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Problem Set 5 Solutions

1. Chapter 6, #2

(a1) In any one year, half the people have 100, and the other half have 200. The formula for the Gini coefficient is

$$G = \frac{1}{2n^2 \mu} \sum_{j=1}^{m} \sum_{k=1}^{m} n_j n_k |y_j - y_k|,$$

where n is the total number of people, n_1 the number of people in income class i, y_i is the income of class i, and μ is the mean income.

In our case, we can take $n = 1$, and the two groups 1 and 2, with $(y_1, y_2; n_1, n_2)$ equal to (100, 200; 1/2, 1/2). Mean income μ is therefore given by 150. Consequently, the Gini in this case is

$$G = \frac{1}{300}\{\frac{1}{4}(200-100) + \frac{1}{4}(200-100)\}$$
$$= 1/6.$$

(a2) Now calculate lifetime income. The expected income in the second period is 150 for everybody (this is because there is a probability of 1/2 of getting a high job and a probability 1/2 of getting the low job in period 2. Thus average income for someone currently holding a low job is $\frac{100+150}{2} = 125$, which for someone holding a high job it is $\frac{200+150}{2} = 175$. Note that there is a narrowing of average incomes relative to part (a), because of the mobility in the economy. You can calculate the Gini just as we did in part (a), and you will see that it is lower.

(b) and (c) I will go ahead and do part (c) because it is a generalization of part (b) (but you should try part (b) separately). If you hold your current job with probability p, then for a low income person (today), the expected income tomorrow is $100p + 200\ (1-p)$. For a high income person (today) it is $100(1-p) + 200p$. Thus expected *average* incomes for the low income person and high income person are $50 + 50p + 100(1-p)$ and $100 + 100p + 50(1-p)$ respectively. The mean income in this society is still 150, as you can easily see by taking the 50-50 average of these two incomes. So the Gini, calculated just as in part (a), is

$$G = \frac{1}{300}\{\frac{1}{4}[50 + 50p - 50(1-p)] + \frac{1}{4}[50 + 50p - 50(1-p)]\}$$

$$= \frac{p}{6}$$

Note that if $p = 1$, this gives you exactly the same answer as in part (a1), while if $p = 1/2$, we get exactly the same answer as in part (a2). This is as it should be. If $p = 1$, there is no mobility at all (why?), so that the answer to overall inequality is the same as the answer to inequality *within* a single time period. In contrast, if $p = 1/2$, there is perfect mobility, which is the case studied in part (a2). As p goes up from 1/2 to 1, mobility becomes progressively lower and lower, and in response the Gini goes up, signaling greater inequality in the presence of lower mobility.

2. Chapter 6, #6

(6) (a) Take the first income distribution, x and let total income (which is just the sum over all the x_k's) be denoted by X. The poorest m people in the population earn $\sum_{k=1}^{m} x_k$, so their *share* of total income is $\sum_{k=1}^{m} x_k / X$. Likewise, the share of the poorest m people in the y distribution is $\sum_{k=1}^{m} y_k / Y$ (again, Y is total income).If the Lorenz curve for x lies inside the one for y, it must be the case that the former share is at least as large as the latter share for *all* m. This means that

$$\sum_{k=1}^{m} x_k / X \geq \sum_{k=1}^{m} y_k / Y$$

for all k, but since $X = Y$ (both distributions have the total income by assumption), it must be that

$$\sum_{k=1}^{m} x_k \geq \sum_{k=1}^{m} y_k$$

(with strict inequality for at least one index k).This is what we needed to prove.

3. Chapter 7, #6

(a) If a person has wealth A, earns w_A and saves a fraction of s_A of income, then total savings is $s_A w_A$. If this (plus existing wealth) earns interest at the rate of r, then next year's wealth must be given by the quantity $[A + s_A w_A](1 + r)$.

(b) Consider the first two groups to start with. If $s_A = s_B = s$ (say), then the ratio of today's weath is A/B (obviously) and the ratio of tomorrow's wealth (using part (a)) is

$$\frac{[A+sw_A](1+r)}{[B+sw_B](1+r)}=\frac{A+sw_A}{B+sw_{B'}}=\frac{A}{B}$$

using the presumption that $w_A / w_B = A/B.$ The same argument also applies to the A and C wealth levels (and to the B and C wealth levels). Therefore by the relative inequality principle, because all ratios of wealth are unchanged, wealth inequality next year must be the same as wealth inequality this year.

(c) Again start with the wealth levels A and B. Let's denote next year's wealth levels by A' and B'. Then by part (a), we know that $A'=[A+s_A w_A](1+r)$ while $B'=[B+s_B w_B](1+r)$. If $s_A < s_{B,}$ then it must be the case that

$$\frac{A'}{B'}=\frac{[A+s_A w_A](1+r)}{[B+s_A w_B](1+r)}$$

$$=\frac{A+s_A w_A}{B+s_B w_B}$$

$$<\frac{A}{B}$$

where we continue to use the assumption that $w_A / w_B = A/B.$ Likewise, it must be the case that $B'/C'<B/C.$ Now it should be easy to check that the Lorenz curves for wealth distribution must worsen next year (do this in detail).

(d) If all wages and savings rates are equal (say to w and to s respectively) then

$$\frac{A'}{B'}=\frac{[A+sw](1+r)}{[B+sw](1+r)}$$

$$=\frac{A+sw}{B+sw}$$

$$>\frac{A}{B},$$

because adding a common constant (sw) to the numerator and denominator of a fraction that's less than one to start with must bring up the value of that fraction. The same argument applies to wealth levels B and C, so now we read the opposite conclusion from that in part (c); wealth inequality as captured by the Lorenz curve must decline.

4. Chapter 8, #5
Here is an example that illustrates the point of the question. Suppose that a moneylender is advancing a loan today of size L. The loan can be used by the

borrower in different projects. Some of them may simply involve present consumption, and some of them might involve actual production activities that bring the borrower greater and greater insulation against a *future* demand for credit. Let us arrange these various projects-say projects 1, 2,…, n — in decreasing order of what they imply for the borrower's future credit demands: call these $D_1 D_2, ... D_n$. [So, for instance, project 1 may involve pure consumption credit so that the borrower's future needs continue to be high. And project n may involve setting up his own self-sufficient business so that the future credit needs shrink to a small number.]

Now suppose that the borrower puts up collateral C and gets a loan for project i. When it comes time to repay the loan, the borrower's cost of repaying is $L(1+r)$ (where r is the rate of interest). If the borrower defaults he loses his collateral and the ability to meet his future demand for credit $D_{i,}$ so his loss is $C + D_{i.}$ He will repay if

$$C + D_i \geq L(1+r)$$

Knowing this, it is clear that the lender will not advance loans for projects such that the above inequality fails. Note that it is more likely to fail for projects with a higher index (and consequently a smaller future demand for loans). Also note that the higher the collateral put up by the borrower, the less the likelihood of default and the better the projects that he can get loans for (in terms of reducing future dependence). It follows that poor borrowers who lack collateral are more likely to receive loans that are for current consumption or for working capital — for projects that do *not* reduce the borrower's future dependence on credit.