1 14.461 Advanced Macro: Additional Problems

Question 1 (Endogenous Growth Without Scale Effects): Consider the following model. Population at time t is $L(t)$ and grows at the constant rate n (i.e., $\dot{L}(t) = nL(t)$). All agents have preferences given by

$$\int_0^\infty \exp(-\rho t) \frac{C^{1-\theta} - 1}{1-\theta} dt, \tag{1}$$

where C is consumption defined over the final good of the economy. This good is produced as

$$Y = \left[\int_0^N y(i)^\beta \, di \right]^{1/\beta}$$

where $y(i)$ is intermediate good i. The production function of each intermediate is

$$y(i) = l(i)$$

where $l(i)$ is labor allocated to this good.

New goods are produced by allocating workers to the R&D process, with the production function

$$\dot{N} = \eta \cdot N^\phi \cdot L_R$$

where $\phi \leq 1$ and L_R is labor allocated to R&D. So labor market clearing requires $\int_0^N l(i) \, di + L_R = L$.

Risk-neutral firms hire workers for R&D. A firm who discovers a new good becomes the monopoly supplier, with a perfectly and indefinitely enforced patent.

1. Characterize the balanced growth path equilibrium in the case where $\phi = 1$ and $n = 0$. Why does the long-run growth rate depend on θ? Why does the growth rate depend on L? Do you find this plausible? Why aren't there any transitional dynamics?

2. Now suppose that $\phi = 1$ and $n > 0$. What happens? Interpret.

3. Now characterize the balanced growth path equilibrium when $\phi < 1$ and $n > 0$. Does the growth rate now depend on L? Does it depend on n? Why? Do you think that the configuration $\phi < 1$ and $n > 0$ is more plausible than the one with $\phi = 1$ and $n = 0$?

Question 2 (Endogenous Skill-Biased Technical Change): There are H skilled and L unskilled workers, and two goods, y_L and y_H. All consumers have instantaneous utility defined over the final good y

$$U = y = \left[y_L^\rho + \gamma y_H^\rho \right]^{1/\rho},$$

and are risk-neutral would discount rate r.

The production function of these two goods are:

$$
\begin{aligned}
y_L &= \left(\int_0^1 q_x(i)\, x(i)^\alpha di \right) l^{1-\alpha} \\
y_H &= \left(\int_0^1 q_z(i)\, z(i)^\alpha di \right) h^{1-\alpha}
\end{aligned}
$$

where l and h are quantities of skilled and skilled labor, $x(i)$ is the quantity of labor-complementary intermediate good i that an unskilled worker produces with, and $z(i)$ is the quantity of skill-complementary intermediate good i that a skilled worker produces. $q_x(i)$ and $q_z(i)$ denote the quality of the highest vintage of machine i used for sector L or H.

The profit function of a labor-intensive firm employing l workers is therefore:

$$
p_L \left(\int_0^1 q_x(i)\, x(i)^\alpha di \right) l^{1-\alpha} - \left(\int_0^1 \chi(i) x(i) di \right) - w_L l
$$

where w_L is unskilled wage, and p_L is the price of the labor intensive good, and $\chi(i)$ is the price of intermediate good $x(i)$. The profit function of a skill-intensive good is similarly defined. Suppose that intermediate goods are supplied by monopolistically competitive firms, which set the prices of skill-intensive intermediates, $\chi(i)$ and $\zeta(i)$.

1. Take the distribution of $q_x(i)$ and $q_z(i)$ as given and assume that all intermediates can be produced at marginal cost equal to 1 in terms of the final good y. Characterize the equilibrium and find the unskilled and the skilled wage w_L and w_H. [**Hint:** final good producers have to make zero-profits].

2. What changes in parameters could increase the skill premium, w_H/w_L, in this economy. In answering this question, distinguish between $\rho > 0$ and $\rho < 0$, and explain why the results differ in these two cases.

3. Now endogenize $q_x(i)$ and $q_z(i)$. Assume that R&D on a machine of quality q costs κq units of the final good, and leads to a new vintage of quality λq. Assume that λ is high enough such that the producer of the new vintage can set the monopoly price (instead of a limit price). Characterize the balanced growth path equilibrium.

4. Can we have $d(w_H/w_L)/d(H/L) > 0$? Give the intuition carefully, and explain why this can never happen when $\rho < 0$.

5. Repeat this exercise when a new vintage in sector x is of quality $\lambda_x q$ while a new vintage in sector z is of quality $\lambda_z q$. Why haven't the results changed much?

Question 3 (Competition and Growth):

1. What is the effect of competition on the rate of growth of the economy in a standard product variety model of endogenous growth? What about the quality-ladder model? Explain the intuition.

2. Now consider the following one-period model. There are two Bertrand duopolists, producing a homogeneous good. At the beginning of each period, duopolist 1's marginal cost of production is determined as a draw from the uniform distribution $[0, \bar{c}_1]$ and the marginal cost of the second duopolists is determined as an independent draw from $[0, \bar{c}_2]$. Both cost realizations are observed and then prices are set. Demand is given by $Q = A - P$.

   (a) Characterize the equilibrium pricing strategies and calculate expected ex ante profits of the two duopolists.

   (b) Now imagine that both duopolists start with a cost distribution $[0, \bar{c}]$, and can undertake R&D at cost k. If they do, with probability λ, their cost distribution shifts to $[0,\ \bar{c} - \alpha]$ where $\alpha < 1$. Find the conditions under which one of the duopolists will invest in R&D and the conditions under which both will.

   (c) What happens when $\bar{c}$ declines? Interpreting the decline in $\bar{c}$ as increased competition, discuss the effect of increased competition on innovation incentives. Why is the answer different from that implied by the standard endogenous growth model?