

Lecture notes 2: Physical Capital, Development and Growth

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Physical capital: machines, buildings, roads, vehicles, computers .

Diff in quantity of capital may explain diff in income among countries.

PC capital in 1990: U.S. = \$34,705, Mexico = \$12,900, India = 1,946.

Figure 1: Relationship striking, explaining diff in Y between countries .

- The Nature of Capital, five key characteristics:

1. Capital is productive, raises output.

2. Capital producible, investment, sacrifice of consumption.

In 1996 US Inv=\$1,091 b, %14 of GDP, Inv corresponds Savings.

3. Capital is “rival” in use: limited users at one time (ideas and knowledge are non-rival).

4. Capital earn a return, incentive for its creation.

5. Capital wears out: depreciation.

2. Capital's Role in Production

Production function: relationship between inputs.

Two inputs, capital, K , labor, L . output, Y :

$$(1) \quad Y = F(K, L)$$

Assumptions: constant returns to scale (CRS)

$$(2) \quad F(zK, zL) = z F(K, L),$$

Diminishing marginal product, MPK , MPL , Figure 2

$$(3) \quad MPK = F(K+1, L) - F(K, L) .$$

CRS implies that the quantity of output per worker depends only on the quantity of capital per worker.

k - capital per worker, y - output per worker,

$$(4) \quad \left(\frac{1}{L}\right)Y = F\left(\frac{K}{L}, \frac{L}{L}\right)$$

$$(5) \quad y = F(k, 1)$$

Using a Cobb-Douglas production function:

$$(6). \quad y = f(k)$$

A Cobb-Douglas production function, A – a measure productivity

$$(7) \quad F(K, L) = AK^\alpha L^{1-\alpha}$$

$$(8) \quad F(zK, zL) = A(zK)^\alpha (zL)^{1-\alpha} = z^{\alpha+1-\alpha} AK^\alpha L^{1-\alpha} = zF(K, L)$$

$$(9) \quad y = \frac{Y}{L} = \frac{F(K, L)}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = A\left(\frac{K}{L}\right)^\alpha \left(\frac{L}{L}\right)^{1-\alpha} = Ak^\alpha$$

$$(10) \quad y = Ak^\alpha$$

$$(11) \quad MPK = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1} L^{1-\alpha}$$

$$(12) \quad \left(\frac{\partial MPK}{\partial L}\right) = (\alpha - 1)\alpha A^{\alpha-2} L^{1-\alpha} < 0$$

Factor Payments and Factor Shares

- Assume factors of production paid their marginal products.
- In a competitive economy, MPK =rental rate per unit of capital,
- $MPK \times K$ = rental rate per unit of capital multiplied by the total quantity of capital, that is, $MPK \times K$.
- **Capital's share of income** is the fraction of national income (Y) that is paid out as rent on capital.

$$(13) \quad \frac{MPK \times K}{Y} = \frac{\alpha AK^\alpha L^{1-\alpha}}{AK^\alpha L^{1-\alpha}} = \alpha$$

- Even if the quantities of capital and labor in the economy may vary, changes in the rental rate of K and wage rate will be such that the shares of national income paid out to each factor of production will be constant.
- The importance of this result: we can estimate the value of α just by looking at capital's share of national income. This number is generally estimated as being close to 1/3.

- **Figure 3**

The Rise and Fall of Capital:

Before the 19th Century, labor and land were most important factors.

Agricultural Land as a Fraction of Total Wealth in the UK:

1688	64%
1798	55%
1885	18%
1927	4%
1958	3%

- Capital replaced land as a key input: changes in technology.
- The Industrial Revolution (beginning in roughly 1760):
- Invention of new technologies, made capital more productive.

Similarly, advances in agricultural technology allowed other inputs (chemical fertilizers) to substitute for land.

- Today, knowledge and skills are taking the place of physical capital as the key.

3. The Solow Growth Model

- Focuses on a single dimension along which countries may differ or along which a single country may change over time: capital per worker.
- L and PF constant over time: no improvement in productivity,
- All action from accumulation of capital,
- Governed by two forces: investment (I) and depreciation (D):

$$(1) \quad \Delta K = I - D$$

In per capita terms:

$$(2) \quad \Delta k = i - d$$

$$(3) \quad i = \gamma y$$

$$(4) \quad d = \delta k$$

.Combining 1-4:

$$(5) \quad \Delta k = \gamma y - \delta k$$

$$(6) \quad \Delta k = \gamma f(k) - \delta k$$

Steady States

- Equation (6) describes how capital evolves over time.
- According to the equation, if investment, $\gamma f(k)$, $>$ depreciation, δk , then Δk , will be positive – capital stock will be growing.
- On the other hand, if $\gamma f(k) < \delta k$, then k will be shrinking.
- If $\gamma f(k)$ is equal to δk – in other words, if investment is equal to depreciation – then the capital stock will not change at all.

Figure 4: analysis of steady state:

y_{ss} , k_{ss} . **Figure 6**

$$(7) \quad \Delta k = \gamma A k^\alpha - \delta k$$

$$(8) \quad \gamma A k^{ss\alpha} = \delta k^{ss}$$

$$(9) \quad k^{ss} = \left(\frac{\gamma A}{\delta} \right)^{\frac{1}{1-\alpha}}$$

$$(10) \quad y^{ss} = A k^{ss} = A^{\frac{1}{1-\alpha}} \left(\frac{\gamma}{\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

The SOLOW model with a Cobb-Douglas production function

The Solow Model as a Theory of Income Differences

- - A higher rate of investment leads to a higher SS level of output.
- - Solow model as a *theory of income differences*.
- - How well this theory fits the data?
- - How actual differences in income compare to differences predicted by Solow model?
- - Consider case where *only* difference between countries is in investment:

$$(1) y_i^{ss} = A^{\frac{1}{1-\alpha}} \left(\frac{\gamma_i}{\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

$$(2) y_j^{ss} = A^{\frac{1}{1-\alpha}} \left(\frac{\gamma_j}{\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

$$(3) \left(\frac{y_i^{ss}}{y_j^{ss}} \right) = \left(\frac{\gamma_i}{\gamma_j} \right)^{\frac{\alpha}{1-\alpha}}$$

if $\alpha=1/3$, so that $\alpha/(1-\alpha) = 1/2$:

$$(4) \left(\frac{y_i^{ss}}{y_j^{ss}} \right) = \left(\frac{0.20}{0.05} \right)^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2$$

Figure 7: a significant relationship between predicted income and actual income:

- All countries predicted poor are indeed poor.
- Most countries predicted rich are indeed rich.
- But not perfect predictions, several predicted rich are poor and vice versa.
- Why not perfect fit? Other factors left out, countries not in SS

The Solow Model as a Theory of Relative Growth Rates:

- In SS, no longer any growth.
- The Solow model will fail to explain growth over long period during which countries should have reached their SS.
- What about relative growth rates – why some countries grow faster than others?

Key point:

Growth while away from SS, so all growth is *transitional*.

Three interesting predictions:

- *If two countries have the same rate of investment, but different levels of income, then the country with lower income will grow faster.*

Since i equal, y in SS equal

- *If two countries have the same level of income, but different rates of investment, then the country with a higher rate of investment will grow faster.*

A higher Investment rate, a higher income per capita in SS

- *A country which raises its level of investment will experience an increase in its growth rate of income.*

4. The Relationship Between Investment and Saving

- Solow model predicts diff in investment rates lead to different SS.
- But why investment rates differ?
- Perhaps investment rates differ between countries because saving rates differ between countries.

Figure 8: strong relationship between saving rates and IPC.

- But be cautious: investment can cross national boundaries, international capital flows.

What determines saving rates?

- **Exogenous factors:**

Government policy, culture, and geography , (income inequality).

Example - government policy: deficit (spending not covered by taxes) is negative S.

Governments can influence the private saving rates by a number of means:

Programs in which individuals fund their own retirements by saving during their working years generate a large quantity of capital. But Pension plans (e.g., Social Security) that are funded by taxes on workers do not generate savings.

Chile:

- private pension programs in the early 1980s in Chile, the private savings rate up from 0 (early 1980s) to 17% by 1991.
- The success of the Chilean program led Argentina, Bolivia, Colombia, Mexico, Peru and Uruguay to adopt similar plans in the 1990s.

Singapore:

- 1950s, workers required to contribute part of their wages to a “central provident fund,” used to finance not only retirement but also medical expenditures and the purchase of housing.
- The government determined the required contribution rate, up to 40% of a worker’s salary in the early 1980s.
- This forced saving policy was an important determinant of Singapore’s phenomenally high saving rate.

Endogenous factors affecting saving

Income level, **Fig 8:**

- Rich countries save more, but saving more make a country rich.
- Constraints argument: at subsistence income people cannot afford to reduce their present consumption in order to save for the future.
- Make sense for Bangladesh (income per capita=\$1,050) but what about Zimbabwe (\$2,280)?
- Voluntary choice argument: Consumption versus Saving is a choice between current and future satisfaction. So a person who does not care much about the future will not save.
- It follows from this theory, that a poor person cares less about the future.

Extension of the Solow model to allow Saving to vary with Income:

What are the implications of this effect for the Solow model?

$$(1) \gamma = s_1 \text{ if } y < y^*$$

$$(2) \gamma = s_2 \text{ if } y \geq y^*$$

Figure 9:

- Two countries could be completely identical in terms of the underlying determinants of their incomes but still end up with different levels of income per capita in the SS. A saving trap with multiple SS:
- Is it an important explanation for differences in income among countries?
- If multiple SS are important, then differences in income per capita between countries do not necessarily arise because of “fundamental” differences among countries, but rather because of self-reinforcing economic dynamics.

A alternative saving-income model:

- Saving rate rises *gradually* as income rises, rather than jumping up suddenly at a particular level of income, as it does in Figure 9.
- One or multiple SS are possible in this model.
- If there is only one steady state, the fact that saving rises with the level of income means that the process of convergence to SS will be slower because as the economy approach y of SS, saving rises which will increase growth and will postpone convergence to SS.