant, show that the equilibrium V is increasing in e. Give the precise intuition for why a multiplicity of equilibria is possible now whereas it was not possible before.

(v) Show that when $\eta = 1$ although both e is increasing in V and V is increasing in e, a multiplicity of equilibria is not possible.