1 14.461 Advanced Macro; Problem Set 2

Question 1 (Complementary Investments): Consider an economy that consists of a continuum of workers and a continuum of firms both normalized to 1 for simplicity. The economy lasts for two periods. In the first period, workers choose the level of their human capital, h incurring a cost $c(h)$ where $c(.)$ is differentiable, convex and increasing and firms choose their capital stock at cost r. There is no discounting in this economy. In the second period, firms and workers are randomly matched 1-to-1 and switching is not allowed so if there is disagreement nothing is produced and both parties obtain zero return. If they agree to produce, output is equal to $F(h, k)$.

1. Assume that wages are determined by an asymmetric Nash bargain where the worker's bargaining power is β. Determine the equilibrium of this economy and show that there is underinvestment in both physical and human capital.

2. Suppose $F(h, k) = Ah^{\alpha}k^{1-\alpha}$ and $c(h) = \frac{1}{\delta}\frac{h^{1+\Gamma}}{1+\Gamma}$. At which value of β is output maximized. Explain.

3. Suppose there are many countries that differ in their labor market institutions thus have different β's. Show that there will be an inverse U-shaped relationship between factor shares of capital and the level of income.

4. Suppose now that countries differ with respect to their cost of human and physical capital investment, but they have the same technology parameter A, and the same α and β. Can the model be consistent with the regression of output on physical and human capital reported in the Mankiw-Romer-Weil regression, while also remaining in line with Mincerian micro-evidence on returns to schooling and aggregate factor share evidence?

5. Show that a multiplicity of equilibria is possible with general technology $F(h, k)$ (Hint: a diagrammatic answer is sufficient).

6. How would the results change if switching between partners were possible during the bargain but was costly?

Question 2 (Directed Search and Bargaining): Consider the Acemoglu-Shimer I AER 1999 model discussed in the lecture. Firms choose their capital stock k. Assume that workers observe the capital stock distribution of the firms before applying good jobs, and can direct their search towards jobs with different levels of capital stocks. Once they match with a firm, there is bargaining with the worker's bargaining power is exogenous and equal to β. Characterize the equilibrium of this economy. Does that exist to value of β that achieves constrained efficiency? What happens if firms can post a level of β and workers can direct their search towards different (k, β) combinations? Why are the equilibria different. Interpret.

Question 3 (Search, Asymmetric Information and Wage Posting): Consider the following economy consisting of a continuum of firms of mass 1 and a continuum of workers again with mass 1. All firms and all workers are homogenous. Time is discrete and infinite, t=0,1,2,..... Both types of agents discount the future at the rate δ . Production requires one firm and one worker. Unemployed workers and vacant firms look for a match. A match arrives with probability $\alpha < 1$ every period.

Each pair produces output equal to y. Workers when they work suffer a disutility of effort equal to η which is again match specific, that is, it is drawn once for every match at the beginning of the relationship. Every draw of η is independent of other draws at the same time and of all past history, and η is only observed by the worker in question. The distribution of is uniform over [0,1]. Each productive partnership faces an exogenous probability of separation equal to $s < 1$ per period.

1. Assume that the government legislates that all jobs must pay a wage $w(< y)$. Thus workers and firms do not have an option to negotiate over the wage. Each worker simply decides whether to accept the match (conditional upon the value of the specific show that he observes) or to continue search.

   a) Write down a Bellman equation characterizing the present discounted value of a searching worker.

   b) Show that workers will choose a cut-off value η^* and accept the match only if $\eta < \eta^*$.

   c) Write down first-order conditions that determine η^*.

2. Now assume that w is not set by government but posted by firms. In doing this firms take the cut-off level of other workers η^* and wage offers of other firms as given [note that different posted wages do not influence the meeting probabilities, just the acceptance probabilities]. Characterize a symmetric equilibrium in which all firms make the same offer w^{**} and all workers use the same cut-off rule η^{**}.

3. Show that the symmetric equilibrium wage offer w^{**} is too low from the viewpoint of maximizing social surplus.

4. Is this model theoretically satisfactory as a model of the working of labor markets under imperfect information [Hint: is this the best way of dealing with the imperfect information?].

Question 4 (Wage Dispersion): Consider the following 1-period economy. There are two types of workers. Fraction λ have a utility function $u(c)$ where u is strictly concave, increasing and differentiable and c is consumption. Fraction $1-\lambda$, have utility given by $v(c) = c$, i.e. are risk-neutral. Both workers produce output $f(k)$ when matched with a firm employing capital k where f

satisfies the regular assumptions on production function. There is a large number of potential firms which can enter, buy capital (at price 1 per unit) and post vacancies.

Matching takes place as follows: first, firms decide to enter, irreversibly buy some capital, and post wages. Then workers observe all wages that have been posted, and decide which wage to apply to. If employed, a worker receives the promised wage, otherwise he receives some unemployment benefit z. *There are matching frictions in that workers make their application decisions without coordination.* In particular, if qN workers apply to N firms (e.g. N firms offer wage w' and qN workers seek wage w'), then, the firm gets a worker with probability $1 - e^{-q}$ and each worker is employed with probability $(1 - e^{-q})/q$ (just for completeness, this is the limit of the urn-ball process for $N \to \infty$). So workers have to tradeoff wages and employment probabilities in their application decisions. Because firms choose their capital before matching, if a firm does not get a worker, its capital is sunk.

Define an equilibrium. Show that the equilibrium can be characterized as a pair of constrained maximization problems. Characterize the equilibrium and show that in equilibrium, generically, there will be an observed wage distribution with two wages w^h and $w^l < w^h$, with respective fractions $1 - \mu$ and μ. Show that $\mu > \lambda$. Is the offered wage distribution different from the observed wage distribution? [The offered distribution is the wage offers among vacancies, the observed distribution is the one among employed workers].

Question 5 (Risk Aversion in Search): Consider the following one period economy. All workers have utility $u(c)$ where c is consumption, and they start with asset level A. There are no private insurance markets, and workers have to apply to jobs in order to be employed. If they obtain the job, they get the posted wage, otherwise they obtain unemployment insurance z. A large number of firms have access to a common technology and decide whether to open a vacancy, what wage to offer and what level of specialization to choose for their job. Workers observe all wage offers and specialization decisions, and decide what jobs to apply to (anticipating the application decisions of other workers). If there are on average q workers to a job with specialization level α, each has probability of getting the job $(1-\alpha)\mu(q)$and each firm has a probability of filling its vacancy of $(1-\alpha)\eta(q)$ where μ is strictly decreasing function and η is a strictly increasing function. The cost of posting a vacancy is γ and a job of specialization level of α produces output $g(\alpha)$ where g is an increasing and concave function.

1. Explain the form of the matching probabilities for workers and firms.

2. Define an equilibrium, paying special attention to expectations about off-the-equilibrium path behavior.

3. Prove that any allocation that is a solution to the maximization problem

$$\max_{w,q,\alpha} (1-\alpha)\mu(q)u(A+w) + [1-(1-\alpha)\mu(q)]u(A+z)$$

subject to

$$(1-\alpha)\,\eta\,(q)\,[g\,(\alpha)-w]=\gamma$$

is an equilibrium, and that any equilibrium allocation is a solution to this maximization problem.

4. Assume $\mu\,(q)=\min\{1,1/q\}$ and $\eta\,(q)=\min\{1,q\}$. Interpret these functions and show that in this case, an increase in z leads to an increase in α. Interpret this result.

5. Now assume that there are N groups of workers, each with different levels of assets. Informally describe how the equilibrium will look.