Better Technologies, Larger Wars, and Influential Persuaders:

A Synoptic View of Warring States History

Abstract: This paper reviews the history of the Warring States period in China (403 B.C. to 221 B.C.) and analyses it under a game-theoretic framework. In particular, it shows why: a) Increased productivity lead to larger scaled wars; b) Coalitions of five or six countries often joined together to fight one single country; c) Countries used Persuaders as a method to coordinate their efforts.

I. Introduction

Warring States (403 B.C. to 221 B.C.) is an extremely important period in Chinese history. During the period, the Chinese culture, its social economic situations, and its political institutions underwent dramatic changes. Tremendous areas of new land were cleared and cultivated; population exploded; technological innovations abounded. Many of the changes were direct consequences of the reforms adopted by the countries of the time. There were seven major countries during the Warring States period, and all of them undertook serious reforms. Although these reforms led to great increases in productivities, they destroyed the balance of powers among the countries. Consequently, wars constantly broke out among the countries.

As suggested by the name of the period, the battles of the Warring States were frequent and fierce, massive and multinational. The size and scale of the wars escalated with time. Some later wars involve about one million people, which was a shockingly large number considering the time they occurred. In major wars, countries formed alliance to fight together, but the coalitions were fleeting and unstable. Nevertheless, one distinct pattern of the coalitions was persistent: it often happened that five or six countries
formed a huge coalition to fight one single country. Furthermore, the coalition was mostly formed by the Persuaders, who attached little loyalty to any of the countries.

Despite its richness, the history of the Warring States had not received much attention. This is partly due to the scarcity of historical records. Most records about the countries in the Warring States were destroyed by the First Emperor of Qin. The few remaining ones were burnt in a fight several decades later. Fortunately, historians of Han Dynasty still reconstructed some pictures of the Warring States. Although we no longer had detailed records for each individual country, the wars between the countries were fully recovered and documented.

These war histories are of great value to applied game theorists and economists in general. Deeper understandings can be gained from observing how the coalitions were formed and dissolved. Furthermore, we can examine how countries cooperated with each other when no contracts could be enforced. The economic community, however, seems to have so far neglected this gold mine. I could only find one paper related to the Warring States from the Econlit. That paper argued verbally how competition among the countries benefited China during the Spring and Autumn (770 B.C to 403 B.C.) and the Warring States period and has little reference to the wars of the period.

In this paper, we build a simple model that attempts to offer a framework for analyzing the wars among the countries during the Warring States. In particular, we showed and explained that a): Increased productivities lead to larger scaled wars; b): Why five or six countries often join together to fight one single country; c) Why countries use Persuaders as a method to coordinate their efforts and conquer moral hazard problems. The model can be enriched to analyze how geographic locations and how the small
countries might affect the formation of coalitions.

The rest of the paper is organized as follows. Section II discusses the background of the seven countries and a detailed history of the major wars occurred during the Warring States period. A simple model is set up in Section III and is applied to explain the escalation of wars and the formation of coalitions. The role of the Persuaders is also discussed in Section II and is formally analyzed in the appendix. We conclude in section IV and discuss some potential extensions of the model.

II. Background

Although there is no consensus among the historians of the starting dates of the period, the most popular view considers it begin in 403 B.C., when Zhao, Wei, and Hann were recognized as independent states. The period ended in 221 B.C., when Qin conquered all of the other countries and united China.

A. The Seven Countries

There were seven major countries during the Warring States period: Qi, Chu, Yan, Hann, Zhao, Wei, and Qin. These countries possess independent central economical, military, and political power. These countries were constantly in wars with each other in order to expand their territories, to compete for submission from smaller countries, or just to prevent one country from becoming dominant. The scales of the wars were significant, and they escalated in the later period of the Warring States. Frequently, there were wars involving six or seven countries, mostly in the form of many countries forming a giant alliance to fight one single country. The frequency of the wars even made people of that time called the period “Warring States”, and this name has passed along.
A. Qi

Qi was a large country on the east coast. Its territory comprised of northern parts of the modern ShanDong province and the southeastern parts of the modern HeBei province. Because of its geographic locations, Qi had been rich in the resources of the sea, and it had a productive agricultural sector. Qi had always been one of the strongest countries among the seven, especially after its reform under King Wei. In 286 B.C., however, Qi was defeated fatally by a coalition formed by six other countries. Qi’s strengthen weakened afterwards and was conquered by Qin in 221 B.C.

B. Chu

Chu had the largest territory among the seven countries. It situated in the south and its territory spread across the modern HuBei, HuNan, JiangXi, AnHui, ShanXi, HeNan, and JiangSu provinces. Chu’s power during the Warring States, however, did not match its territory size. Many internal crises weakened the power of Chu. Chu’s reform was also the least successful among the seven countries. As a result, it never became a superpower, and Chu’s strength was further weakened after its defeat by an alliance led by Qi in 301 B.C. Chu was annihilated by Qin in 226 B.C.

C. Yan

Yan located in the far northeast. Its geographical remoteness prevented Yan from involving in most wars. It was also the last country to emerge as a major power in the Warring States. Yan bordered Zhao on the west and Qi on the south. As a result, Yan was often invaded by the two countries. Although Yan managed to acquire huge areas of land from Qi in the war in 286 B.C., many of the lands were soon lost. Yan was eliminated by Qin in 222 B.C.
D. Hann

Together with Wei and Zhao, Hann originated from the partition of Jin, which was one of the largest countries in the Spring and Autumn period. Hann had the smallest territory among the seven countries and its power had been weak throughout. Hann was a loyal ally of Wei most of the times, and its power declined with Wei’s fall. Hann was the first of the six countries that were conquered by Qin.

E. Zhao

Zhao situated in the north, and its territory spread across the modern Shanxi, Shannxi, HeBei, HeNan, and ShanDong province. Together with Wei and Hann, it was also a successor of Jin. The three countries formed a strong alliance in the early days of the Warring States and gained many lands. Zhao, however, did not receive much gain because it was in the north and the gained lands were in the south. This led Zhao to initiate a war with Wei, which caused the dissolution of the three country coalition. Years later, Zhao benefited from the reform of King WuLing, and it was the only power that could fight Qin after the defeat of Qi in 286 B.C. Zhao, however, lost to Qin in the war of ChangPing, in which more than 400,000 Zhao soldiers were killed. This was a fatal blow for Zhao, and Zhao was eliminated by Qin in 222 B.C.

F. Wei

Wei was the strongest country in the early days of the Warring States. Wei’s power partly benefited from its central location. Since Wei’s territory was surrounded by Qin, Zhao, Han, and Qi, Wei easily attracted scholars and talented people of all countries. Wei’s reform was the earliest of the seven countries, and many of the former Wei residents helped other countries reform. Although Wei’s location was a blessing, it was a
curse as well. As its neighboring countries grew and became more aggressive, Wei was constantly troubled by wars at all sides of its border. After successive lost from 350 B.C. on, Wei’s power declined gradually. Wei was conquered by Qin in 225 B.C.

G. Qin

Qin was the final winner of the seven countries. Qin located in the west and was protected by the Xiao Mountain. Qin had rich natural resources and had always been a superpower of the seven. Qin’s reform was the most dramatic of the seven countries, but it was the most effective as well. Qin’s power rose rapidly after the reform and increased steadily. From 232 B.C. to 221 B.C., Qin conquered the other six countries and united the China.

Figure 1: Maps of the Warring States.
(Sources: The Cambridge History of Ancient China p.594)
Although the wars were destructive, their occurrences infused a sense of competition into the minds of the emperors. The competition manifested most in the fight for human resources. All the emperors were extremely friendly to scholars and competed to meet the famous scholars of the time. As a result, numerous schools of thought bloomed: Mozi, the Sophists, Zhuangzi, Xunzi, Han Feizi, and many other philosophers were celebrities of the time and were extremely influential. Confucius’ thought was further developed by his most distinguished successor, Mencius.

These scholars not only had ideas, but also put their ideas into practice by actively participating in political activities. It was not uncommon for an ordinary scholar to become a powerful minister of a country, once after the emperor recognized the talent of the scholar. For scholars in ministerial positions, they often offered advice and proposals to the emperors on new ways to manage the country, so that the country could thrive and become stronger in face of new economic situations. Apparently, the emperors of the time were very open-minded. They trusted and listened to the advice of the scholars. Consequently, reforms flourished in the land of China. They did not happen in one country alone or two; reforms occurred in all seven countries!

Wei had the earliest reform among the seven countries. Surrounded by major powers, Wei Wen Hou, the emperor of Wei, realized that Wei could not survive unless it became a superpower. Appointing Li Kui as its Prime Minister, Wei Wen Hou adopted three major policies. First, Wei replaced the heritage system of ministerial positions with a system based on meritocracy. Second, Wei changed its tax policy to provide better incentives to the peasants. Third, Li Kui constructed the first comprehensive judicial system in Wei, so that decisions could be made according to the law. These policies
boosted the productivity of Wei tremendously and made Wei a superpower in the early days of the Warring States.

Wei’s reform not only had deep impact on its own, it also positive externalities on neighboring countries. Some residents of Wei immigrated to other countries and initiated reforms there. Wu Qi, a former minister in Wei, moved to Chu and helped it reform. Shang Yang, also a former resident of Wei, helped Qin to start its reform. Zhao and Qi started their reforms a bit later, but their reforms were very successful.

B. The Wars

Since Wei’s reform occurred earliest among the seven countries, it became the strongest country in the earlier periods of Warring States. Wei formed a coalition with Zhao and Hann, probably because the three countries share the origin and the emperors had good friendships. The coalition was extremely powerful and won a sequence of wars against Qin, Qi, and Chu. These victories brought large areas of land to the three countries, but the division of the gains was hardly even. In particular, Zhao received the smallest gain as it situated in the north and the defeated countries were in the south.

To expand its territory, Zhao attacked Wey and forced it to become Zhao’s subordinate. Wey, however, was previously a subordinate country of Wei, so Wei attacked Zhao in 354 B.C. Wei soon besieged the capital of Zhao, and Zhao asked for help from Qi and Chu. Meanwhile, Qin also attacked Wei. This caused a first large scale war among the countries. Although Wei defeated the joint force of Qi, Zhao, and Chu on the east, it lost many lands to Qin on the west.

Although Wei remained strong, many other countries, particularly Qin and Qi, also rose to power after effective reforms. The change in power balance led to many
wars. Since Wei’s territory was surrounded by Qin, Zhao, Qi, and Chu’s, it was most severely inflicted by wars. In 341 B.C, Wei was simultaneously attacked by Qin, Zhao, and Qi. Wei lost in all sides and its prince was captured. This had been the largest defeat Wei ever experienced, and Wei’s power waned afterwards. Wei constantly lost to Qin in many of the small wars ensued, and its territory diminished gradually.

The decline of Wei was accompanied by the rise of Qin and Qi. While Qi was relatively peaceful, Qin was very aggressive and posed a great threat to Wei. Consequently, Wei formed a coalition with Hann, Zhao, Chu, and Yan. In 318 B.C, these five countries joined forces and attacked Qin. The joint forces won some early victories, but the coalition was not robust. Chu and Yan never fought against Qin seriously. Zhao, Wei, and Hann were defeated by Qin in 317 B.C. This led to the dissolution of the coalition. Despite its failure, this was the first major coalition of countries fighting one superpower. This pattern recurred in many of the subsequent wars.

The defeat of the joint force further weakened the power of Wei. Once the strongest country, Wei could only struggle for its survival and faced continual diminution of territory. Its central location among the countries made Wei a target of many superpowers. Together with Hann, a weak country also located in the center, the two countries frequently had to join a superpower in fighting other countries. In 312 B.C, Wei and Hann joined Qin to fight Chu. Later the year, the three countries fought Qi again. Both Chu and Qi were defeated, while Qin acquired all of the gain. In 301 B.C, Wei and Hann joined Qi to fight Chu. This was a debacle for Chu, as it lost huge areas of land. In 298 B.C, Wei and Hann again joined Qi to fight Qin. After three years of war, the joint force conquered HanGu Guan, the most important garrison of Qin. Qin begged
for peace and gave up large areas of land. After defeating Qin, the three countries attacked Yan in the north and destroyed the military force of Yan.

Despite its success, coalition of Hann, Wei, and Qi dissolved in 294 B.C. Qin took advantage of this and defeated Hann and Wei decisively in the war of YiQue. Two 240,000 soldiers from Hann and Wei were killed. Hann and Wei also lost huge areas of land. As the result, Wei asked for help from Zhao, hoping Zhao could protect Wei from Qin. Since Zhao was rarely involved in the early wars, its participation added great instability to the relations among the countries.

288 B.C. was an extremely unusual year for coalition formation of the countries in the Warring States. The whimsy of the countries paled the friendships to the desires for profits. Earlier of the year, Zhao led Chu, Wei, Hann, and Qi to form a coalition in order to attack Qin. None of the countries, however, was willing to be the first one to fight Qin, so the joint force was stuck in ChenGao and had retreat. In October, Qin joined to Qi to form a coalition, which also included Chu, Wei, and Hann in order to attack Zhao together. Contracts were written, and the date of attack was set. Qi, however, broke the contract. In December, Qi instead joined the rest of the countries to fight Qin. Qin begged for peace and gave up large areas of land.

The victory of the joint force further increased Qi’s power. Qi began to attack Song, a small country that has DingTao, the most important commercial city of the time. Qi conquered Song in 286 B.C., and this troubled the rest of the countries. In 284 B.C, Qin formed a giant coalition with the rest of the countries to attack Qi. This delivered a fatal blow to Qi. Yan obtained more than seventy cities from Qi; Qin acquired DingTao;
Wei, Zhao, and Chu also gained large areas of land from Qi. After the war, Qi lost more than sixty percent of its territory. Qi never recovered from the defeat.

After the debacle of Qi, Zhao became the only country that could fight Qin. Zhao engaged in one of the largest battles in the Warring States with Qin in 262 B.C. For three years, the two countries fought with each other at Changping, and no side had significant advantage. The emperor of Zhao, however, mistakenly replaced the commander of the Zhao army, General Lian Po, with an inexperienced General, Zhao Huo. Zhao Huo was soon tricked by the Qin army and lost the war. 400,000 soldiers from Zhao were massacred by Qin.

After the loss of Zhao, no other country had power to fight against Qin anymore. Wei’s power had waned due to successive losses since 350 B.C, especially after the war in 341 B.C. Chu suffered from a serious loss to the coalition led by Qi in 301 B.C, and its strength was further reduced from many internal crises. Although Qi had always been strong, its debacle to the six-country coalition was fatal. The two countries left, Yan and Hann, had always been small and weak throughout. Therefore, Qin started its great unification program in 230 B.C. Within ten years, all six countries were eliminated by Qin. Qin’s conquer of Qi in 221 B.C. marked the end of the Warring States period.

The high frequency of wars among the countries is certainly the most distinctive feature of the Warring States, as implied by the name of the time period. But there are a few other interesting patterns of the wars. First, countries formed coalitions throughout the period. Except for the War of Changping, nearly all of the major wars involved strategic coalitions. Second, the powers on the two sides of the fight were often very imbalanced, especially in large scaled wars. Often five or six countries formed a giant
coalition to fight one single country. Third, the coalitions were not very stable. A giant coalition could collapse within a month and was replaced by another one. Forth, even if contracts were written to restrain the actions of the countries, they are often violated easily and no punishment was imposed to the violators. Finally, many coalitions are initiated by Persuaders, who travel around to convince the emperors to form certain alliances. Once a coalition was formed, the persuader who joined the participating countries together could often become the Prime Minister of all the participating countries. This is a phenomenon particular to the history of Warring States.

Table I summarizes the major wars of the period:

<table>
<thead>
<tr>
<th>Time of the War (B.C.)</th>
<th>The Winners of the Wars</th>
<th>The Defeated Countries of the Wars</th>
</tr>
</thead>
<tbody>
<tr>
<td>354-353</td>
<td>Zhao, Qi</td>
<td>Wei</td>
</tr>
<tr>
<td>341-340</td>
<td>Qi, Qin, Zhao</td>
<td>Wei</td>
</tr>
<tr>
<td>318</td>
<td>Qin</td>
<td>Wei, Zhao, Hann, Chu, Yan</td>
</tr>
<tr>
<td>312</td>
<td>Qin, Wei, Hann</td>
<td>Chu, Qi</td>
</tr>
<tr>
<td>301</td>
<td>Qi, Wei, Hann</td>
<td>Chu</td>
</tr>
<tr>
<td>298-296</td>
<td>Qi, Wei, Hann</td>
<td>Qin</td>
</tr>
<tr>
<td>288(1)</td>
<td>Zhao, Qi, Chu, Wei, Hann</td>
<td>Qin</td>
</tr>
<tr>
<td>288(2)</td>
<td>Zhao, Qi, Chu, Wei, Hann</td>
<td>Qin</td>
</tr>
<tr>
<td>285</td>
<td>Qin, Chu, Yan, Hann, Zhao, Wei</td>
<td>Qi</td>
</tr>
<tr>
<td>262</td>
<td>Qin</td>
<td>Zhao</td>
</tr>
</tbody>
</table>

III. The Model

Conflicts among the states intensified in the later Warring States period, and the scale of the wars escalated. Before 500 B.C, most wars included no more than hundreds
of chariots followed by tens of thousands of men, who were feebly armed and had little training. In some of the largest wars from 400 to 300 B.C, the total number of men involved was less than one hundred thousand. From 300 B.C. on, however, many wars involved about one million people. In 251 B.C, the army attacking Zhao from Yan had more than 600,000 men. In the War of Changping, Qin killed 400,000 soldiers from Zhao in 260 B.C.

There is no theory, however, to explain why the scale of the wars increased so dramatically. Most authors simply documented the increase in size and scale of the wars and offered no explanation. (Geret, Lewis, Walker, and etc). Yang briefly mentioned in a three-line paragraph that the escalation of the wars comes from a): an increase of population, and b): the establishment of a complete conscription system. We argue here that the advancement in technology also contributed significantly to the increase in the scale of the wars.

Although the military technology had certainly improved during the Warring States period, we focus on the technology of goods production here. Indeed, we saw in the Warring States period development of irrigation system, revolution in agriculture production techniques (especially in using iron-plow), and the widespread of iron-casting. These technological innovations boosted the labor productivity and thus increased the revenues of the emperors.

If the objective of an emperor is simply to maximize his revenues, naïve reasoning would suggest that the scale of the war should decrease, rather increase as the technology progresses. The reasoning is that the cost of the war is to forgo the labors that could otherwise stay home and produce goods. Now that the labor productivity increases,
the opportunity cost of the war also goes up. Since the war becomes more costly, the emperors will be less willing to wage a large-scaled war.

The above reasoning is erroneous because it fails to distinguish labor productivity from marginal labor productivity. Even if the technological progress will boost the productivity, it may decrease the marginal productivity of labor. For instance, if the peasants learn to use the iron plow, so that a land that requires ten people in the past needs only one person now. Then the marginal productivities of the rest nine people drop down to zero. Once the marginal productivity drops down, the cost to fight a war is in fact lower, not higher as reasoned in the previous paragraph. As a result, the scale of the wars should escalate.

Indeed, in the earlier centuries of the Warring States period, many new lands were cultivated. There also emerged many independent peasants who discovered unused land and farmed on it. These are evidences that the labor productivity had increased so much that that there was an excess supply of labor in the old lands. Therefore, countries all spent great effort in search of new lands, and wars often broke out in fighting for new lands.

Another advantage of new lands resulted from the emergence of the cities. Cities were small in the Spring and Autumn period, but they had grown in the Warring States period to ten times as large as they were before. Numerous craftsmen resided in the cities permanently, and commercial activities were blooming. Since the emperor collected the sale tax, a large city can be of great value to him. Many wars were centered on the acquisition of a large commercial city.
The analysis above shows that the technological improvements increased the marginal value of the land. Therefore, it became an increasingly important task for the emperor to acquire more land. Once no new lands were available, countries fought each other to expand their territory. The higher the marginal value of the lands, the greater the wars. In fact, we show in the model below that under a Cobb-Douglas production function and reasonable assumptions of the wars, technology improvement leads to larger wars.

We look at a simple two period model with two countries, 1 and 2. Denote $N_1$ and $N_2$ the number of people in country 1 and 2 respectively. $N = N_1 + N_2$. Denote $L_1$ and $L_2$ the amount of land in country 1 and 2, and we let $L = L_1 + L_2$.

We assume that the population density is the same in both countries, so we have $d = \frac{N_1}{L_1} = \frac{N_2}{L_2} = \frac{N}{L}$, where $d$ is the population density. This assumption is made mostly for tractability, so the model can be solved analytically. On the other hand, this is not a bad assumption realistically, if we think of the land not as the area of the territory of a country, but as the amount of land that is being farmed. As the states in the Warring States period have close ties, there is no evidence that some country has superior technology than others.

The production function is Cobb-Douglas, so the output $Y = L^\alpha N^{1-\alpha}$. Note that we could write $Y = KL^\alpha N^{1-\alpha}$, but the constant $K$ will have no effect in the analysis, so we omit it. The larger the $\alpha$, the greater the marginal value of the land is.
If a war involves \( K_1 \) people from country 1, and \( K_2 \) people from country 2, we assume that the probability country 1 wins the war is \( \frac{K_1}{K_1 + K_2} \). Even if some countries might just be intrinsically better at fighting, adding those components in to the model can be ad hoc and unnecessarily complicates the analysis. We maintain that the countries are equal in their technologies, so that the idea of technology improvement leads to larger war will not be confounded with other factors.

Once a country wins a war, we assume that it gains \( \frac{K_1 + K_2}{2} \) of men from the other country. Or equivalently, it wins \( \frac{K_1 + K_2}{2d} \) amount of land from the other country. The equivalence follows from the earlier assumption that the population density is the same in two countries. The idea that the winning country can acquire land from the lost country is at the heart of the analysis and that is why we use \( \frac{K_1 + K_2}{2d} \) as the gain from the war. Another plausible assumption is the winning country, say country 1, will gain \( K_2 \), the number of men involved in the war from country 2. The problem of this assumption is that it gives insurance to the weak country. For example, if country 1 is a lot stronger than country 2, then country 2 might choose \( K_2 = 0 \) to avoid the war and any losses in men and land. Often, war is not a voluntary activity, but a forced one. Therefore, we believe that our assumption is more natural.

To complete the model, we assume that the emperor acquires all the goods that are produced in the state and his objective is to maximize his income. In the Warring States period, the income of an emperor is mainly composed of a): the rent of the land,
and b): various different taxes on “house”, “sales”, and etc. Therefore, it is reasonable to model the emperor’s earning as the proportion of the goods produced in his land. It turns out the proportion plays no role in the analysis, so for convenience we assume the emperor takes all of the outputs in his land.

To summarize, we have made the following assumptions:

**Assumption 1**: The population density is the same in both countries: \( \frac{N_1}{L_1} = \frac{N_2}{L_2} = \frac{N}{L} \).

**Assumption 2**: The production function is Cobb-Douglas: \( Y = L^{\alpha} N^{\beta} \).

**Assumption 3**: If a war involves \( K_1 \) people from country 1, and \( K_2 \) people from country 2, then the probability country 1 wins the war is \( \frac{K_1}{K_1 + K_2} \).

**Assumption 4**: Once a country wins a war, we assume that it gains \( \frac{K_1 + K_2}{2} \) of men and the associated land from the other country.

**Assumption 5**: The emperor acquires all of the goods produced in the state and his objective is to maximize his income.

To solve the model, we look for a pair of \( (K_1^*, K_2^*) \) as the Nash Equilibrium level of soldiers conscripted by the emperors to fight the war.

If the emperor of country 1 draft \( K_1 \) men and country uses \( K_2 \) men in the war, then in the first period, the emperor’s income is \( L_1^* (N_1 + \frac{K_1 + K_2}{2}) \). The probability that country 1 wins the war is \( \frac{K_1}{K_1 + K_2} \). If country 1 wins, then it will have \( N_1 + \frac{K_1 + K_2}{2} \) men and \( L_1^* \frac{K_1 + K_2}{2d} \) land. Note that \( d(L_1^* \frac{K_1 + K_2}{2d}) = N_1 + \frac{K_1 + K_2}{2} \), so the income of
the emperor in the second period is

\[(L_1 ? K_1 ? K_2) \two / 2d (N_1 ? K_1 ? K_2) \two \] given he wins the war.

Similarly, we can calculate that the emperor’s income in the second period is

\[d \two (N_1 ? K_1 ? K_2) \two \] if he loses the war. This happens with probability \[K_2 \two \]

Therefore, the expected income of the emperor in the second period is

\[\frac{K_1}{K_1 ? K_2} d \two (N_1 ? K_1 ? K_2) + \frac{K_2}{K_1 ? K_2} d \two (N_1 ? K_1 ? K_2) = d \two (N_1 + \frac{K_1}{K_2} \two ).\]

If the emperor has a discount rate of \(? ?, then his objective is to

\[Max L_i (N_1, K_i) \two \] + \[d \two (N_1 + \frac{K_1}{K_2} \two ).\]

The first order condition shows that \[d \two (1 ? ? ) = 0.\] In other words, we can explicitly solve \[K_1^* = L_2[1 ? \left( \frac{2(1 ? ? )}{? ? \two } \right)^1].\] Similarly, we can solve that

\[K_2^* = L_2[1 ? \left( \frac{2(1 ? ? )}{? ? \two } \right)^1].\] Note that the optimal number of men in the war is not just a Nash Equilibrium solution, but in fact a dominant strategy. The number of men employed in a war is proportion to the population of a country, where the proportion is determined by the production function.

**I. The Scale of the Wars**

From the production function, we observe that the marginal productivity of the land is \[\frac{Y}{L} = \frac{Y}{L} \]. As the marginal productivity of the land increases, \[? \] goes up. Note
that \((1 ? ? \, )^{\frac{1}{7}}\) is a decreasing function in \([0, 1]\). Therefore, as \(?\) increases, so does \(K_1^{\ast}\) and \(K_2^{\ast}\).

Now we have established our first result: Technological progress leads the war to escalate. To examine the expression \(K_1^{\ast} = L_d[1 ? \left(\frac{2(1 ? ?)}{?}\right)^{\frac{1}{7}}]\), we note also that the number of people in the war is proportional to the density of population. This is also an intuitive result.

One interesting result relates to how forward-looking the emperors are. The more forward-looking the emperor is, the greater the \(?\). As a larger \(?\) leads to a larger \(K_1^{\ast}\), we conclude that a farsighted emperor seems to be more aggressive. If we believe a great emperor is one with vision, this can be an accurate statement of the emperors in the Warring States. From Wei Wen Hou to First Emperor of Qin, great emperors initiated far more wars than ordinary ones.

As the wars escalated, it became more likely that one country could eliminate all of the others. The model confirms such intuition. If there are two countries with the same population, we can indeed show that as \(?\) increases, the expected time that one country conquers the other (so that one country has all the land and people) decreases in the model. This helps to explain why there could be several ups and downs of a country in the Spring and Autumn period, during which the productivity was still low. In the Warring States period, however, once a country was seriously defeated, it became very hard to recover. This facilitated Qin’s unification of China.

\textit{II. Coalition formations}
Wars in the later period of the Warring States were often huge in scale and involved almost all of the countries. Countries formed coalitions to fight together. There were, however, very few wars involving coalitions fighting against coalitions. The two sides of the war were very unequal in power. In most cases, three, five, or six countries formed large coalitions to fight against one single country. Even if the single country were usually a superpower, its force could hardly match the sum of the rest, because there were always at least two superpowers in the seven countries until the defeat of Zhao in 259 B.C.

The model could be used to explain why giant coalitions form. In particular, we could show that if two countries are at war, the third country can in general gain more from joining the stronger country in the war. The stronger country also benefits from the third country’s alliance. The same logic implies that once a small but strong enough coalition is formed to fight against one particular country, the rest of the countries prefer to join the coalition. Since the members in the coalition benefit from the outside allies, they also welcome the outside allies to join the coalition. As a result, the size of the coalition may keep growing. In fact, if two countries are at war with each other, the only Nash Equilibrium for the rest of the countries is that they coordinate their effort to help one country.

To show that the third country will in general help the stronger country, we need one more assumption to make the formal analysis possible:
**Assumption 6:** Suppose country 1 and 2 forms a coalition. If country 1 sends $K_1$ people and country 2 sends $K_2$ people in the joint force, then country 1 receives (suffers)

$$\frac{K_1}{K_1 \div K_2}$$ from gain (loss).

This assumption states that the countries divide the profit or loss from the wars according to the percentage of people they have in the joint force.

Now suppose country 1 and 2 are at war. If country 1 has $K_1$ people and country 2 has $K_2$ many, such that $K_1 > K_2$. Then we claim that country three gain more from joining country 1 unless $K_3 > K_1 + K_2$.

In fact, if country 3 joins country 1, its emperor will choose to send an optimal $K_3$ number of soldiers to the war such that $K_3$ maximizes its total payoff.

If we follow the Cobb-Douglas production function in the model, the emperor’s payoff in the first period is $L_3(N_3 \div K_3)^{1/2}$. The discounted payoff of the second period equals

$$d^{1/2} \frac{K_1}{K_1 \div K_3} \left[ \frac{K_1 \div K_3}{K_1 \div K_2 \div K_3} \frac{K_1 \div K_2 \div K_3}{2} \frac{K_2}{K_2 \div K_3 \div K_1} \frac{K_2 \div K_3 \div K_1}{2} \right].$$

The sum of the two terms after some simplification is equal to

$$L_3(N_3 \div K_3)^{1/2} + \alpha d^{1/2} \frac{K_3}{K_1 \div K_3} \frac{K_1 \div K_3}{2}.$$ (1)

Similarly, if the emperor of country 3 joins country 2, his payoff will be

$$L_3(N_3 \div K_3)^{1/2} + \alpha d^{1/2} \frac{K_3}{K_2 \div K_3} \frac{K_1 \div K_3}{2}. \quad (2)$$
If \((K_1^*, K_2^*, K_3^*)\) forms a Nash Equilibrium at which country 3 joins country 1, we need to have \((1) \geq (2)\) evaluated at \((K_1^*, K_2^*, K_3^*)\). For this to hold, it is equivalent to have
\[
(K_1^+ ? K_2^+)(K_3^+ ? K_1^+ K_2^+) \leq 0.
\]
This inequality says that unless country 3 sends a troop with a size greater than the sum of the other two countries’, it is to his advantage to join a country that has a larger army.

The analysis above establishes formally why a third country will in general join a stronger country at war, and therefore why in a war we often see a huge coalition forming to fight one single country. The analysis, however, is subject to at least two important critiques. First, the analysis simplistically assumes that the third country makes a decision based only upon the possible gain from the war, and that countries at war passively accept country three’s decision. It can be argued that a better way to model the war should assume that country one and two compete for country three’s support. For example, they could both promise lands to country three in exchange for its help. And the promises can be kept by writing a contract with country three.

It indeed happened especially in the earlier centuries of the Warring States that one country promised to give lands to another country in exchange for either its military support or its eschewal from the war. In fact, one of the most famous stories in Warring States told that Qin promised Chu six hundred Square Li of land, as long as Chu agreed to stay away from Qin’s potential war with Qi. Chu accepted the offer. Once Chu broke its tie with Qi, however, Qin said that it only promised six Square Li of land, instead of six hundred. Chu never received the land.

The story suggests that agreements were very informal during the Warring States. No written contract, but an oral agreement instead, was used in an issue of great
importance. Furthermore, even if there were records showing that countries sometimes wrote contracts to restrain their decisions, the contracts were not well enforced. For example, in 288 B.C. the contract that six countries would send force on a specified date to fight Zhao was broken without consequence. Instead, five countries joined Zhao to fight Qin. Therefore, contracts were very difficult to enforce during the Warring States period. Furthermore, these contracts are subject to severe moral hazard problems, especially if the contract asks the countries to join force to fight a war. Therefore, the competition-contract approach is not feasible during the Warring States period, and we choose to model the wars in a more simplistic way.

The second critique is more serious and focuses on the implementation of the giant coalitions. There are at least two problems with the formation of an effective large coalition. First, if two countries are at war and the rest of the countries need to decide simultaneously which country to help, it is difficult to coordinate their effort to help one single country due to information problems. Potentially, many different coalitions are possible. Second, once a coalition is formed, there exist huge moral hazard problems in the joint forces. In particular, probably none of the countries wants to be the first one to fight the enemy.

The countries in the Warring States, however, managed to partially solve the above problems through the use of the Persuaders, who were at the heart of alliances. Persuaders were people who travel around to convince the emperors to form certain coalitions. Once an alliance was formed, the persuader who joined coalition often became the Prime Minister of all the participating countries. The persuader’s ability crucially determined the success of the coalition.
A successful persuader possesses at least three important characteristics. First, a persuader needs to be an outstanding debater. There exist many detailed records showing how the Persuaders debate with and convinced the emperors and their ministers on the court. Many of their arguments are even in the textbooks for Chinese students today. Indeed, as the emperors were surrounded by ministers of varying interests, it is an extremely arduous task for the Persuader to convince the emperors to join a coalition.

Second, the Persuaders are required to have little loyalty to any particular country. Otherwise, the emperors would probably suspect that the coalition was only for the benefit of the country the persuader is loyal to. Therefore, the persuader’s credential will be severely damaged and his argument weakened. Even if the persuader managed to form a coalition, the alliance was often weak and teemed with suspects.

Third, the persuaders need to have military talents. After the formation of a coalition, the persuader is often the best candidate for the position of commander-in-chief. For a joint force, the role of commander-in-chief is crucial in forming a complete military plan and in coordinating the decisions of different armies. Without a commander-in-chief, the joint military action can be disastrous. If the role is taken by a general of a particular country, however, other countries might suspect that the military decisions are favoring that particular country. Those countries are then less willing to contribute. Therefore, unless the persuader takes the role of commander-in-chief, it is hard for the coalitions to be successful. The military talent of the persuader then affects the emperors’ estimate of the prospect of the coalition. A militarily-talented persuader certainly facilitates the formation of a coalition.
Although it is very difficult to find people with all of the three characteristics, there emerged many successful persuaders in the Warring States. Su Qin, Zhang Yi, and GongSun Yan are still familiar names to the Chinese today. Many of the persuaders emerged from humble families, so their successes marked them the role models of people of the Warring States. In fact, ZhanGuoCe is a book that documented the arguments of successful persuaders for persuaders-to-be of those days to emulate. This book has also become a classic in Chinese literature.

To formalize the effects of the Persuaders, we provide one example to illustrate how the Persuaders might help the emperors solve the coordination problems. Similar examples can be constructed to account for how the Persuaders solve the moral hazard problems in a giant coalition. Those examples would be in the spirit of Holmstrom’s moral hazard in team problem, but there might be a sequential structure to them. Since the example is of pure theoretic interests, it is included in the Appendix.

IV. Conclusion and Discussion

In this paper, we use a simple model to analyze the wars during the Warring States period. We showed first that the technological progress led to increases in productivity. The productivity increase created an excess supply of labor and made the land more valuable. As a result, the scale of the war escalated. Second, we showed that when two countries were at war, the third country in general would like to join the stronger country. Therefore, large coalitions are more likely to emerge. Third, Persuaders were needed to coordinate the decisions and actions of the countries to make the large coalitions possible and successful.
Since the model was very simple, it could be enriched to incorporate more reality and to further explain some interesting phenomena. First, the model neglects the geographic location of the countries. In reality, Qin located in west and Chu in south; Qi was in the east and Yan in the northeast; Zhao was in the north, and Wei and Hann are in the center. The fact that Yan was free from most of the large wars except for those to Qi and Zhao, which are its neighbors, is probably due to its remoteness in location. By incorporating geographic components into the model can certainly help us understand the formation of coalitions better.

Second, other than the seven major countries, there are still a few small countries during the Warring States period. We neglected them for simplicity of the model. In reality, however, they could play vital roles in the formation of coalition. Major countries fought with each other to win the submission of these smaller countries. In fact, Qi’s debacle in 286 B.C. to the six country coalition was caused by Qi’s conquer of Song, a small sized country. By adding these small countries into the picture could further our understanding of the formation of coalitions as well.

Bibliography:


Appendix: The Coordination Problem

Consider the following coordination game: There is a set $I=\{1, 2, \ldots, I\}$ of players. For each player $i \in I$, she chooses $s(i) \in \{0, 1\}$. The players choose their action simultaneously. The payoff function of the players is as follows: If all of the players choose the same action, i.e. $s(i)=s(j)$ for some $i, j \in I$, then $u(i)=1$ for all $i$. Otherwise, $u(i)=0$ for all $i$.

In this game, if all of the players coordinate on the same action, they receive a positive payoff of 1 each. Otherwise, no one receives anything. Clearly, there are two pure strategy Nash Equilibria in this game: all the players choose 0 and all the player choose 1. There is also a unique mixed strategy equilibrium: each player chooses 0 with probability $\frac{1}{2}$, and 1 with probability $\frac{1}{2}$.

Although there has been no consensus in the selection of multiple equilibria, it is reasonable to think that everyone chooses 0 and everyone chooses 1 are identical equilibria in this game. If a large number of players have to choose their actions simultaneously without previous communication, the most likely outcome is probably that about half of the players choose 0 and the other halves choose 1. In other words, that
everyone plays the mixed equilibrium is probably the best description of the outcome of the game.\footnote{Of course, this can also be understood as a B.N.E. as in Harsanyi (1967).}

If one believes the previous paragraph, then it is reasonable to define a value for this game. The value of the game for a particular player is her expected payoff from the game. In this game, if the player expects everyone to play the mixed strategy, then her expected payoff is $2^{1/2}$.

Now suppose we introduce another strategy into the game. Instead of choosing 0 or 1, the players can choose to give their rights of choice to an outside player I+1 for a price of p. The outside player will guarantee that he will choose the same action for all of the players who give the rights to him. As long as p<=1/2, there is a unique equilibrium in which all of the players give their rights to the outside player. Furthermore, this is a focal equilibrium that Pareto dominates the mixed equilibrium above. The player’s payoff in this equilibrium is 1-p, which is a higher than $2^{1/2}$, for I>1.

Although this is an abstract game, it captures the situation in the Warring States. If two countries with similar strength are at war, the rest of the countries need to decide which country to help. If there are no persuaders, the likely outcome is that countries will randomly choose a country to help with. This, however, is a bad outcome for the countries which joined a weaker coalition, and every country has ex ante a probability of $1/2$ to join a weaker coalition.

No countries, however, need to worry about joining the weaker coalition if there are persuaders. The persuaders will coordinate the decisions of the countries once they acquired the trust to the emperors. The emperors could be assured to join a stronger
coalition so that their interests are satisfied. The preferences of the persuaders, however, determined which coalition will be chosen.