1 The Family

- A family consists two people F and M with utility functions $U_F(\mathbf{x}, \mathbf{a}), U_M(\mathbf{x}, \mathbf{a})$, where $\mathbf{x} = (\mathbf{x}_F, \mathbf{x}_M)$ is a vector of amounts of consumption goods for the two people and $\mathbf{a} = (\mathbf{a}_F, \mathbf{a}_M)$ is a vector of actions that they each can take.

- Let $\widetilde{\mathbf{p}} = (\mathbf{p}, \mathbf{p})$ be the vector of all the commodity prices. Then the budget constraint for the family:

$$\widetilde{\mathbf{p}}\mathbf{x} = y = y_F(\mathbf{a}) + y_M(\mathbf{a}) + \phi$$

  where ϕ is a shock., $y_F(\mathbf{a})$ and $y_M(\mathbf{a})$ are the incomes assigned by the existing system of property rights to F and M. Does not mean M earns the income that is assigned to him.

- Special case: $\mathbf{a}$ is **pure investment**. In this case $\partial U_F(\mathbf{x}, \mathbf{a})/\partial \mathbf{a}$ and $\partial U_M(\mathbf{x}, \mathbf{a})/\partial \mathbf{a}$ are both zero. $y(\mathbf{a}) = F(\mathbf{a}) - r(\mathbf{a})$, where $F(\mathbf{a})$ is a production function and $r(\mathbf{a})$ is interest cost of investing an amount a.Note that we do not assume perfect capital markets.

1.1 The Unitary Model (Becker)

- $U_F(\mathbf{x}, \mathbf{a}) \equiv U_M(\mathbf{x}, \mathbf{a}) = \mathbf{U}(\mathbf{x}, \mathbf{a})$, i.e identical preferences.

- The consumption decision: Maximize $\mathbf{U}(\mathbf{x}, \mathbf{a})$ over $\mathbf{x}$ subject to $\widetilde{\mathbf{p}}\mathbf{x} = y_F(\mathbf{a}) + y_M(\mathbf{a}) + \phi$. Pareto Optimality by construction.

- FOC

$$\partial U(\mathbf{x}, \mathbf{a})/\partial \mathbf{x} = \boldsymbol{\lambda} \ \widetilde{\mathbf{p}} \qquad (1)$$
$$y_F(\mathbf{a}) + y_M(\mathbf{a}) + \phi = \widetilde{\mathbf{p}}\mathbf{x} \qquad (2)$$

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$$\frac{\partial U(\mathbf{x}, \mathbf{a})/\partial x_i}{\partial U(\mathbf{x}, \mathbf{a})/\partial x_j} = \frac{p_i}{p_j}$$

1.1.1 The unitary model continued

- We can solve these equations to get $\mathbf{x}$ but $\mathbf{x}$ *will depends on* $\mathbf{a}$ as well as y. Unfortunately $\mathbf{a}$ is not typically easily measured.

- A more convenient characterization comes from assuming $\partial U(\mathbf{x}, \mathbf{a})/\partial x_i = u_i(\mathbf{x})g(\mathbf{a})$, i.e. separability. In this case $\mathbf{x}$ depends only on total family income, $y_F(\mathbf{a}) + y_M(\mathbf{a}) + \phi$. *The intra-family distribution of income does not matter.* Distributin neutrality

1.2 The bargaining model (Chiappori)

- $U_F(\mathbf{x}, \mathbf{a}) \neq U_M(\mathbf{x}, \mathbf{a})$, i.e non-identical preferences.

- The family maximizes $U_F(\mathbf{x}, \mathbf{a}) + \mu U_M(\mathbf{x}, \mathbf{a})$, where μ is bargaining weight.

- The key assumption is that μ is independent of $\mathbf{x}, \mathbf{a}$. One possible scenario is that μ is chosen first, then all the other decisions are taken.

1.2.1 The bargaining model (contd)

- Given that μ is a constant, the decision taken by the family is obviously Pareto efficient.

- The family's decision: Maximize $U_F(\mathbf{x}, \mathbf{a}) + \mu U_M(\mathbf{x}, \mathbf{a})$ over $\mathbf{x}, \mathbf{a}$ subject to $\tilde{\mathbf{p}}\mathbf{x} = y_F(\mathbf{a}) + y_M(\mathbf{a})$. FOC

$$\frac{\partial U_F(\mathbf{x}, \mathbf{a})}{\partial \mathbf{x}} + \mu \frac{\partial U_M(\mathbf{x}, \mathbf{a})}{\partial \mathbf{x}} = \lambda \tilde{\mathbf{p}} \qquad (3)$$

$$\frac{\partial U_F(\mathbf{x}, \mathbf{a})}{\partial \mathbf{a}} + \mu \frac{\partial U_M(\mathbf{x}, \mathbf{a})}{\partial \mathbf{a}} = \lambda y'(\mathbf{a}) \quad (4)$$

$$y_F(\mathbf{a}) + y_M(\mathbf{a}) + \phi = \tilde{\mathbf{p}}\mathbf{x} \qquad (5)$$

- In general both the choice of $\mathbf{x}$ and $\mathbf{a}$ depends on $\mu, y'(\mathbf{a})$.

1.2.2 A special case

- No family public goods and separability $U_F(\mathbf{x}, \mathbf{a}) = U_F(\mathbf{x}_F, \mathbf{a}_F), U_M(\mathbf{x}, \mathbf{a}) = U_M(\mathbf{x}_M, \mathbf{a}_M)$

- In this case by separability the FOC reduces to

$$g_F(\mathbf{a})\partial U_F(\mathbf{x}_F)/\partial \mathbf{x}_F = \boldsymbol{\lambda}\mathbf{p} \qquad (6)$$
$$g_M(\mathbf{a})\boldsymbol{\mu}\partial U_M(\mathbf{x}, \mathbf{a})/\partial \mathbf{x}_M = \boldsymbol{\lambda}\mathbf{p} \qquad (7)$$
$$y_F(\mathbf{a}) + y_M(\mathbf{a}) + \phi = \tilde{\mathbf{p}}\mathbf{x} \qquad (8)$$

- Therefore $\frac{\partial U_F(\mathbf{x}_F)/\partial x_{iF}}{\partial U_F(\mathbf{x}_F)/\partial x_{jF}} = \frac{p_i}{p_j} = \frac{\partial U_M(\mathbf{x}_M)/\partial x_{iM}}{\partial U_M(\mathbf{x}_M)/\partial x_{jM}}$,which implies that the marginal rates of substitution between any two goods is independent of who has bargaining power, as long as there is efficient bargaining. Used for tests of efficiency.

- Only y matters for the level of $\mathbf{x}_F$ and $\mathbf{x}_M$ for fixed μ : $Conditional$ distribution neutrality.However the level of $\mathbf{x}_F$ and $\mathbf{x}_M$ still depend on μ, for any given y. Shifts in bargaining power, keeping income fixed, do affect the pattern of consumption.

1.2.3 Another special case

- The investment model. Satisfies separability but much more special.

- At the optimum it must be that $y' = 0$.

- The level of investment is independent of bargaining power.

1.3 The incomplete contract approach (Maher-Wells)

- One good investment model: $U_F = u(x_F), U_M = u(x_M)$.

- The budget constraint is that

$$x_F + x_M = y_F(a_F) + y_M(a_M).$$

- If μ is, as before, fixed, then the family will set $y_F'(a_F) = y_M'(a_M) = 0$ and then distribute consumption.

- Now let μ be dtermined after the investment is made but before consumption is chosen. Let $\mu(\frac{a_F}{a_M})$ and $\mu' < 0$. If the woman invests then she become more powerful. One reason may be that she can just walk off with her $y(a_F)$. i.e her outside option is $u(y(a_F))$ and the bargaining has to give her at least this.

1.3.1 The incomplete contract approach continued

- Maximizing $u(x_F) + \mu(a_F/a_M)u(x_M)$ subject to the budget constraint yields:

$$u_F^*(a_F/a_M, y_F(a_F) + y_M(a_M)) \qquad (9)$$
$$u_M^*(a_F/a_M, y_F(a_F) + y_M(a_M) \qquad (10)$$

- F chooses a_F to maximize u_F^* and likewise for M. We assume non-cooperative behavior.

- Since u_F^* is increasing in a_F and u_M^* is increasing in a_M, both F and M will over-invest, i.e. $y_F' < 0$ and $y_M' < 0$.