1 14.461 Advanced Macro: Additional Problems

Question 1 (Endogenous Growth Without Scale Effects): Consider the following model. Population at time \( t \) is \( L(t) \) and grows at the constant rate \( n \) (i.e., \( \dot{L}(t) = nL(t) \)). All agents have preferences given by

\[
\int_0^\infty \exp(-\rho t) \frac{C^{1-\theta} - 1}{1-\theta} \, dt,
\]

where \( C \) is consumption defined over the final good of the economy. This good is produced as

\[
Y = \left[ \int_0^N y(i)^\beta \, di \right]^{1/\beta}
\]

where \( y(i) \) is intermediate good \( i \). The production function of each intermediate is

\[
y(i) = l(i)
\]

where \( l(i) \) is labor allocated to this good.

New goods are produced by allocating workers to the R&D process, with the production function

\[
\dot{N} = \eta \cdot N^\phi \cdot L_R
\]

where \( \phi \leq 1 \) and \( L_R \) is labor allocated to R&D. So labor market clearing requires

\[
\int_0^N l(i) \, di + L_R = L
\]

Risk-neutral firms hire workers for R&D. A firm who discovers a new good becomes the monopoly supplier, with a perfectly and indefinitely enforced patent.

1. Characterize the balanced growth path equilibrium in the case where \( \phi = 1 \) and \( n = 0 \). Why does the long-run growth rate depend on \( \beta \)? Why does the growth rate depend on \( L \)? Do you find this plausible? Why aren’t there any transitional dynamics?

2. Now suppose that \( \phi = 1 \) and \( n > 0 \). What happens? Interpret.

3. Now characterize the balanced growth path equilibrium when \( \phi < 1 \) and \( n > 0 \). Does the growth rate now depend on \( L \)? Does it depend on \( n \)? Why? Do you think that the configuration \( \phi < 1 \) and \( n > 0 \) is more plausible than the one with \( \phi = 1 \) and \( n = 0 \)?

Question 2 (Endogenous Skill-Biased Technical Change): There are \( H \) skilled and \( L \) unskilled workers, and two goods, \( y_L \) and \( y_H \). All consumers have instantaneous utility defined over the final good \( y \)

\[
U = y = [y_L^{\rho} + \gamma y_H^{\rho}]^{1/\rho},
\]

and are risk-neutral would discount rate \( r \).
The production function of these two goods are:

\[ y_L = \left( \int_0^1 q_x(i) x(i)^{\alpha} di \right)^{1-\alpha} \]
\[ y_H = \left( \int_0^1 q_z(i) z(i)^{\alpha} di \right)^{1-\alpha} \]

where \( l \) and \( h \) are quantities of skilled and skilled labor; \( x(i) \) is the quantity of labor-complementary intermediate good \( i \) that an unskilled worker produces with, and \( z(i) \) is the quantity of skill-complementary intermediate good \( i \) that a skilled worker produces. \( q_x(i) \) and \( q_z(i) \) denote the quality of the highest vintage of machine \( i \) used for sector \( L \) or \( H \).

The profit function of a labor-intensive firm employing \( l \) workers is therefore:

\[ p_L \left( \int_0^1 q_x(i) x(i)^{\alpha} di \right)^{1-\alpha} - \left( \int_0^1 \chi(i) x(i) di \right) - w_L l \]

where \( w_L \) is unskilled wage, and \( p_L \) is the price of the labor intensive good, and \( \chi(i) \) is the price of intermediate good \( x(i) \). The profit function of a skill-intensive good is similarly defined. Suppose that intermediate goods are supplied by monopolistically competitive firms, which set the prices of skill-intensive intermediates, \( \chi(i) \) and \( \zeta(i) \).

1. Take the distribution of \( q_x(i) \) and \( q_z(i) \) as given and assume that all intermediates can be produced at marginal cost equal to 1 in terms of the final good \( y \). Characterize the equilibrium and find the unskilled and the skilled wage \( w_L \) and \( w_H \). [Hint: final good producers have to make zero-profits].

2. What changes in parameters could increase the skill premium, \( w_H/w_L \), in this economy. In answering this question, distinguish between \( \rho > 0 \) and \( \rho < 0 \), and explain why the results differ in these two cases.

3. Now endogenize \( q_x(i) \) and \( q_z(i) \). Assume that R&D on a machine of quality \( q \) costs \( q \) units of the final good, and leads to a new vintage of quality \( \lambda q \). Assume that \( \lambda \) is high enough such that the producer of the new vintage can set the monopoly price (instead of a limit price). Characterize the balanced growth path equilibrium.

4. Can we have \( d(w_H/w_L)/d(H/L) > 0 \)? Give the intuition carefully, and explain why this can never happen when \( \rho < 0 \).

5. Repeat this exercise when a new vintage in sector \( x \) is of quality \( \lambda_x q \) while a new vintage in sector \( z \) is of quality \( \lambda_z q \). Why haven’t the results changed much?

Question 3 (Competition and Growth):

2. Now consider the following one-period model. There are two Bertrand duopolists, producing a homogeneous good. At the beginning of each period, duopolist 1’s marginal cost of production is determined as a draw from the uniform distribution \([0, \bar{c}_1]\) and the marginal cost of the second duopolist is determined as an independent draw from \([0, \bar{c}_2]\). Both cost realizations are observed and then prices are set. Demand is given by \(Q = A - P\).

(a) Characterize the equilibrium pricing strategies and calculate expected ex ante profits of the two duopolists.

(b) Now imagine that both duopolists start with a cost distribution \([0, \bar{c}]\), and can undertake R&D at cost \(k\). If they do, with probability \(\lambda\), their cost distribution shifts to \([0, \bar{c} - \alpha]\) where \(\alpha < 1\). Find the conditions under which one of the duopolists will invest in R&D and the conditions under which both will.

(c) What happens when \(\bar{c}\) declines? Interpreting the decline in \(\bar{c}\) as increased competition, discuss the effect of increased competition on innovation incentives. Why is the answer different from that implied by the standard endogenous growth model?