14.461 PROBLEM SET 7 SOLUTIONS

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1. Endogenous Growth without Scale Effects

(i) Begin by calculating the demand for an intermediate good. We find this by solving the problem of a final good producer at any time t:

$$\max_{y_t(i)} \left\{ \left[\int_0^{N_t} y_t(i)^\beta \, di \right]^{\frac{1}{\beta}} - \int_0^{N_t} p_t(i)\, y_t(i)\, di \right\}$$

The first order condition is

$$y_t(i)^{\beta-1} \left[\int_0^{N_t} y_t(i)^\beta \, di \right]^{\frac{1}{\beta}-1} = p_t(i)$$

which can be rearranged to yield the demand for intermediate good i as

$$y_t(i) = p_t(i)^{\frac{1}{\beta-1}}\, Y_t.$$

Each intermediate monopolist takes this iso-elastic demand curve as given and solves

$$\max_{p_t(i)} \{ y_t(i) (p_t(i) - w_t) \}$$

which results in them setting price as a constant percentage markup over their marginal cost

$$p_t(i) = \frac{w_t}{\beta}.$$

From this we can express the flow profit to intermediate producer i as

$$\pi_t(i) = Y_t (1-\beta) \left(\frac{w_t}{\beta} \right)^{\frac{\beta}{\beta-1}}. \tag{1}$$

We can also determine the amount of $y_t(i)$ that the monopolist will sell

$$y_t(i) = \left(\frac{w_t}{\beta} \right)^{\frac{1}{\beta-1}} Y_t.$$

Substituting this back into the production function we get that

$$Y_t = \left[\int_0^{N_t} \left(\left(\frac{w_t}{\beta} \right)^{\frac{1}{\beta-1}} Y_t \right)^\beta di \right]^{\frac{1}{\beta}}$$

which pins down the equilibrium wage in terms of the number of goods produced

$$w_t = \beta N_t^{\frac{1-\beta}{\beta}}. \tag{2}$$

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Next let's calculate L_t^P the demand for labor to produce intermediate goods. This will be

$$L_t^P = \int_0^{N_t} y_t(i)\, di = Y_t N_t^{\frac{\beta-1}{\beta}}$$

Next let's turn to the free entry condition which governs the growth of the intermediate sector. Let V_t denote the present discounted value of an intermediate good producer at time t. Next consider a firm undertaking R&D to create a new intermediary. Spending $w_t L_{Rt}$ on R&D will generate $\dot{N}_t$ new units each of which will be worth V_t. The free entry condition requires that entry to occur up to the break even point so that we must have

$$V_t \dot{N}_t - w_t L_{Rt} = 0$$

which (using $\dot{N}_t = \eta N_t L_{Rt}$) requires that

$$w_t = \eta N_t V_t.$$

We can combine this with (2) to yield

$$V_t = \left(\frac{\beta}{\eta}\right) N_t^{\frac{1-2\beta}{\beta}}.$$

Since we are looking for the balanced growth path I am now going to guess that the range of product varieties is growing at a constant rate and we will verify this later (see below). In particular suppose that the rate of growth of N_t is x so that

$$N_t = N_0 e^{xt} \tag{3}$$

Using our guess we will have that

$$V_t = \left(\frac{\beta}{\eta}\right) N_0^{\frac{1-2\beta}{\beta}} e^{x\left(\frac{1-2\beta}{\beta}\right)t} \tag{4}$$

which implies that the growth of V_t must be

$$\frac{\dot{V}_t}{V_t} = x\left(\frac{1-2\beta}{\beta}\right). \tag{5}$$

Armed with this expression we can then make use of the Bellman equation which governs the optimal path for any intermediate good producer. This is simply

$$rV_t - \dot{V}_t = \pi_t$$

Combining this with (1) (2) (4) and (5) we get that equilibrium value of final output at time t must be

$$Y_t = \left(\frac{1}{1-\beta}\right)\left(r - x\left(\frac{1-2\beta}{\beta}\right)\right)\left(\frac{\beta}{\eta}\right) N_0^{\frac{1-\beta}{\beta}} e^{x\left(\frac{1-\beta}{\beta}\right)t} \tag{6}$$

which implies that the growth rate of output and hence consumption is

$$\frac{\dot{Y}_t}{Y_t} = x\left(\frac{1-\beta}{\beta}\right).$$

Next we make use of the consumer's optimization to pin down the interest rate. Since all output is consumed then it must be that the interest rate is such that induces the consumer to adopt and growth rate of consumption equal to the growth rate of output. The consumer's Euler equation is

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r-\rho).$$

So in equilibrium the interest rate must be such that

$$x\left(\frac{1-\beta}{\beta}\right)=\frac{1}{\theta}\left(r-\rho\right). \tag{7}$$

The next task we have is to determine what x is in equilibrium. The equation governing the growth of product varieties gives us that

$$\frac{\dot{N}_t}{N_t}=x=\eta L_{Rt} \tag{8}$$

which allows us to write (7) as

$$\eta L_{Rt}\left(\frac{1-\beta}{\beta}\right)=\frac{1}{\theta}\left(r-\rho\right) \tag{9}$$

and thus we need to pin down r and L_{Rt}. The equilibrium value of L_{Rt} can be found from the labor market clearing condition $L_{Rt}+L_{Pt}=L$. And we calculated equilibrium demand for production workers as $L_{Pt}=Y_tN_t^{\frac{\beta-1}{\beta}}$. When combined with (3) and (6) this yields

$$L_{Pt}=\left(\frac{1}{1-\beta}\right)\left(r-x\left(\frac{1-2\beta}{\beta}\right)\right)\left(\frac{\beta}{\eta}\right)$$

So labor market clearing combined with (8) requires that

$$L_{Rt}=L-\left(\frac{1}{1-\beta}\right)\left(r-\eta L_{Rt}\left(\frac{1-2\beta}{\beta}\right)\right)\left(\frac{\beta}{\eta}\right). \tag{10}$$

The system is then pinned down by solving (9) and (10) simultaneously for L_{Rt} and r. This yields

$$L_{Rt}=\frac{\left(1-\beta\right)\left(L-\frac{\rho\beta}{\eta}\right)}{\beta+\theta\left(1-\beta\right)}$$

$$r=\frac{L\eta\theta\left(1-\beta\right)^2+\rho\beta^2\left(1+\left(1-\beta\right)\theta\right)}{\beta\left(\beta+\theta\left(1-\beta\right)\right)}.$$

Now we can verify our initial guess that the growth of product varieties was constant. This is easily seen by noting that $\frac{\dot{N}_t}{N_t}=\eta L_{Rt}$ so that

$$\frac{\dot{N}_t}{N_t}\equiv x=\frac{\left(1-\beta\right)\left(L\eta-\rho\beta\right)}{\beta+\theta\left(1-\beta\right)}.$$

Finally, we can write the growth of output (and hence consumption) as

$$\frac{\dot{Y}_t}{Y_t}=\frac{\left(1-\beta\right)^2\left(L\eta-\rho\beta\right)}{\beta\left(\beta+\theta\left(1-\beta\right)\right)}.$$

This is equal to the growth of wages. The growth of V_t is equal to the growth of profits and is

$$\frac{\dot{V}_t}{V_t}=\frac{\dot{\pi}_t}{\pi_t}=\left(\frac{1-2\beta}{\beta}\right)\left(\frac{\eta L\left(1-\beta\right)-\rho\beta}{2-3\beta+\theta\left(1-\beta\right)}\right)$$

The growth rate depends on θ (growth is lower the higher is θ). To understand this recall that θ is the intertemporal elasticity of substitution. The lower it is the closer the utility function is to being linear. Note that in equilibrium consumption is growing over time. Individuals with a high θ have a stronger desire to bring forward future consumptions and so (for a given level of output growth) require higher equilibrium rates of interest to counter such a tendency. So higher θ leads to higher r. Output growth is decreasing in r. To understand why note that the reward to innovation V_t is the present discounted value of the profit flows generated by a new intermediate good. The larger is r the more will future profits be discounted and hence the less incentive there is for the creation of additional intermediate goods.

The growth rate depends on L because it is the size of work force which determines how much R&D is undertaken in the economy.

There aren't any transitional dynamics because the equilibrium labor allocation is also the steady state allocation.

(ii) This will yield explosive growth - this will be clear in Part 3.

(iii) Now we consider the case where $\phi < 1$ and $n > 0$. Our calculations in Part 1 up until the free entry condition will be the same. The free entry condition now implies that

$$V_t \eta N_t^{\phi} L_{Rt} - w_t L_{Rt} = 0.$$

Using (2) this becomes

$$V_t = \left(\frac{\beta}{\eta}\right) N_t^{\frac{1-\beta}{\beta} - \phi}. \tag{11}$$

Again I employ the guess and verify method. Let x be the constant growth rate of N_t on the balanced growth path.This then implies that

$$\frac{\dot{V}_t}{V_t} = x\left(\frac{1-\beta-\beta\phi}{\beta}\right). \tag{12}$$

Next consider the Bellman equation for a intermediate producing firm

$$rV_t - \dot{V}_t = \pi_t$$

Combining this with (1) (11) and (12) we get that the growth rate of output is

$$\frac{\dot{Y}_t}{Y_t} = x\left(\frac{1}{\beta} - \phi\right)$$

Recall that total employment in the production sector $L_{Pt} = Y_t N_t^{\frac{\beta-1}{\beta}}$ and so the growth rate of employment in this sector must be

$$\frac{\dot{L}_{Pt}}{L_{Pt}} = \frac{\dot{Y}_t}{Y_t} + \left(\frac{\beta-1}{\beta}\right)\frac{\dot{N}_t}{N_t} = x\left(1-\phi\right)$$

Next consider the equation which governs the growth in the number of product varieties. Combining this with labor market clearing we have that

$$\dot{N}_t = \eta N_t^{\phi}\left(L_t - L_{Pt}\right) \tag{13}$$

Suppose that the economy began on the balanced growth path at time zero. We can then write

$$\begin{aligned} N_t &= N_0 e^{xt} \\ L_t &= L_0 e^{nt} \\ L_{Pt} &= L_{P0} e^{x(1-\phi)t} \end{aligned}$$

which allows us to re-write (13) as

$$N_0 x e^{xt} = \eta N_0 e^{\phi x t}\left(L_0 e^{nt} - L_{P0} e^{x(1-\phi)t}\right).$$

We need this to hold if our guess that the economy is on a balanced growth path (x is constant) is to hold. This will only occur if

$$\frac{\dot{N}_t}{N_t} \equiv x = \frac{n}{1-\phi}$$

This condition automatically pins down the growth rate of the economy as

$$\frac{\dot{Y}_t}{Y_t} = \left(\frac{n}{1-\phi}\right)\left(\frac{1}{\beta} - \phi\right)$$

The rest of the analysis to pin down the steady growth rates of the other variables is then straight forward and follows the same logic we applied in Part 1.

2. Endogenous Skill-Biased Technical Change

(i) Begin by considering the optimal choice of y_L and y_H in forming the composite good y. This will solve

$$\max\left[y - p_L y_L - p_H y_H\right].$$

The first order conditions give us that

$$\frac{p_L}{p_H} = \left(\frac{1}{\gamma}\right)\left(\frac{y_L}{y_H}\right)^{\rho-1} \tag{14}$$

which indicates how the relative demand for each good will depend upon their relative prices.

Next consider optimization in each sector. Let's characterize the problem from the L sector and the solution for the H sector will be identical in form. Firms producing y_L will choose $x(i)$ and l to solve

$$\max\left\{p_L\left(\int_0^1 q_x(i)\, x(i)^{\alpha}\, di\right) l^{1-\alpha} - \int_0^1 \chi(i)\, x(i)\, di - w_L l\right\}. \tag{15}$$

The first order condition of (15) with respect to $x(i)$ yields the demand function faced by the producer. This is

$$x(i) = \left(\frac{\chi(i)}{\alpha p_L q_x(i)}\right)^{\frac{1}{\alpha-1}} L$$

where I have substituted the labor market equilibrium condition that $l = L$. The monopoly producer of good i will solve

$$\max_{\chi(i)}\left(\left(\frac{\chi(i)}{\alpha p_L q_x(i)}\right)^{\frac{1}{\alpha-1}} L\left(\chi(i) - 1\right)\right)$$

which, given the iso-elastic demand curve admits the simple solution that

$$\chi(i) = \frac{1}{\alpha}$$

Armed with this we can return to the problem in (15) and characterize the equilibrium demand for labor. This is found from the first order condition with respect to l which we evaluate using the labor market clearing condition. From this we can calculate the equilibrium wage rate in the L sector in terms of the price p_L as

$$w_L = p_L^{\frac{-1}{\alpha-1}}(1-\alpha)\,\alpha^{\frac{-2\alpha}{\alpha-1}}\int_0^1 q_x(i)^{\frac{-1}{\alpha-1}}\, di \tag{16}$$

and similarly we can calculate the level of output in terms of p_L

$$y_L = Lp_L^{\frac{-\alpha}{\alpha-1}} \alpha^{\frac{-2\alpha}{\alpha-1}} \int_0^1 q_x(i)^{\frac{-1}{\alpha-1}}\,di.$$

At this point we now combine the results from the L sector with those from the H sector. To simplify expressions define the following

$$\Phi_x \equiv \int_0^1 q_x(i)^{\frac{-1}{\alpha-1}}\,di$$
$$\Phi_z \equiv \int_0^1 q_z(i)^{\frac{-1}{\alpha-1}}\,di$$

Thus dividing y_L by y_H will give

$$\frac{y_L}{y_H} = \left(\frac{L}{H}\right)\left(\frac{p_L}{p_H}\right)^{\frac{-\alpha}{\alpha-1}}\left(\frac{\Phi_x}{\Phi_z}\right)$$

To close our solution we combine this ratio - which has been derived from the supply side of the model - with (14) which was found by considering the relative demand of each good for a given set of relative prices. Combining these we find that the equilibrium price ratio is

$$\frac{p_L}{p_H} = \left[\left(\frac{1}{\gamma}\right)\left(\frac{L}{H}\right)^{\rho-1}\left(\frac{\Phi_x}{\Phi_z}\right)^{\rho-1}\right]^{\frac{\alpha-1}{\alpha\rho-1}}$$

Finally, we can use (16) and its counterpart from the H sector in conjunction with this equilibrium price ratio to arrive at the relative wages between the two sectors

$$\frac{w_H}{w_L} = (\gamma)^{\frac{1}{1-\alpha\rho}}\left(\frac{H}{L}\right)^{\frac{\rho-1}{1-\alpha\rho}}\left(\frac{\Phi_z}{\Phi_x}\right)^{\frac{\rho(1-\alpha)}{1-\alpha\rho}}$$

which is the equilibrium skill premium.

(ii) Looking at our expression for the equilibrium skill premium we have that

$$sign\left\{\frac{\partial\left(\frac{w_H}{w_L}\right)}{\partial\left(\frac{H}{L}\right)}\right\} = sign\left\{\frac{\rho-1}{1-\alpha\rho}\right\}$$

which is positive if and only if $\rho \in \left[1, \frac{1}{\alpha}\right]$ and we should rule out $\rho \geqslant 1$ so this simply tells us that the premium earned by one factor falls when its relative supply is increased.

Next consider the effect of technological change upon the skill premium

$$sign\left\{\frac{\partial\left(\frac{w_H}{w_L}\right)}{\partial\left(\frac{\Phi_z}{\Phi_x}\right)}\right\} = sign\left\{\frac{\rho(1-\alpha)}{1-\alpha\rho}\right\}$$

This is positive if and only if $\rho \in \left[0, \frac{1}{\alpha}\right]$. So (since we are imposing $\rho \leqslant 1$) it hold that technology improvements in the H sector will increase the skill premium provided that $\rho > 0$. Why? Note first that when $\rho = 0$ the production function of the composite good is Cobb-Douglas - and so expenditure is divided equally between the two. When $\rho > 0$ then a fall in the price of one of the two good will cause a greater percentage increase in the demand for that good - the effect of a price fall in good H will be to increase expenditure on good H. Conversely when $\rho < 0$ the opposite will hold. In the extreme the production function will be leontieff. It is for this reason that the effect of technology change upon the skill premium depends on the value of ρ - noting that a technology change works effectively to supply the good at a lower price.

(iii) I am only going to sketch a solution here. To begin the question needs to be amended from the originally posted version because a quality ladders model requires new entry to be stochastic (it is the probability of being a monopolist that gives the incentive to innovate t begin with). Let's assume that doing an amount of R&D of κ when the current quality of good is q will cost κq. This will be successful with probability $\kappa\theta$ and will lead the innovator to be able to sell a good of quality $q\lambda$. If the existing technology is $q_{xt}(i)$ then an innovator will enjoy flow profits of

$$\pi_t = \left(P_{Lt} q_{xt}(i) \alpha^2\right)^{\frac{1}{1-\alpha}} L \left(\frac{1}{\alpha} - 1\right) \lambda^{\frac{1}{1-\alpha}}$$

If we let V_t be the present discounted value of expected profits for an innovator then we can write the Bellman equation as

$$rV_t - \dot{V}_t = \pi_t - xV_t$$

where x is the flow probability at which the monopolist will be taken over by another firm. Finally the free entry condition implies

$$\kappa\theta V_t - \kappa q = 0$$

From this we will be able to determine the rate at which innovation occurs and thus we can determine how Φ_x and Φ_z will evolve in steady state.

3. Competition and Growth

(i) Generally competition is bad for growth in endogenous growth models. This is because growth comes from individual agents undertaking costly R&D activities, the reward for which comes from the monopoly rents they will earn following a successful innovation. Competition decreases these rents and as such lowers the incentive for R&D activities. A good framework to think about this in the simple quality ladders model.

(ii) (a) Without loss of generality suppose that $\bar{c}_1 \leqslant \bar{c}_2$. Assume that $\bar{c}_1$ and $\bar{c}_2$ are sufficiently high relative to A so that we can focus purely on pricing equilibria which are characterized by limit pricing. As such we will have that the firm with the lowest marginal cost will satisfy all demand and will set their price equal to the marginal cost of their rival. As such if the realizations of marginal costs are such that $c_i < c_j$ then firm i will earn profits of

$$\pi_i = (A - c_j)(c_i - c_j).$$

Now to calculate the expected profits of each firm. For a given value of c_2 the expected value of π_1 is

$$E(\pi_1|c_2) = \begin{cases} \frac{1}{\bar{c}_1}\int_0^{c_2} (A - c_2)(c_2 - x)\,dx \text{ if } c_2 \leqslant \bar{c}_1 \\ \frac{1}{\bar{c}_1}\int_0^{\bar{c}_1} (A - c_2)(c_2 - x)\,dx \text{ if } c_2 > \bar{c}_1 \end{cases}$$

which integrates to give

$$E(\pi_1|c_2) = \begin{cases} \frac{1}{\bar{c}_1}(A - c_2)\left(\frac{c_2^2}{2}\right) \text{ if } c_2 \leqslant \bar{c}_1 \\ (A - c_2)\left(c_2 - \frac{\bar{c}_1}{2}\right) \text{ if } c_2 > \bar{c}_1 \end{cases}$$

We now integrate this over all possible values for c_2 to give $E(\pi_1)$:

$$E(\pi_1) = \int_0^{\bar{c}_2} \frac{1}{\bar{c}_2} E(\pi_1|c_2)\, dc_2$$

which we can write as

$$E(\pi_1) = \frac{1}{\bar{c}_2}\left[\int_0^{\bar{c}_1} \frac{1}{\bar{c}_1}(A-x)\left(\frac{x^2}{2}\right)dx + \int_{\bar{c}_1}^{\bar{c}_2}(A-x)\left(x-\frac{\bar{c}_1}{2}\right)dx\right].$$

I am going to leave this expression like this for now since it will minimize mess later. Now let's calculate the expected profits for firm 2:

$$E(\pi_2|c_1) = \frac{1}{\bar{c}_2}\int_0^{c_1}(A-c_1)(c_1-x)\,dx = \frac{1}{\bar{c}_2}(A-c_1)\left(\frac{c_1^2}{2}\right)$$

and so we have

$$E(\pi_2) = \frac{1}{\bar{c}_1}\int_0^{\bar{c}_1} E(\pi_2|c_1)\, dc_1$$

which we can write as

$$E(\pi_2) = \frac{1}{\bar{c}_1\bar{c}_2}\int_0^{\bar{c}_1}(A-x)\left(\frac{x^2}{2}\right)dx.$$

(b) Using our work from the previous section we can define the following four objects:

$$\Pi^{NN} \equiv \frac{1}{\bar{c}^2}\int_0^{\bar{c}}(A-x)\left(\frac{x^2}{2}\right)dx \tag{17}$$

$$\Pi^{YN} \equiv \frac{1}{\bar{c}}\left[\int_0^{\bar{c}-\alpha}\frac{1}{(\bar{c}-\alpha)}(A-x)\left(\frac{x^2}{2}\right)dx + \int_{\bar{c}-\alpha}^{\bar{c}}(A-x)\left(x-\frac{(\bar{c}-\alpha)}{2}\right)dx\right] \tag{18}$$

$$\Pi^{YY} \equiv \frac{1}{(\bar{c}-\alpha)^2}\int_0^{\bar{c}-\alpha}(A-x)\left(\frac{x^2}{2}\right)dx \tag{19}$$

$$\Pi^{NY} \equiv \frac{1}{\bar{c}(\bar{c}-\alpha)}\int_0^{\bar{c}-\alpha}(A-x)\left(\frac{x^2}{2}\right)dx. \tag{20}$$

These expressions are the expected profits of a firm given a particular outcome from the R&D stage. So if both firms do R&D and they both succeed then they will both earn Π^{YY} (gross of the R&D cost k). Conversely if a firm successfully carries out R&D and its rival does not (either it fails or it doesn't attempt R&D at all) then it will has expected profits of Π^{YN}. So the first superscript refers to my R&D outcome and the second superscript refers to the outcome of my rival. The expect payoff to undertaking R&D is dependant upon the action taken by the rival. Undertaking R&D when ones rival does yields an expected payoff of

$$\lambda^2\Pi^{YY} + \lambda(1-\lambda)\left(\Pi^{NY}+\Pi^{YN}\right) + (1-\lambda)^2\Pi^{NN} - k$$

Undertaking R&D when one's rival does not has an expected payoff of

$$\lambda\Pi^{YN} + (1-\lambda)\Pi^{NN} - k$$

Not undertaking R&D when one's rival does has an expected payoff of

$$\lambda\Pi^{NY} + (1-\lambda)\Pi^{NN}$$

And finally when neither firm undertakes R&D their expected payoffs are both Π^{NN}.
From here we can now form best response conditions. Doing R&D is a best response to No R&D if

$$\Pi^{YN} - \Pi^{NN} \geqslant \frac{k}{\lambda} \tag{21}$$

Doing R&D is a best response to ones rival doing R&D if

$$\lambda \left(\Pi^{YY} - \Pi^{NY}\right) + (1-\lambda)\left(\Pi^{YN} - \Pi^{NN}\right) \geqslant \frac{k}{\lambda}. \tag{22}$$

Note that we can expand these conditions by writing

$$\begin{aligned} \Pi^{YN} - \Pi^{NN} &= \frac{\alpha}{\bar{c}^2 (\bar{c}-\alpha)} \int_0^{\bar{c}-\alpha} (A-x)\left(\frac{x^2}{2}\right) dx \\ &\quad + \int_{\bar{c}-\alpha}^{\bar{c}} \frac{(A-x)}{2\bar{c}} \left(2x - \bar{c} + \alpha - \frac{x^2}{\bar{c}}\right) dx \end{aligned}$$

$$\Pi^{YY} - \Pi^{NY} = \frac{\alpha}{\bar{c}\,(\bar{c}-\alpha)^2} \int_0^{\bar{c}-\alpha} (A-x)\left(\frac{x^2}{2}\right) dx$$

So both firms carrying out R&D is an equilibrium if (22) holds. Conversely we will have an equilibrium in which only one firm does R&D when (21) holds but (22) does not. A necessary condition for this asymmetric equilibrium is that

$$\Pi^{YN} - \Pi^{NN} \geqslant \frac{k}{\lambda}$$

$$\Pi^{YY} - \Pi^{NY} < \frac{k}{\lambda}$$

(c) Let's answer this without doing all the integration. Begin by considering $\Pi^{YN} - \Pi^{NN}$. This is the expected return to being successful in R&D conditional on the fact that ones rival is not. When α is small (or the larger is $\bar{c}$ for a given value of α) this difference will be positive but small - because an innovation doesn't get you much. So having $\bar{c}$ decrease for a given value of α will increase the return to successfully carrying out R&D.
Next consider $\Pi^{YY} - \Pi^{NY}$. This is the expected return to successfully completing R&D conditional on ones rival having successfully innovated. As we argued in the previous paragraph innovation won't get you much if α is small (or the larger is $\bar{c}$ for a given value of α). But this is a non-monotonic relationship. To see this suppose that $\bar{c}$ is as small as it can be relative α - this will occur when $\bar{c} = \alpha$. In this instance successfully innovating means that a firms marginal cost falls to zero with certainty. Now given that ones rival has zero marginal cost then the expected profits to innovating is zero - all marginal cost gains will be competed away. So as α moves over the interval $(0, \bar{c})$ (or conversely as $\bar{c}$ moves over the interval (α, ∞)) we will have that $\Pi^{YY} - \Pi^{NY}$ is positive and obtains a value of zero at each of the interval.
We can then comment upon the effect of increased competition. Increasing competition will always increase the incentive to undertake R&D if ones rival is not. However the effect of competition upon inducing an equilibrium in which both firms innovate will be non-monotonic - it will provide a positive effect for when competition is slack and but this will be reversed if competition is already very aggressive.

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