14.461 PROBLEM SET 5 SOLUTIONS

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1. Complementary Investments

(i) A firm that chooses capital k gets expected gross return

$$(1-\beta)\int F(h,k)\,dH(h) - rk$$

where $H(\cdot)$ is the distribution of human capital chosen by workers. Similarly, a worker that chooses human capital h gets expected gross return

$$\beta\int F(h,k)\,dK(k) - c(h)$$

where $K(\cdot)$ is the distribution of physical capital chosen by firms. By symmetry and concavity of F, each worker chooses the same h and each firm the same k, so H and K are degenerate and firms choose k to maximize

$$(1-\beta)F(h,k) - rk$$

while workers choose h to maximize

$$\beta F(h,k) - c(h)$$

This leads to FOC

$$F_k(h,k) = \frac{r}{1-\beta} \tag{1}$$

$$F_h(h,k) = \frac{c'(h)}{\beta}. \tag{2}$$

The social planner solves

$$\max_{h,k} F(h,k) - rk - c(h)$$

with FOC

$$F_k(h,k) = r \tag{3}$$

$$F_h(h,k) = c'(h). \tag{4}$$

Comparing FOC: (1) and (3) imply that for a given h, the social planner sets a higher k than firms do, while (2) and (4) imply that for a given k, the social planner sets a higher h than workers do. Thus there's underinvestment in the decentralized economy.

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(ii) If $F(h,k) = Ah^\alpha k^(1-\alpha)$, then the FOC for the social planner's problem become:

$$(1-\alpha)A\left(\frac{h}{k}\right)^\alpha = \frac{r}{1-\beta} \tag{5}$$

$$\alpha A\left(\frac{k}{h}\right)^{1-\alpha} = \frac{c'(h)}{\beta}. \tag{6}$$

We can rearrange to get:

$$c'(h) = \alpha\beta A\left[\frac{(1-\alpha)(1-\beta)A}{r}\right]^{\frac{1-\alpha}{\alpha}} \tag{7}$$

$$k = \left[\frac{(1-\alpha)(1-\beta)A}{r}\right]^{\frac{1}{\alpha}} h. \tag{8}$$

which implies that

$$F(k,h) = A\left[\frac{(1-\alpha)(1-\beta)A}{r}\right]^{\frac{1-\alpha}{\alpha}} [c']^{-1}\left(\alpha\beta A\left[\frac{(1-\alpha)(1-\beta)A}{r}\right]^{\frac{1-\alpha}{\alpha}}\right).$$

To maximize this, just take the first order condition. Note the maximand for β lies between 0 (which maximizes $1-\beta$ for $\beta \in [0,1]$) and α (which maximizes $\beta(1-\beta)^{\frac{1-\alpha}{\alpha}}$). The exact optimum depends on the functional form for c.

If $c = c_0 h^{1+\Gamma}$, it's easy to check that $[c']^{-1}(h) \propto h^{\frac{1}{\Gamma}}$, which leads immediately to

$$F(h,k) \propto \beta^{\frac{1}{\Gamma}}(1-\beta)^{\frac{1+\Gamma}{\Gamma}\frac{1-\alpha}{\alpha}}$$
$$\Rightarrow \beta^* = \frac{\alpha}{1+\Gamma(1-\alpha)}.$$

Why does β need to be intermediate? If $\beta = 0$, workers have no incentive to invest in human capital, while if $\beta = 1$, firms have no incentive to invest in physical capital. We need to balance these two problems with an intermediate β.

(iii) This question asks us to show that

$$F(\beta) \equiv F(h^*(\beta), k^*(\beta))$$

is inverse U-shaped (since $1-\beta$ is the capital share). We showed in the previous part that for general $c(\cdot)$, $F(\beta)$ is zero if $\beta = 0$ or $\beta = 1$, and $F(\beta)$ is maximized for some $\beta \in [0,\alpha]$. It's hard to say more in general, although in some abstract sense this is already enough to call the graph of $F(\beta)$ an inverse U. If we specialize to $c(h) = c_o h^{1+\Gamma}$ then we already showed the FOC for β has a unique solution, so $F(\beta)$ is indeed inverse U-shaped.

(iv) Roughly speaking, the micro evidence on human capital suggests that the return to schooling is around 10% while countries differ by at most 12 years of schooling, indicating that human capital varies across countries by up to $1.1^12 \approx 3$. Also, the evidence on factor shares pins down β, the bargaining parameter (and not α, the Cobb-Douglas exponent) as $\beta \approx 2/3$. Thus we can generate large differences in output by magnifying the differences in human capital by an appropriate choice of α.

(v) See handwritten sheet for one example. Another example is provided by a Leontief technology, which will provide a range of pairs (h,k) with $h = k$ all of which are equilibria.

(vi) The implication of being able to switch partners at a small cost depends on exactly how this bargaining game is formulated. One possibility is that we now have again a generalized Nash bargain with different threat points: so

$$r_K = V_K + (1-\beta)(y - V_K - V_L)$$
$$w_L = V_L + \beta(y - V_K - V_L)$$

Assuming the outside option of a factor is increasing in the ex ante investment, workers and firms now receive a greater share of the marginal product of their investment and so higher investment levels can be supported than before. If we formalize the bargaining game differently, the results may be different (for example, now equilibria of the form (h^*, k^*, β) can be supported by threshold strategies of the form 'we match and I demand share $1-\beta$ (alternatively, β) of the product unless you chose $h < h^* - \delta$ (alternatively, $k < k^* - \delta$) or you demand share more than β (alternatively, $1-\beta$)'. Here δ is an increasing function of the switching costs. Note this also allows more allocations to be supported in equilibrium than before, but it's no longer exactly the Nash bargaining framework.

2. Directed Search and Bargaining

This question is answered in section 6 of the paper, so I'll refer you there for a detailed discussion. I'll quote what Acemoglu and Shimer say (equations have been renumbered and added at the appropriate places below):

1. *Wages are determined by ex post Nash bargaining, as in Section 4, but workers are able to observe firms' capital investment and direct their search appropriately, as in Section 5.* The fundamental condition of Section 5 which determines applications decisions will once again be an equilibrium condition: workers must have the same expected utility at all jobs (with positive queue length). As a result, the equilibrium is characterized by the constrained optimization problem

$$\max_{w,k,q} \frac{\mu(q)}{r+s+\mu(q)} w \tag{9}$$

subject to

$$\frac{\eta(q)}{r+s+\eta(q)} \cdot \frac{f(k)-w}{r+s} \geqslant pk,$$

with one additional constraint: wages are not a choice variable, but instead are set ex post by Nash bargaining. So in this hybrid environment, wages must satisfy the additional constraint

$$w(k^B) = \frac{\beta(r+s+\mu(Q))f(k^B)}{r+s+\beta\eta(Q)+(1-\beta)\eta(Q)}. \tag{10}$$

It is straightforward to see that constraint (10) does not bind at the equilibrium (efficient) values of k^S; w^S and q^S of Sections 3 and 5 and (i.e. at the efficient allocation) if and only if workers' bargaining power β is equal to the elasticity of the matching function η. In other words, when the Hosios "bargaining power equals elasticity" condition holds, we obtain the efficient allocation as the equilibrium of this hybrid environment. Therefore, when workers can direct their search towards firms with more capital, holdups are avoided if only if the Hosios condition is satisfied.

3. *Firms can commit to and advertise a bargaining rule β before the matching stage. Workers can observe each firm's bargaining rule and capital investment, and they direct their search accordingly.* The equilibria of this economy coincide with the efficient equilibria characterized in Sections 3 and 5. A choice of β conditional on the capital investment is formally equivalent to a wage commitment.

3. Search, Asymmetric Information and Wage Posting

(i) (a) Denote by J^U the value of being an unemployed worker and by $J^E(\eta)$ the value of being an employed worker with job disutility η. The Bellman equation for an unemployed worker is

$$J^U = b + \delta(1-\alpha)J^U + \delta\alpha \int_0^1 \max\left\{J^U, J^E(\eta)\right\} d\eta. \tag{11}$$

(b) The Bellman equation for an employed worker is

$$J^E(\eta) = w - \eta + \delta(1-s)J^E(\eta) + \delta s J^U. \tag{12}$$

Rearrange:

$$J^E(\eta) = \frac{w - \eta + \delta s J^U}{1 - \delta(1-s)}.$$

It's clear that a worker should accept a job with disutility η iff $J^E(\eta) > J^U$, which immediately implies a cutoff rule for η. A worker should accept any job with disutility $\eta < \eta^*$ where

$$J^U = J^E(\eta^*) = \frac{w - \eta^* + \delta s J^U}{1 - \delta(1-s)}. \tag{13}$$

(c) The equilibrium value of η^* is characterized by (11) and (13) which are two equations for the two variables J^U and η^*. These can be further simplified if desired. Note these are not first order conditions in any obviously sensible way.

(ii) We can analogously write the Bellman equations for firms with a vacancy and with a worker to whom they are paying w as

$$J^F(w) = y - w + \delta(1-s)J^F(w) + \delta s J^V \tag{14}$$

$$J^V = \delta\alpha \max_w \left[\int_0^{\eta^*+w-w^*} J^E(w)\, d\eta + \int_{\eta^*+w-w^*}^1 J^V\, d\eta\right]. \tag{15}$$

The $\eta^* + w - w^*$ terms come from the fact that a worker receiving an offer of w^* is indifferent between accepting and rejecting it if she draws $\eta = \eta^*$.

Rearranging (14), we get

$$J^F(w) = \frac{y - w + \delta s J^V}{1 - \delta(1-s)}.$$

We can then simplify (15) to get

$$J^V = \delta\alpha \max_w \left[(1 - \eta^* - w + w^*)J^V + (\eta^* + w - w^*)\frac{y - w + \delta s J^V}{1 - \delta(1-s)}\right]. \tag{16}$$

This has FOC for w given by

$$-J^V - (\eta^* + w - w^*) + \frac{y - w + \delta s J^V}{1 - \delta(1-s)} = 0$$

so in a symmetric equilibrium, we have

$$-J^V - \eta^* + \frac{y - w^* + \delta s J^V}{1 - \delta(1-s)} = 0$$

or

$$w^* = y - \eta^*(1 - \delta(1-s)) - (1-\delta)J^V. \tag{17}$$

Substituting this in (16) gives

$$J^V = \delta\alpha \left[(1-\eta^*)J^V + \eta^* \frac{y - w^* + \delta s J^V}{1 - \delta(1-s)} \right] \tag{18}$$

(17) and (18), together with (11) and (13), determine the equilibrium values of w^* and J^V, together with η^* and J^U.

(iii) For efficiency it's clear we need $w^* = y$. This implies $J^V = 0$ and $\eta^* = 0$ from (17) and (18). But this doesn't satisfy (generically) (11) and (13). Thus since $w \leqslant y$ and we showed that $w^* \neq y$, it follows that the equilibrium wage is inefficiently low.

(iv) It's not a great model - one thing clearly lacking is that conditional on agreeing to accept a job, all workers are identical as far as firms are concerned. Asymmetric or imperfect information about productivity or skill would be a desirable feature. Exogenous separations are as usual problematic. There are many other possible objections.

4. Wage Dispersion

An equilibrium is a set of wage offers and capital choices $\mathcal{W} = \{w, k\}$ for firms and utilities (U^h, U^l) and queuing functions $q^h(w)$ and $q^l(w)$ for each type of workers such that where $q(w) \equiv q^h(w) + q^l(w)$:

- for all $(w,k) \in \mathcal{W}$, $(1 - e^{-q(w)})(f(k) - w) = k$;
- for all (w,k), $(1 - e^{-q(w)})(f(k) - w) \leqslant k$;
- $U^h = \sup_w \frac{1-e^{-q(w)}}{q(w)} u(w) + \left(1 - \frac{1-e^{-q(w)}}{q(w)}\right) u(z)$;
- $U^l = \sup_w \frac{1-e^{-q(w)}}{q(w)} w + \left(1 - \frac{1-e^{-q(w)}}{q(w)}\right) z$;
- $q^h(\cdot)$ satisfies with complementary slackness:
  - $U^h \geqslant \sup_w \frac{1-e^{-q(w)}}{q(w)} u(w) + \left(1 - \frac{1-e^{-q(w)}}{q(w)}\right) u(z)$
  - $q^h(w) \geqslant 0$.
- $q^l(\cdot)$ satisfies with complementary slackness:
  - $U^l \geqslant \sup_w \frac{1-e^{-q(w)}}{q(w)} w + \left(1 - \frac{1-e^{-q(w)}}{q(w)}\right) z$
  - $q^l(w) \geqslant 0$.

I claim that an equilibrium can be characterized by the two maximization problems

$$\max_{w,q,k} \frac{1-e^{-q}}{q} u(w) + \left(1 - \frac{1-e^{-q}}{q}\right) u(z) \tag{19}$$

subject to

$$(1 - e^{-q})(f(k) - w) = k$$

and

$$\max_{w,q,k} \frac{1-e^{-q}}{q} w + \left(1 - \frac{1-e^{-q}}{q}\right) z \tag{20}$$

subject to

$$(1 - e^{-q})(f(k) - w) = k$$

I'll leave the argument for this slightly informal. The first step is to show that the set of wages offered in equilibrium is partitioned into two sets, $\mathcal{W}^l$ and $\mathcal{W}^h$ whose intersection contains at most one point, and such that the risk neutral agents apply only to wages in $\mathcal{W}^h$ and risk averse agents apply only to wages in $\mathcal{W}^l$. To see this, suppose the risk neutral agent applies to two wages w and $w' > w$ in equilibrium. Then

$$\frac{1-e^{-q(w)}}{q(w)}u(w)+\left(1-\frac{1-e^{-q(w)}}{q(w)}\right)u(z)=\frac{1-e^{-q(w')}}{q(w')}u(w')+\left(1-\frac{1-e^{-q(w')}}{q(w')}\right)u(z).$$

From the concavity of u it follows immediately that the risk averse agent strictly prefers to apply to w than to w'. The easiest way to see this is to implicitly differentiate the utility function to see that the slopes of the indifference curve through (q, w) for the two agents are given by

$$\left(\frac{\partial w}{\partial q}\right)^l=\frac{u(w)-u(z)}{u'(w)}\frac{1-e^{-q}-qe^{-q}}{q(1-e^{-q})} \qquad \text{and} \qquad \left(\frac{\partial w}{\partial q}\right)^h=(w-z)\frac{1-e^{-q}-qe^{-q}}{q(1-e^{-q})}$$

By concavity of u, $\frac{u(w)-u(z)}{w-z} > u'(w)$ (use the mean value theorem or draw a graph). Also one can check that $1-e^{-q}-qe^{-q}$ and $q(1-e^{-q})$ are positive. Hence the indifference curve of the risk averse agents through (q, w) is steeper than the indifference curve of the risk neutral agents through the same point. Note also that both curves are upward sloping in (q, w) space. One can also check that the indifference curves for both types of agents are convex, although this requires some algebraic effort.

Now, suppose there are at least two wages which the risk neutral agents apply to in equilibrium. By the above argument, at least one of these wages is only applied to by risk neutral agents (the higher one). Suppose this wage did not solve (20). Since in equilibrium, firms maximize profits, we must have that the capital choice k is given by the first order condition

$$1 = (1-e^{-q})f'(k).$$

This defines $k = k(q)$ in equilibrium. We can therefore write the zero profit constraint for firms as

$$(1-e^{-q})(f(k(q)) - w) = k(q).$$

This defines a zero profit locus for firms along which all equilibrium allocations must lie in (q, w) space. This isn't concave in general. As usual, if the equilibrium (q, w) doesn't satisfy this, it's easy to see there's a profitable deviation for some firms (more on this in recitation). Thus there are at most two wages to which the risk neutral agents apply in equilibrium, one solving (**??**) and another to which both types of agents apply.

A similar argument applies to wages to which the risk averse agents apply in equilibrium.

It remains to rule out that some agents of each type apply to the same wage in equilibrium. I leave it to you to show this is impossible by constructing an appropriate deviation by firms. The key property is that the indifference curves of the two types of agents at this (q, w) have different slopes, so firms can deviate and offer at least one type of agent a (q, w) pair yielding the same utility and giving higher profits.

Thus (19) and (20) characterize the equilibrium. Note we showed that any wage the risk neutral agents apply to in equilibrium is at least as high as any wage the risk averse agents apply to, showing that the equilibrium wages $w^h > w^l$ respectively receive $1-\lambda$ and λ applications. We also know that since $w^h > w^l$, we must have $q^h > q^l$ also (or else both types of agents would apply to w^h). Thus we observe $\frac{1-e^{-q^h}}{q^h}(1-\lambda)$ workers in jobs paying w^h and $\frac{1-e^{-q^l}}{q^l}\lambda$ workers in jobs paying w^l. Since the function $q \mapsto \frac{1-e^{-q}}{q}$ is decreasing, it follows that

$$\mu \equiv \frac{\frac{1-e^{-q^l}}{q^l}\lambda}{\frac{1-e^{-q^l}}{q^l}\lambda + \frac{1-e^{-q^h}}{q^h}(1-\lambda)} > \frac{\frac{1-e^{-q^l}}{q^l}\lambda}{\frac{1-e^{-q^l}}{q^l}\lambda + \frac{1-e^{-q^l}}{q^l}(1-\lambda)} = \lambda.$$

The offered wage distribution is that $(1-\lambda)/q^h$ firms offer w^h and λ/q^l firms offer w^l. Comparing this to the observed distribution above, we observe that since $1-e^{-q^h} < 1-e^{-q^l}$, the offered wage distribution has a larger fraction of firms offering w^h than the observed distribution.

5. Risk Aversion in Search

(i) There's not much to explain - when a firm chooses a job of specialization α, there's a probability of $1-\alpha$ that any given worker will be suitable for the job. This suitability is not observable either to the firm or the worker ex ante. Thus if q workers apply then each has a probability $\mu(q)$ of meeting the firm, and then a total probability $(1-\alpha)\mu(q)$ that this match will be suitable. The firm has a probability $\eta(q)$ of meeting a worker, and then a total probability $(1-\alpha)\eta(q)$ that this match will be suitable.

(ii) An equilibrium is a set of wage offers and specializations $\mathcal{W} = \{w, \alpha\}$ offered by firms and a utility U and a queuing function $q(w,\alpha)$ used by workers such that:

- for all $(w,\alpha) \in \mathcal{W}$, $(1-\alpha)\eta(q(w,\alpha))[g(\alpha) - w] = \gamma$;
- for all (w,α), $(1-\alpha)\eta(q(w,\alpha))[g(\alpha) - w] \leqslant \gamma$;
- $U = \sup_{w,\alpha}(1-\alpha)\mu(q(w,\alpha))u(A+w) + [1-(1-\alpha)\mu(q(w,\alpha))]u(A+z)$;
- $q(\cdot)$ satisfies with complementary slackness:
  - $U \geqslant (1-\alpha)\mu(q(w,\alpha))u(A+w) + [1-(1-\alpha)\mu(q(w,\alpha))]u(A+z)$;
  - $q(w,\alpha) \geqslant 0$.

(iii) Given the definition of equilibrium, this is clear. First suppose (w^*, q^*, α^*) satisfies

$$\max_{w,q,\alpha}(1-\alpha)\mu(q)u(A+w) + [1-(1-\alpha)\mu(q)]u(A+z)$$

subject to

$$(1-\alpha)\eta(q)[g(\alpha) - w] = \gamma.$$

Then we can define the queuing function required by the definition of equilibrium by first letting $\tilde{q}(w,\alpha)$ solve

$$\begin{aligned}(1-\alpha)\mu(\tilde{q}(w,\alpha))u(A+w) + [1-(1-\alpha)\mu(\tilde{q}(w,\alpha))]u(A+z) \\ = (1-\alpha^*)\mu(q^*)u(A+w^*) + [1-(1-\alpha^*)\mu(q^*)]u(A+z)\end{aligned}$$

and then setting

$$q(w,\alpha) = \begin{cases} \tilde{q}(w,\alpha) & \text{if } w > z \\ 0 & \text{if } w \leqslant z. \end{cases}$$

One can check this is possible and satisfies the complementary slackness conditions assuming $\mu(q) \to 1$ as $q \to 0^+$ and $\mu(q) \to 0$ as $q \to \infty$. A slight alteration is required if these conditions don't hold. The other conditions follow immediately from the definition of (w^*, q^*, α^*).

Conversely, the result follows using the concavity of the utility function.

(iv) The functions $\mu(q) = \min\{1, 1/q\}$ and $\eta(q) = \min\{1, q\}$ are the frictionless matching functions where the number of matches is determined by the short side of the market, and every agent on the long side gets to match with the same probability.

First, note that in this case in equilibrium we must have that $q = 1$. There are two cases. If $q \geqslant 1$ then the problem is

$$\max_{w,\alpha,q\geqslant 1} \frac{1-\alpha}{q} u(A+w) + \left(1 - \frac{1-\alpha}{q}\right) u(A+z)$$

subject to

$$(1-\alpha)(g(\alpha) - w) = \gamma$$

which is clearly maximized for $q = 1$ assuming that $w \geqslant z$ (and if $w < z$ then workers should optimally not apply for any jobs). If $q \leqslant 1$ then the problem is

$$\max_{w,\alpha,q\leqslant 1} (1-\alpha)u(A+w) + (1-(1-\alpha))\, u(A+z)$$

subject to

$$(1-\alpha)q(g(\alpha) - w) = \gamma.$$

If $q < 1$ then we can increase w and q until $q = 1$, keeping the constraint satisfied and increasing the objective. Thus $q = 1$ at any optimum.

Now, the easiest way to establish the desired comparative statics is to draw the indifference curves in (w, α) space for workers and firms.

A worker's indifference curve is given by

$$(1-\alpha)u(A+w) + \alpha u(A+z) = c.$$

Note that $c \geqslant u(A+z)$ (or else the worker shouldn't apply for this job) and similarly $w \geqslant z$. Implicit differentiation gives that

$$\frac{\partial \alpha}{\partial w} = \frac{u'(A+w)(c - u(A+z))}{(u(A+w) - u(A+z))^2}.$$

Thus the worker's indifference curve is positively sloped in (w, α) space. Also note that $u'(A+w)$ is decreasing in w and the denominator is increasing in w, so also $\frac{\partial^2 \alpha}{\partial w^2} < 0$ and so the worker's indifference curve defines α as a concave increasing function of w; that is, it defines w as a convex increasing function of α. Finally observe that we can also write

$$\frac{\partial w}{\partial \alpha} = \frac{u(A+w) - u(A+z)}{(1-\alpha)u'(A+w)}$$

so

$$\frac{\partial}{\partial z}\left[\frac{\partial w}{\partial \alpha}(\alpha, w; z)\right] < 0; \tag{21}$$

that is, the indifference curve passing through a given (w, α) becomes less steeply sloped as z increases.

The firm's zero profit condition can be rewritten as

$$w = g(\alpha) - \frac{\gamma}{1-\alpha}.$$

It's easy to see this defines w as a concave function of α. w is decreasing in α for α sufficiently close to 1, and for the problem to be interesting, w must be increasing in α for α sufficiently close to 0. Thus the equilibrium is unique and is characterized by the tangency of the zero-profit condition with an indifference curve for the workers (unique because of the concavity and convexity of the two curves by a standard argument). See Figure 1 (the case shown is $u(c) = \log(c)$, $g(\alpha) = \sqrt{\alpha}$, $\gamma = 0.05$, $A = 1$ and $z = 0.2$).

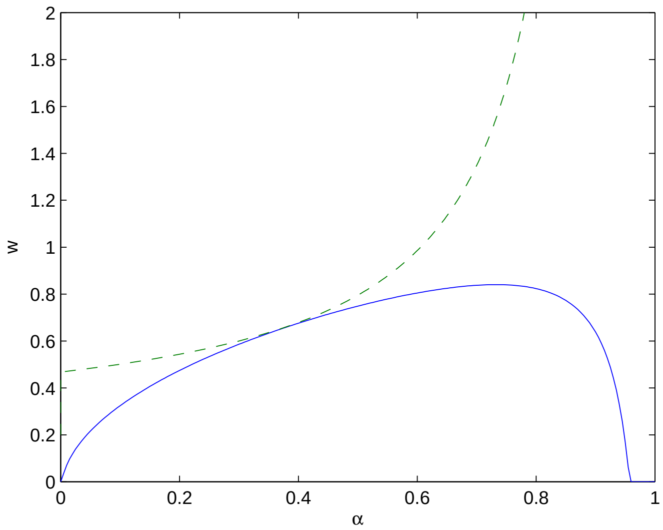

FIGURE 1. Consumer indifference (dashed) and zero profit (solid)

An increase in z leads to an increase in the equilibrium α. Since we showed in equation (21) that increasing z makes the indifference curve through a given (w, α) flatter, it's clear that the indifference curve for $z' > z$ through the equilibrium (w^*, α^*) associated with z crosses the firm's zero profit condition from above. It follows that the indifference curve tangent to the zero profit condition under z' must be tangent to the zero profit condition to the right of the old (w^*, α^*). That is, the equilibrium α^* increases. This is shown in Figure 2. The dotted line corresponds to $z = 0.4$, while the remainder of the graph is as in Figure 1.

(v) This will be quite similar to question 4. The market will endogenously subdivide, so that (generically) there will be N groups of firms, each equal to the size of the N groups of workers (since we have $q = 1$ in equilibrium). Each group i of firms will offer the (w, α) that would be the equilibrium if only group i of workers was in the market. One can show, although doing it by brute force is not easy, that if u exhibits decreasing absolute risk aversion then (w^*, α^*) is such that α^* is increasing in A (and w^* is too). Under increasing absolute risk aversion, the ordering is reversed. If u is CARA, then all firms will offer the same (w^*, α^*). This should be intuitive since the size of the absolute risk involved here is constant in A.

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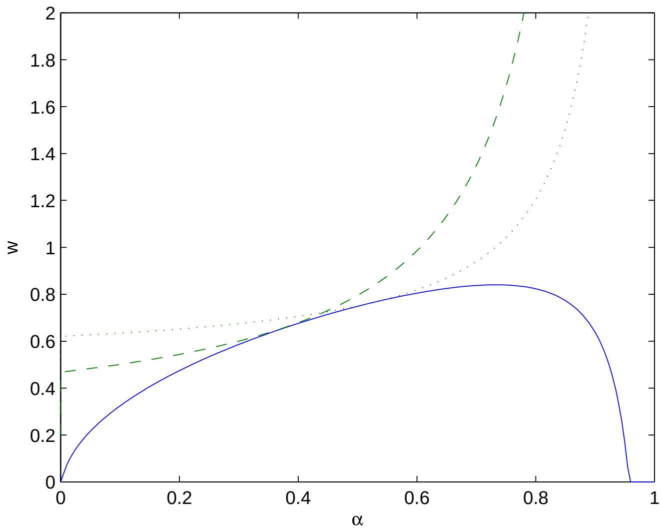

FIGURE 2. The effect of higher z (dots versus dashes)