

**Political Origins of Dictatorship and
Democracy.
Chapter 4:
An Introduction to Models of Democratic
Politics**

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Abstract

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1 Introduction

In this chapter we begin our analysis of the factors that lead to the creation of democracy. As discussed earlier, our approach is based on conflict over political institutions, in particular democracy vs. nondemocracy. This conflict results from the different economic consequences that follow from these regimes. In other words, different political institutions lead to different outcomes, creating different economic winners and economic losers. Realizing these consequences, various groups have preferences over these political institutions.

Therefore, a first step towards our analysis of why and when democracy is created is to build a model of collective decision-making in democracy and nondemocracy. The literature on collective decision-making in democracy is vast (with a smaller companion literature on decision-making in nondemocracy). Our purpose here is not to survey this literature, but to bring out the essential points on how individual preferences and various types of distributional conflicts are mapped into economic and social policies. In this chapter, we start with an analysis of collective decision-making in democracies, turning to nondemocratic politics in the next chapter.

To focus the discussion, it is useful to take a simple example. Imagine that political choice is about the level of (redistributive) taxation. The government levies a proportional (linear) income tax, and uses the proceeds to finance a public good that is equally valued by all citizens. Each citizen evaluates different tax policies according to their implications for his income and utility, and votes for (or supports) the policy that he most prefers. Since taxes are proportional to income, they are redistributive: rich people pay more in taxes than poor people. Therefore, the rich prefer low tax rates while the poor favor high taxes. Whose preferences will prevail? The answer is different in democracy from what it is in dictatorship. In the Introduction, we argued that it seems likely that the poor segments of society fare better under democracy than under various types of authoritarian regimes, and Chapter 2 showed that available evidence is indeed consistent with this. Likewise, dictatorships often promote the interests of narrow groups, elites or cliques. But exactly how does democracy promote the interests of the mass of the populace, and how do the interests of an elite map into political outcomes in a dictatorship?

The most basic characteristic of a democracy is that all individuals (above a certain age) can vote and voting determines which policies get adopted. In a direct democracy, the populace would vote directly over the policies. In a representative democracy, the voters choose the government, which then decides what policies to implement. In the most basic model of democracy, political parties, who wish to come to office, offer tax policies, and voters elect political parties, thus indirectly choosing policies. This interaction between voters' preferences and parties' policy platforms determines what the tax rate will be in a democracy. One party wins the election and implements the tax policy that it promised. This approach builds on a body of important research in economics and political science, most notably that due to Hotelling (1929), Black (1948) and Downs (1957).

Undoubtedly, in the real world there are important institutional features of democracies missing from our model and their absence makes our approach only a crude approximation to reality. Parties rarely make a credible commitment to a policy, and run not on a single issue, but on a broad platform. In addition, parties may have partisan (ideological) preferences as well as simply a desire to be in office. Voters might also have preferences over parties' ideologies, as well as their policies. There are various electoral rules, for example proportional representation vs. majoritarian electoral systems, and these determine in different ways how votes translate into seats and therefore governments. Some democracies have presidents, while others are parliamentary. There is often divided government, with policies determined by legislative bargaining between various parties, or by some type of deal between presidents and parliaments, and not by the specific platform offered by any party in an election. Last but not least, interest groups influence policies through non-voting channels, including lobbying and in the extreme, corruption. Many of these features can be added to our models, and these refined models often make different predictions over a range of issues. For example, it appears to be the case that, in line with plausible theoretical expectations, electoral systems with proportional representation lead to greater income redistribution than majoritarian institutions (see Milesi-Feretti, Perotti, and Rostago, 2002, Persson and Tabellini, 2003). Nevertheless, our intention here is not to compare various different types of democracies, but to understand the major differences between democracies and nondemocracies. Our focus

will therefore be on simpler models of collective decision-making in democracies, bringing out the common elements of democracies, not their various institutional differences. For this purpose, we will emphasize that democracies are, relatively speaking, situations of *political equality*. Each citizen has one vote. As a result, in democracy the preferences of all citizens matter in the determination of political outcomes. In nondemocracy, this is not the case since only some subset of people have political rights, and we will argue, this subset will generally be the richer, economically and socially more powerful, segments of society. Therefore, by and large, we will treat nondemocracy as the opposite of democracy: while democracy approximates political equality, nondemocracy is typically a situation of *political inequality*, with more power in the hands of the rich who we shall usually take to be the critical elite and therefore the most privileged group in society.

Bearing this contrast in mind, our treatment in this section will try to bring out some common themes in democratic politics, and emphasize how the preferences of the poorer segments of society influence economic policies.

2 Aggregating Individual Preferences

In this section, we begin with some of the concepts and problems faced by the theory of social choice, which deals with issue of how to aggregate individual preferences into “society’s preferences”. This will be an introduction to some of the issues that arise in democratic politics.

2.1 Arrow’s General Possibility Theorem

To fix ideas it is useful to think of government policy as a proportional (linear) tax rate on incomes, and some way of redistributing the proceeds from taxation. Generally, individuals differ in their tastes and their incomes, and thus will have different preferences over policies, for example, over the level of taxation, redistribution, public good provision, etc. However, even if people are identical in their preferences and incomes this does not mean that there is no conflict over government policy. In a world where individuals want to maximize their income, each person would have a very clear preference: impose a relatively high tax rate on all incomes, and then redistribute all the proceeds to themselves! How

do we then aggregate these very distinct preferences? Do we choose one individual who receives all the revenues? Or will there be no redistribution of this form? Or some other outcome altogether?

This question is indirectly addressed by Arrow's (1951) seminal study of collective decision making. The striking, but upon reflection reasonable, result that Arrow derived is that under very weak assumptions, a democracy will be unable to make coherent decisions in situations like this. In fact Arrow showed that the only situation where it made sense to talk about the preferences of society was one where only one person's preferences counted - a dictatorship. More precisely, Arrow established a *possibility theorem*, showing that it is not possible to aggregate individual preferences to determine what would happen in a democracy; put differently, there does not exist a "social welfare" function summarizing "society's preferences". Arrow's general possibility theorem is a fundamental, and quite deep, result in political science (and economics). It builds on a very important, and much simpler, feature of politics: *conflict of interest*. Different allocations, and therefore different economic policies leading to different allocations, create *winners* and *losers*. The difficulty in forming social preferences is how to aggregate the wishes of different groups, some of whom prefer one policy, while some others prefer another. For example, how do we aggregate the preferences of the rich segments of society who dislike high taxes versus the poor segments who like high taxes that redistribute to themselves? Conflicts of interest between various social groups, in particular between the poor and the rich, will underlie all of the results and discussion in this book. In fact, the contrast we draw between democracy and nondemocracy precisely concerns how they tilt the balance in favor of the poor or the rich.

However, there is much more to Arrow's general possibility theorem than conflict of interest. Although a full statement, and proof, of this theorem would be too much of a diversion for our purposes, it is useful to give the basic idea. Arrow started from well-defined individual preferences, which here we can define as a preference ordering \succeq_j^P over economic allocations, for each individual j in a society. For two allocations, x and y , (these could be levels of income or consumption) the notation $x \succeq_j^P y$ means that individual j prefers x to y . Since different policies lead to different economic allocations, i.e., different

levels of income and utility for the individuals, these preferences are also indirectly over policies. So if a tax rate of τ leads to the allocation x and τ' results in the allocation y , then $x \succeq_j^P y$ implies $\tau \succeq_j^P \tau'$ (with a slight abuse of notation, since the preference ordering \succeq_j^P is first defined over allocations, and it is now being used over policies). The basic assumptions we make on preference orderings in the rational choice approach to social sciences are completeness and transitivity. Completeness means that for any two allocations x and y , we have either $x \succeq_j^P y$ or $y \succeq_j^P x$, or both. Transitivity means that for any three allocations x , y and z , if we have $x \succeq_j^P y$ and $y \succeq_j^P z$ then it must also be true that $x \succeq_j^P z$. Arrow asked the question of whether in a society with at least three individuals and at least three policy choices, and preferences satisfying completeness and transitivity, we can derive a social preference ordering \succeq^S , with the property that if $x \succeq^S y$, we can say that “society prefers x to y ”. In addition, Arrow imposed that this social ordering, \succeq^S , satisfy the following reasonable conditions:

1. Monotonicity: suppose that $y \succeq_j^P x$ and $x \succeq^S y$, then if we keep all other individual orderings the same, but change individual j 's preferences so that $x \succeq_j^P y$, then it must still be the case that $x \succeq^S y$.
2. Unanimity: if $x \succeq_j^P y$ for all j in society, then $x \succeq^S y$.
3. Independence from irrelevant alternatives: let X be the set of available allocations, and suppose that $x \succeq_j^P z$ and $y \succeq_j^P z$. Then $x \succeq_j^P y$ with $\{x, y\} \in X$ if and only if $x \succeq_j^P y$ with $\{x, y, z\} \in X$. In other words, the addition of an irrelevant alternative should not change the ordering between x and y .

Arrow's general possibility theorem is that there exists no social ordering that is non-dictatorial that satisfies these three conditions. A dictatorial ordering is one where \succeq^S is identical to \succeq_j^P for some individual j in this society—in other words, the social ordering simply reflects the preferences of a specific voter, without any reference to the preferences of all other citizens.

This powerful theorem implies that if we want to aggregate individual preferences to arrive to social preferences, then we have to restrict either the way we aggregate

these preferences, i.e., impose specific institutional rules for collective decision-making, or restrict the potential choices or the types of ‘admissible’ preferences, by imposing further conditions than completeness and transitivity.

What about voting? Would the process of voting always choose a policy among the set of available policies? One might conjecture that voting is a specific institution that can aggregate individual preferences into a consistent and transitive social ordering. Unfortunately, Arrow’s theorem implies that this conjecture is false. The best way to see why, and also get a sense of the reasoning underlying Arrow’s theorem, is to turn to an even older example, the so-called the Condorcet Paradox, of the 18th-century revolutionary French philosopher, the Marquis de Condorcet.

2.2 Condorcet’s Paradox

Consider a society consisting of three individuals, A , B and C , choosing between three allocations, x , y , and z . We assume that all three individuals have preferences that are complete and transitive. To illustrate the results, suppose that these individuals’ preferences are

$$\begin{aligned} x &\succeq_A^P y \succeq_A^P z \\ y &\succeq_B^P z \succeq_B^P x \\ z &\succeq_C^P x \succeq_C^P y \end{aligned}$$

In other words, A likes x the best, y next best and z the worst, while B prefers y to z and z to x . Finally, C ’s ordering is z , x and y . For concreteness, you might want to think that these three allocations result from different tax policies. For example, x might correspond to low taxes, y to medium taxes, and z to high taxes. These preferences imply that individual A prefers lower taxes to higher taxes.

Now we imagine that our society of three individuals is a direct democracy where individuals vote over policies— without parties or politicians. It is a majoritarian voting system, so if in a pairwise contest of x and y , x receives two or more votes, it is the winner. But how does society decide which pairwise contest to run? Here, the assumption is that there is *open agenda*, which will be key to Condorcet’s Paradox. An open agenda means

that any of the individuals can propose that the three of them vote between the current (status quo) policy and any other candidate policy. Thus, we can think of the society as first voting on two of the alternatives. After this the winner is voted against another alternative, if such an alternative is proposed, and so on.

Suppose we start with a vote of x against y . Clearly A and C prefer x while the opposite is true for B . Therefore in a pairwise vote A and C vote for x , which becomes the winner. Now imagine that C proposes that x be voted on against z . A prefers x but B and C prefer z , so z wins. Now note however that if y and z are voted upon y will win. Finally, we know that x defeats y under majority voting so we can ‘cycle’ back to where we started. Notice moreover that there is always an individual who has an incentive to propose another alternative, since each person has different favorite policies, thus ensuring that none of the policies is a winner and the society will keep on cycling. For example, if the current policy is x , individual C has an incentive to propose z as an alternative, forcing a pairwise contest between x and z , and inducing his most preferred policy.

This is Condorcet’s Paradox. Even though individuals have complete and transitive preferences, all that we require of rational choice at the individual level, society cannot make “rational choices”. For any proposed alternative which wins a majority vote, there is another which will defeat it in another vote. Voting then just cycles and cycles. At a more technical level, cycling is just an interpretation of what happens when there is non-existence of Nash equilibrium. Recall that a Nash equilibrium is a strategy combination where no individual has an incentive to deviate. Here, such a strategy combination does not exist, since given any choice, one of the individuals has an incentive to propose an alternative, and improve his welfare. One can think of this endless process of changing strategies as ‘cycling’ because one can construct examples where you can change your strategy all the way back to where you started.

There are different reactions to this issue. The first is that the assumption of open agenda might not be reasonable in many cases. It is in fact straightforward to show that if you allow one of the agents to monopolize the agenda and decide on the order and number of votes, then cycling goes away, and we can determine society’s preferences under a specific institution. For example, suppose that individual A is the agenda-setter,

and decides the order in which there will be two pairwise contests to eventually determine society's choice. Notice that A 's most preferred policy is x . Therefore, he will set the following vote-ordering: first, there is a vote between y and z , and the winner is voted against x . Moreover, assume that there is sincere voting, in the sense that in a vote between two alternatives individuals simply vote for the alternative they prefer without taking into account the implications that this vote will have on subsequent votes and outcomes down the line.¹ Then, as noted above, y will win against z , and then in the pairwise contest between y and x , A 's most preferred policy, x , will be the winner.

Many scholars in the subfield of American politics, where the structure of the Congress often gives agenda-setting power to certain groups, have adopted and developed this approach. Nevertheless, it would often be an undesirable feature of a 'general theory' to rely on very specific agenda-setting procedures. Moreover, in many situations relevant to this book, an open agenda may be entirely sensible, especially if there is free entry by political parties or organizations into the political arena. For instance, the process by which the Democratic and Republican parties offer platforms to voters in elections seems to mirror a situation of an open agenda. To illustrate this point, and also to make the above choices, x , y , and z , more concrete, let us now consider a more specific situation featuring distributional conflict.²

Suppose that there are three politicians, A , B and C who have to divide \$1,000. There is open agenda. The politicians propose splits of the money after which the new split is voted on against whichever split won the last vote. Imagine that there is some status-quo

¹It is straightforward to construct the same argument with 'strategic' voting. Strategic voting means that individuals anticipate the ramifications of their votes today, and may vote for an outcome that they don't prefer, in order to eventually induce a more desirable outcome. Game theoretically, strategic voting amounts to solving for the subgame perfect equilibrium of a dynamic voting game. In the example discussed in the text, agent B has an incentive to vote strategically. If he were to vote sincerely, the ultimate outcome would be x , which is his least preferred allocation. In the first vote, if he changes his vote from y to z , then it will be z against x , and the second stage, and z will win receiving votes from B and C . Therefore, agent B should strategically vote for z over y . However, the argument of agenda-setting generalizes to environment with strategic voting. For example, if A is again the agenda-setter and anticipates strategic voting, he would first set a vote between x and z . He will himself vote for x , and agent B will vote for z . Agent C , if he were to vote sincerely, would choose z . But this would lead to a contest between y and z , which y would win, and y is the least preferred outcome for C . Therefore, C will strategically vote for x , and in the second stage x would compete against y and win. Hence, A can easily design an agenda, which, with strategic voting, leads to his most preferred outcome.

²This example is due Ken Shepsle (Shepsle and Bonchek, 1997).

distribution of the money, which we simply assume is complete equality,

$$\left(333\frac{1}{3}, 333\frac{1}{3}, 333\frac{1}{3}\right).$$

However, politician A quickly proposes the split,

$$(500, 500, 0)$$

which means that A and B share all the money. This defeats the status-quo under majority voting since A and B prefer it. However, C , who gets nothing in this split, can propose to cut A out with the split,

$$(0, 600, 400)$$

and this wins against $(500, 500, 0)$ since A and C prefer it. Now however, A can retaliate by proposing,

$$(300, 700, 0)$$

which wins against the previous split since B votes with A again and against C . Imagine C now proposes,

$$\left(333\frac{1}{3}, 333\frac{1}{3}, 333\frac{1}{3}\right)$$

which is supported by A and C . A cycle! Though this example is simple it captures something of the flavor of ‘log-rolling’ or the type of legislative deal making that occurs with redistributive politics. We can also interpret it in terms of electoral politics. Think of two parties competing for three votes in an election. The parties offer policies which have distributional implications and we can think of this as corresponding to dividing the \$1,000. Hence, if a party proposes the split $\left(333\frac{1}{3}, 333\frac{1}{3}, 333\frac{1}{3}\right)$ this is a policy which favors all voters equally. If a party offered such a platform in an election it would lose to a party that offered the division $(500, 500, 0)$. A party which initially offered equality would then be induced to offer $(0, 600, 400)$ etc. Thus the model of log-rolling can also approximate what is possible in electoral contests where parties compete for support. Cycles seem endemic in redistributive politics.

Overall, Condorcet’s Paradox shows the classic result that even if individual preferences are transitive, social choices need not be. Moreover, Condorcet’s Paradox also

encapsulates the intuition underlying Arrow's theorem. There will often exist allocations that benefit some individuals, while harming others, and there is no coherent way of aggregating these individual preferences into a social ordering.

3 Single-Peaked Preferences and the Median Voter Theorem

3.1 Single-Peaked Preferences

Let us now return to Condorcet's Paradox and try to understand the mathematical intuition with the help of diagrams. These diagrams will not only reveal the intuition, but also point to a strategy for removing the problems caused by the presence of voting cycles. Figure 4.1 draws the preferences of the three citizens in our above example. On the vertical axis, we plot the utility of different alternatives with $x \succeq_A^P y$ implying that x gives A more utility than y . The alternatives are arrayed along the horizontal axis (thus we are implicitly taking an ordering over the allocations x , y and z , which is natural when they correspond to tax rates etc.). In the figure, we have drawn the graphs of the utility functions which we can think of as capturing the preference orderings of the individuals. For example, since $x \succeq_A^P y \succeq_A^P z$, we know that A 's utility is highest at x and lowest at z , and we can think of utility as decreasing in-between.

Now look at the preferences of C . Imagine again that these were tax rates, and think of a rich person as liking a low tax rate, a middle-class person an intermediate tax rate and a poor person a high tax rate. In this case A is a rich person who likes low taxes the best. B is a middle class person who likes intermediate tax rates. Does such a person prefer high to low tax rates or vice versa? The answer is not obvious; either ranking is reasonable since B 's preferences would depend on how much she benefits from what happens to tax revenues (public goods, schools, health care etc.). What about individual C ? The fact that C prefers z best suggests that he is a poor person. However, the rest of his preferences are hard to interpret because he seems to prefer low to intermediate taxes, which is 'strange', or somewhat irregular.

Imagine, now, we change C 's preferences so they are not irregular. For example, let

us suppose that C prefers higher taxes to lower taxes, so the set of preferences changes to (see Figure 4.2):

$$\begin{aligned} x &\succeq_A^P y \succeq_A^P z \\ y &\succeq_B^P z \succeq_B^P x \\ z &\succeq_C^P y \succeq_C^P x \end{aligned}$$

Is cycling still possible? The answer is no. Now, y receives a majority of votes both against z (from A and from B) and against x (from B and from C), so is always the majority winner, or in the terminology of social choice, it is the *Condorcet winner*.

What has happened here? When we changed C 's preferences between Figure 4.1 and Figure 4.2 we changed them to what are known as 'single peaked preferences'. In Figure 4.1, C 's preferences had two peaks: at z and x , and a plateau at the middle point, y . Put differently, the preference function of C was non-concave (or non-quasi-concave). Concavity or quasi-concavity is a restriction we normally place on preferences over economic choices. The discussion so far suggests that a similar restriction is useful when it comes to social or political choices.

As a digression, it is useful here to remind the reader of the concepts of concavity and quasi-concavity. The function u (for 'utility') is concave if and only if

$$u(\lambda x + (1 - \lambda)y) \geq \lambda u(x) + (1 - \lambda)u(y), \quad (4-1)$$

for all x and $y \neq x$ in the domain of the function, and for all $\lambda \in (0, 1)$ (i.e., $0 < \lambda < 1$). The function u is strictly concave if inequality in (4-1) is strict.

Here $\lambda x + (1 - \lambda)y$ is called a 'convex combination' of x and y . This algebraic definition captures the idea that the graph of a concave function is like a hill where the maximum is the peak of the hill—the highest point (see Figure 4.3). In terms of calculus the important observation is that the slope of a concave function is falling everywhere, or equivalently the function has a negative second derivative (i.e., u is concave if and only if $u'' \leq 0$, and u is strictly concave if and only if $u'' < 0$). The opposite of the concept of concavity, is that of convexity, which is a notion we will also make use of. We say that a function u is convex if and only if

$$u(\lambda x + (1 - \lambda)y) \leq \lambda u(x) + (1 - \lambda)u(y), \quad (4-2)$$

for all x and $y \neq x$ in the domain of the function, and for all $\lambda \in (0, 1)$, with strict convexity defined analogously. Similar to concavity, convexity is also related to the second derivative of the function. In particular, u is convex if and only if $u'' \geq 0$, and u is strictly convex if and only if $u'' > 0$.

The notion of a quasi-concave function is weaker than that of a concave function. More specifically, u is quasi-concave if and only if $u(x) \geq u(y)$ implies that

$$u(\lambda x + (1 - \lambda)y) \geq u(y). \quad (4-3)$$

for all $0 < \lambda < 1$. Strict quasi-concavity applies when the inequality in (4-3) is strict.

Intuitively, quasi-concavity means that if the value of the function at x is greater than it is at y then this implies that the value of the function at any point in a convex combination of x and y is also greater than the value at y . A function like this is in Figure 4.4. Notice that this definition does not imply that the derivative is always falling. Of course if u were concave it would be true that (4-3) would be satisfied. Thus concave functions are always quasi-concave. However, it is clear that functions that satisfy (4-3) do not have to satisfy (4-1) and are not necessarily concave. This is the sense in which quasi-concavity is weaker than concavity. As in many optimization problems, here too, the less restrictive concept of quasi-concavity, (4-3), is sufficient (a result shown generally for optimization problems by Kenneth Arrow and Alan Enthoven in the late 1950's).

Another way of thinking of quasi-concavity, which will be useful for our purposes, is that a function is quasi-concave if it has a single peak. A function like the one depicted in Figure 4.5 is not quasi-concave because it has two peaks, and convex combinations of those two peaks lie above the function. Therefore, an important implication of quasi-concavity is the following property: take any three choices, x , y , and z , with a particular ordering, " $>$ ", say $x > y > z$. Quasi-concavity implies that if an individual has $x \succeq_A^P y$, then he also has $y \succeq_A^P z$. In other words, as we move further and further from the more preferred points of an individual, the choices become less and less preferred. Absence of single-peakedness, on the other hand, implies that after the peak at x (relative to y), we can have another peak at z (i.e., $z \succeq_A^P y$).

Now we can more formally define single-peaked preferences. First, let us define q as the policy choice, Q as the set of all policy choices, with an ordering " $>$ " over this set (again,

if these choices are simply uni- dimensional, e.g., tax rates, this ordering is natural), and $V^i(q)$ as the indirect utility function of individual i . The ideal point (political bliss point) of this individual, q^i , is such that $V^i(q^i) \geq V^i(q)$ for all $q \in Q$. Single-peaked preferences can be more formally defined as

Definition 4.1: (Single-Peaked Preferences) Policy preferences of voter i are single peaked if and only if:

$$\begin{aligned} &\text{If } q'' < q' < q^i \text{ or, if } q'' > q' > q^i, \text{ then} \\ &V^i(q'') < V^i(q'). \end{aligned}$$

Our above discussion of the implications of quasi-concavity implies that the strict quasi-concavity of $V^i(q)$ is sufficient for it to be single peaked.³

It is also useful to define the median voter. As individual M is the median voter, if there are exactly as many individuals with $q^i < q^M$ as with $q^i > q^M$ where q^M is the political bliss point of the median voter.

To assume that people have single-peaked preferences is a restriction on the set of admissible preferences. It is important to note here that the restriction to single peaked preferences is not really about the form or nature of people's intrinsic tastes. It is a statement about people's induced preferences over policy outcomes (the choices over which people are voting, such as tax rates). Is the restriction to single-peaked preferences reasonable?

The discussion we just had might suggest that it is reasonable. After all, you might think it is strange for C to prefer low and high taxes to medium taxes. However, the assumption is not as innocent as it appears, and in fact, much more restrictive than the standard assumptions of concavity or quasi-concavity over economic choices. Guaranteeing that induced preferences over policies are single peaked entails making important restrictions on the set of alternatives that voters can vote on. These restrictions often

³It is possible to state the definition of single-peaked preferences with weak inequalities, e.g. If $q'' \leq q' \leq q^i$ or, if $q'' \geq q' \geq q^i$, then $V^i(q'') \leq V^i(q')$. In this case the corresponding concept would be weak quasi-concavity. Such a formulation allows for a lot of indifference over policy choices (the utility function could be flat over a range of policies). We find it more intuitive to rule out this case which will not be relevant for the models we study in this book.

need to take the form of restricting the types of policies that the government can use, in particular, ruling out policies where all individuals are taxed in order to redistribute the income to one individual, or ruling out person-specific transfers as was the case in the example of the three politicians in the previous paragraph.

Although the restriction to single-peaked preferences, and the restriction of the policy space that this entails, is often not ideal, a large political science literature focuses on such single-peaked preferences. This is because single-peaked preferences generate the famous and powerful *Median Voter Theorem (MVT)*, which constitutes a simple way of determining equilibrium policies from the set of individual preferences. In this book, we will either follow this practice of assuming single-peaked preferences and making use of the Median Voter Theorem, or simply focus on a polity that consists of a few different groups, e.g., the rich and poor, where cycling does not occur (see Section 5.2 below). This is because we do not want to focus on specific democratic institutions that could solve cycling problems in the presence of non-single-peaked preferences. Instead, we want to generate some general implications for democratic politics.

But delving deeper into the Median Voter Theorem, and various specific models of democratic politics, it is useful to give a real world example where preferences are single-peaked. For this reason, we turn to a model, which we will often use in the rest of this book, the model of public-good finance previously discussed by Romer (1975), Roberts (1976) and Bergstrom (1979).

3.2 Example of Single-Peaked Preferences

Suppose that there is a group of n people who must decide on the level of a public good, such as roads, parks or maybe military expenditures. Assume, for reasons that will become clear shortly, that n is an odd number. Index individuals with the letter i , where $i = 1, 2, \dots, n$, and assume that each individual has a utility function $U(c^i, G)$, defined over the level of spending on the public good, G and their own consumption c^i . Notice that this utility function is taken to be the same for all individuals, though of course individual i cares about her own consumption, c^i . Let's assume that these utility functions are linear

in consumption of the private good, hence

$$U(c^i, G) = c^i + v(G), \quad (4-4)$$

where v is an increasing and strictly concave function with positive first and negative second derivatives, i.e., $v' > 0$ and $v'' < 0$. These latter conditions embody the intuitive idea that marginal utility is positive (more is better) but diminishing (more is less better the more you have). For simplicity v is the same for all individuals. Note that G is not indexed by i because it is a public good. The provision of a public good simultaneously benefits all individuals. Unlike G , c^i is a private good, if i consumes a private good nobody else can. Each individual i has a fixed income of y^i , and let's assume that to finance the public good the government taxes all individuals at the same rate τ which is just sufficient to pay for the public good.

In this chapter, and indeed much of the book, we take citizens' incomes as exogenous. Incomes are generated from assets, such as land, physical capital and human capital and we can think of these factors generating income in the background. Later, these assets come to the forefront of our analysis because the form in which people hold their assets will be of great significance for redistributive politics. For example, land is immobile, while physical capital to some extent is mobile. This gives the owner of physical capital an option that a land-owner does not have. Similarly, land is much easier to redistribute than physical capital—it is relatively easy to divide a large farm between the workers, while it is much harder to divide a factory between its' workers. Human capital, the skills, education and experience embodied in people, on the other hand, is practically impossible to redistribute. Here these issues are not central, since we assume that assets are not directly taxable, and the government simply taxes income, irrespective of how it is generated, and individuals do not have the option of using their human capital or physical capital in other activities (hence, income is generated 'inelastically').

In this model the only policy decisions are the tax rate and the level of public good provision. There are as yet no economic decisions such as how many hours to work or how to allocate income between different types of goods. Thus c^i will be determined simply by individual i 's income level and the level of the tax rate. In fact, for individual i with income level, y^i , we have that his consumption is $c^i = (1 - \tau)y^i$, where τ is the rate of

proportional income taxation. Moreover, we shall assume that the government can only spend on G what it raises in tax revenue, thus there is the government budget constraint (recall there are n people)

$$G = \tau n \bar{y}, \quad (4-5)$$

where $\bar{y} \equiv \frac{1}{n} \sum_{i=1}^n y^i$ is average (mean) income, thus $\tau n \bar{y}$ is total tax revenue. Now we can write an individual's utility as a function of the government policy variables and his income:

$$V^i(\tau, G) = V(y^i | \tau, G) = (1 - \tau)y^i + v(G). \quad (4-6)$$

(4-6) is called the indirect utility function of i . This is simply the maximized value of utility given particular values of the policy variables. Here there are no real economic decisions that individuals take, so once we know τ and G , we know indirect utility. More generally, however, individuals will also make economic choices which depend on the policy variables. In this case to construct V^i , we first need to solve for individual i 's optimal economic decisions given the policy vector, and then define the induced preferences over policies given these optimally-taken decisions. We shall come to examples of this shortly.

Note that (4-6) expresses individual utility as a function of government policies, τ and G . But these two policies are not independent. They are linked by the government budget constraint (4-5). Now, simply substituting for G from (4-5) into (4-6), we obtain the indirect utility function over policy variables, here simply the linear tax rate, τ :

$$V^i(\tau) = (1 - \tau)y^i + v(\tau n \bar{y}). \quad (4-7)$$

In this simple example it is easy to check that $V^i(\tau)$ is strictly concave because $v(G)$ is strictly concave. Mathematically, we can establish this by taking the first and second derivatives of (4-7). The first derivative may be either positive or negative, but to check whether or not $V^i(\tau)$ is strictly concave, it is the second derivative that is important. This second derivative is clearly negative since $v'' < 0$. Namely, the two derivatives are:

$$\frac{\partial V^i(\tau)}{\partial \tau} = -y^i + v'(\tau n \bar{y})n \bar{y}, \text{ and } \frac{\partial^2 V^i(\tau)}{\partial \tau^2} = v''(\tau n \bar{y})(n \bar{y})^2 < 0.$$

That the second derivative of $V^i(\tau)$ is negative establishes that it is concave, and therefore quasi-concave. This then implies that preferences are single peaked. If, for any individual i and for any three tax rates $\tau > \tau' > \tau''$, we have $\tau \succeq_i^P \tau'$, then we also have $\tau' \succeq_i^P \tau''$.

3.3 The Median Voter Theorem

Let's now move to an analysis of the famous Median Voter Theorem (MVT). We can use the above restrictions on preferences to show in a general way that transitive social choices exist (Austen-Smith and Banks, Theorem 4.1 page 96). However, this does not help us much in practice because we'd like a sharper prediction for society's choice. This is exactly what the MVT does. It tells us not only that there will be no cycling, but also that the outcome of majority voting in a situation with single-peaked preferences will be the ideal point of the 'median voter'.

Proposition 4.1: (The Median Voter Theorem) Consider a set of policy choices $Q \subset \mathfrak{R}$, let $q \in Q$ be a policy vector, and let M be the median voter. If all individuals have single-peaked preferences over Q , then majority voting with an open agenda always has a determinate policy outcome and it coincides with the ideal point of the 'median voter,' q^M .

Single peaked preferences have more general implications than those in Proposition 4.1. Indeed, they guarantee an outcome for *all voting* schemes, not just simple majority voting—in the abstract settings of social choice we can say much more than this. However, for our purposes this level of generality is fine since we will typically be using the result in situations where a society of individuals has to determine a political outcome via majority voting.

To see the argument imagine the individuals are voting in a contest between q^M and some policy $\tilde{q} > q^M$. Because preferences are single peaked, all individuals who have ideal points less than q^M strictly prefer q^M to \tilde{q} . This follows because indirect utility functions fall monotonically as we move away from the bliss points of individuals. In this case, since the median voter prefers q^M to \tilde{q} , this individual plus all the people with ideal points smaller than q^M constitute a majority, so q^M defeats \tilde{q} in a pairwise vote. This argument is easily applied to show that any \tilde{q} where $\tilde{q} < q^M$ is defeated by q^M (now all individuals with ideal points greater than q^M vote against \tilde{q}). This type of argument can be applied to show that if voting is over some pair of policies, say \tilde{q} and \hat{q} such that $\tilde{q} < \hat{q} < q^M$ then \hat{q} surely wins because a majority of voters have ideal points to the right

of \hat{q} . However, \hat{q} itself is beaten by any policy $q \in (\hat{q}, q^M]$ and since some agent has an ideal point in this interval they have an incentive to propose such a policy. Using this type of reasoning we can see that the equilibrium policy must be q^M —this is the ideal point of the median voter who clearly has an incentive to propose this policy.

In the conditions of Proposition 4.1 we stated that policies must lie in a sub-set of the real numbers ($Q \subset \Re$). Why? This is because, as we shall see, although the idea of single-peaked preferences extends very naturally to higher dimensions of policy, the MVT does not. For the MVT to hold policies must be uni-dimensional.

The MVT therefore makes sharp predictions about equilibrium policies when preferences are single-peaked, and the society is a direct democracy with an open agenda. By restricting the form of preferences we can begin to talk about society's preferences. We next turn to the implications of the MVT in the presence of different forms of democratic politics

3.4 Downsian Party Competition and Policy Convergence

The above story was based on an institutional setting where individuals directly vote over policies—in other words, a direct democracy. In practice, most democratic societies are better approximated by representative democracy, where individuals vote for parties in elections, and the winner of the election then implements policies. What does the MVT imply for party platforms?

To answer this question, imagine a society with two parties competing for an election by offering policies. Individuals vote for parties, and the policy promised by the winning party is implemented. The two parties only care about coming to office. This is essentially the model considered in the seminal study by Downs (1957), though his argument was anticipated to a large degree by Hotelling (1929).

How will the voters vote? They anticipate that whichever party comes to power, their promised policy will be implemented. So imagine a situation in which two parties, A and B , are offering two tax rates q_A and q_B —in the sense that, they have made a *credible commitment* to implementing the tax rates q_A and q_B , respectively. Let $P(q_A, q_B)$ be the probability that party A wins power when the parties offer the policy platform (q_A, q_B) .

B , naturally, wins with probability $1 - P(q_A, q_B)$. Both parties want to come to power. If the majority of the population prefer q_A to q_B , then they will vote for party A , and we will have $P(q_A, q_B) = 1$. If they prefer q_B to q_A , then they will choose party B , so we have $P(q_A, q_B) = 0$. Finally, if the same number of voters prefer one policy to the other, we might think either party is elected with probability $1/2$, so that $P(q_A, q_B) = 1/2$ (though the exact value of $P(q_A, q_B)$ in this case is not important).

Since preferences are single peaked, from Proposition 4.1, we know that whether a majority of voters will prefer tax rate q_A or q_B depends on the preferences of the median voter. More specifically, let the median voter be denoted by superscript M , then Proposition 4.1 immediately implies that if $V^M(q_A) > V^M(q_B)$, we will have a majority for party A over party B . The opposite obtains when $V^M(q_A) < V^M(q_B)$. Finally, given single-peaked preferences $V^M(q_A) = V^M(q_B)$ is only possible if $q_A = q_B$, and in this case all voters will be indifferent between the two parties, and one of them will come to power randomly. Therefore, we have

$$P(q_A, q_B) = \begin{cases} 1 & \text{if } V^M(q_A) > V^M(q_B) \\ \frac{1}{2} & \text{if } V^M(q_A) = V^M(q_B) \\ 0 & \text{if } V^M(q_A) < V^M(q_B) \end{cases} \quad (4-8)$$

What about the parties? We have already stated that parties choose their respective policies in order to come to power. Using the terminology we already established, we can introduce a simple objective function for the parties: each party gets a rent $R > 0$ when it comes to power and 0 otherwise. Neither party cares about anything else. More formally parties choose policy platforms to solve the following pair of maximization problems,

$$\begin{aligned} \text{Party } A & : \max_{q_A} P(q_A, q_B)R \\ \text{Party } B & : \max_{q_B} (1 - P(q_A, q_B))R \end{aligned} \quad (4-9)$$

Given this, we can state the Policy Convergence Theorem, originally formulated by Downs (though conjectured by Hotelling), showing that with two-party competition, policies are going to converge the preferences of the median voter:

Proposition 4.2: (Downsian Policy Convergence Theorem) Consider a set of policy choices $Q \subset \mathfrak{R}$, two parties A and B that only care about coming to office, and

can commit to policy platforms. Let M be the median voter, with the ideal point of the ‘median voter,’ being q^M . Then, in the unique equilibrium both parties will choose the platforms $q_A = q_B = q^M$.

To see the intuition for why there will be policy convergence to the preferences of the median voter, imagine a configuration where the two parties have offered policies q_A and q_B such that $q^A < q^B \leq q^M$. In this case, we have $V^M(q_A) < V^M(q_B)$ by the fact that the median voter’s preferences are single peaked, and there will be a clear majority favor the policy of party B over party A , and hence $P(q_A, q_B) = 0$, and party B will win the election. Clearly, A has an incentive to increase q_A to some $q \in (q^B, q^M)$ if $q^B < q^M$, to win the election, and to $q = q^M$ if $q^B = q^M$ to have a $1/2$ probability of winning the election. Therefore, a configuration of platforms such that $q^A < q^B \leq q^M$ cannot be an equilibrium. The same argument applies: if $q^B < q^A \leq q^M$ or if $q^A > q^B \geq q^M$, etc.

Next consider a configuration where $q_A = q_B < q^M$. Could this be an equilibrium? The answer is no: if both parties offer the same policy then $P(q_A, q_B) = \frac{1}{2}$ (hence $1 - P(q_A, q_B) = \frac{1}{2}$ also). But then if A increases q_A slightly so that $q_B < q_A < q^M$ then $P(q_A, q_B) = 1$. Clearly, the only equilibrium involves $q_A = q_B = q^M$ with $P(q_A = q^M, q_B = q^M) = \frac{1}{2}$ (hence $1 - P(q_A = q^M, q_B = q^M) = \frac{1}{2}$).

As we noted, the MVT does not simply entail the stipulation that people’s preferences are single-peaked. We require that the policy space be uni-dimensional. Nevertheless, there are various ways to proceed. Following Plott (1967) we can make assumptions about the distribution of individuals’ ideal points and establish something of a general analogue to the MVT (Austen-Smith and Banks, 1999, Chapter 5 provide a detailed treatment). Alternatively, there are ideas related to single peaked preferences, particularly the idea of value restricted preferences, which do extend to multi-dimensional policy spaces. Finally, once we introduce uncertainty into the model, equilibria often exist even if the policy space is multi-dimensional. In any case, it turns out that in models we will study, even when the policy space is multi-dimensional, the MVT does usually apply essentially because we impose restrictions both on the form that policies can take and because the type of heterogeneity between voters is limited.

Also notice that we are referring to the type of political competition in this section as

Downsian political competition. The key result of this section, Proposition 4.2, resulting from this type of competition contains two important results. First, policy convergence, that both parties will choose the same policy platform, and second that this policy platform coincides with the most preferred policy of the median voter. We will see below that in non-Downsian models of political competition, for example with ideological voters or ideological parties, there may still be policy convergence, but this may not be to the most preferred policy of the median voter.

4 Applications of the Median Voter Theorem

4.1 The Meltzer-Richard Model of Redistribution

Let's begin by applying the MVT to our public good example from above. In the process, we will also like to discuss one of the famous applications of the MVT by Meltzer and Richard (1981) who introduced political economy considerations into the area of the provision of public goods and redistribution more generally.

Meltzer and Richard asked why was it that government expenditure had been increasing as a proportion of national income in the United States during the 20th century. A standard economic approach to this question, derived from welfare economics and public finance, would be to argue that the socially efficient level of expenditure had gone up for some reason (possibly people valued public goods more and the government is supposed to supply public goods, or with the development of economy and technology, market failures have become more rampant) and that the government had therefore raised taxes and increased expenditure. Meltzer and Richard rejected this type of argument saying that basically it was about political power in democracy, who had it, and what they wanted to do with it. Their argument was that when there is income inequality in society the poor are relatively numerous and in a democracy they can use these greater numbers to tax the rich. They explained the increase in redistribution over time by the fact that the voting franchise had been extended (a topic closely related to our focus here).

Meltzer and Richard, in constructing their argument, used a pure model of the redistribution: tax revenues were collected by proportional income taxation and redistributed

lump sum to all people in society. In this section, we start with a model where tax revenues are used to finance a public good. However, the level of provision of the public good is determined by the same considerations that affect the overall amount of redistribution in the pure redistribution model we will analyze below, namely distributional conflict between the relatively rich who pay more in taxes and the relatively poor who pay less.

We will see that what matters for the equilibrium level of taxation is the relationship between the income of the median voter and the average income in society. The effect of increasing the franchise—adding poor people who were previously disenfranchised—is that it reduces the income of the median voter relative to the mean, and this tends to increase the preferred extent of redistribution that the median voter desires.

This Meltzer-Richard model was also picked up on to explain the cross-country relationship between income distribution and redistribution (e.g., Persson and Tabellini, 1994, Alesina and Rodrik, 1994). It implies that the greater is inequality in society, the lower the income of the median voter relative to the mean income, the greater will be the tax rate preferred by the median voter. Thus, other things equal, the model predicts that democracies where there is more inequality, should have higher tax rates and more income redistribution.

To start with, let us order people from poorest to richest and let us think of the median person as being the person with median income, denoted y^M . More specifically, suppose that there are n individuals in the society, and n is an odd number. Then, given that we are indexing people according to their incomes, the person with the median income is exactly individual $M = \frac{n}{2} + 1$. Recall that \bar{y} is average or mean income, thus

$$\bar{y} = \sum_{i=1}^n y^i / n, \tag{4-10}$$

and $V^i(\tau)$, as given in (4-7), still defines the indirect utility of voters over the linear tax rate, τ . It is straightforward to derive each individual i 's ideal tax rate from this indirect utility function. Recall that this is defined as the tax rate τ^i that maximizes $V^i(\tau)$. This tax rate can be found simply from an unconstrained maximization problem, so we need to set the derivative of $V^i(\tau)$ equal to zero. In other words, τ^i needs to satisfy the first-order

condition:

$$-y^i + v'(\tau^i n \bar{y}) \bar{y} n = 0 \text{ or } \frac{y^i}{\bar{y}} = v'(\tau^i n \bar{y}) n \quad (4-11)$$

This first-order condition, (4-11), says that the ideal tax rate of voter i has the property that its marginal cost to individual i is equal to its marginal benefit. The marginal cost is measured by y^i , individual i 's own income, since an incremental increase in the tax rate leads to a decline in the individual i 's utility proportional to his income (consumption). The benefit, on the other hand, is $v'(\tau^i n \bar{y}) n \bar{y}$, which comes from the fact with higher taxes there will be more public good provision. Why is this benefit exactly $v'(\tau^i n \bar{y}) n \bar{y}$? Recall that total tax revenue is $\tau n \bar{y}$, so a marginal increase in the tax rate starting from τ^i increases the tax revenue by $n \bar{y}$. At $\tau = \tau^i$, total tax revenue and therefore the level of the public good is $\tau^i n \bar{y}$, hence the marginal increase in the individual's utility from public good provision is $v'(\tau^i n \bar{y})$. The product of these two terms gives the marginal benefit from higher taxation. Put differently, individual i typically has an interior ideal tax rate (i.e., $0 < \tau^i < 1$), since, though he dislikes paying taxes, he does value the public good. The ideal point of individual i , τ^i , comes from weighing these costs against these benefits. Note that, holding \bar{y} constant, the higher is y^i the lower is τ^i . This follows because when y^i rises, the left side of (4-11) increases, hence for (4-11) to hold, τ^i must change so that the right side, $v'(\tau^i n \bar{y}) n$, must also increase. Since v is a strictly concave function the greater is τ^i the lower is v' , thus for $v'(\tau^i n \bar{y}) n$ to increase, τ^i must fall. As y^i rises, the costs of taxation go up but the benefits do not change. This implies the intuitive result that rich people prefer lower tax rates and less expenditure on public goods than poor people. This distributional conflict, the difference in the attitudes of the relatively rich and the relatively poor towards taxes, is the source of most of the results in this book

Now the MVT implies that the tax rate determined by majority voting, τ^M , will satisfy the equation

$$\frac{y^M}{\bar{y}} = v'(\tau^M n \bar{y}) n \quad (4-12)$$

where M refers to the median voter. Recall that the median voter is the person with tax preferences exactly at the middle point of other individuals's preferences. The above argument established that tax preferences are monotonic in income, therefore, the median voter is the person who is exactly at the middle of the income distribution.

The tax rate determined by (4-12) is the basic prediction for what happens in a democracy. The usefulness of the MVT is not simply that it produces a sharp prediction for the outcome of voting, it allows us to build models which have empirical content/predictions. Embedded within (4-12) are important comparative static predictions, in particular, showing how equilibrium policies change with underlying parameters, especially the distribution of income in this society. We will discuss these comparative static results below.

Often, instead of assuming that there are a finite number of people (and worrying whether this number is odd or even), it is convenient to assume that there is an infinite number, in fact a continuum, of people. In much of this book, we will be working with models that are populated with a continuum of individuals. For this purpose, it is useful to review how this changes some notation in the context of this simple model. Utility is exactly as before, given by equation (4-4). Each individual has income y^i which is now distributed according to the cumulative distribution function $F(y)$ with associated density function $f(y)$, over the support $[0, \infty)$, where we allow the upper support to be infinity so as to include unbounded distributions such as the log normal distribution, which provides a good approximation to many real world income distributions. In addition, we normalize the population, n , to 1.

We can now write mean and total income as:

$$\int_0^{\infty} y dF(y) = \int_0^{\infty} y f(y) dy = \bar{y},$$

which is similar to the equation defining mean income above, equation (4-10), but the integral has replaced the summation sign. The median of this distribution is now defined such that $F(y^M) = 1/2$, i.e., exactly half of individuals are below this income level.

As before, each individual is taxed at rate τ and all the resources are used to provide public goods. Thus, the government budget constraint is,

$$G = \int_0^{\infty} \tau y dF(y) = \tau \int_0^{\infty} y dF(y) = \tau \bar{y}$$

since τ is the same for all individuals and can be brought in front of the integral. An individuals' indirect utility is now exactly as before with ideal point defined by (4-12).

Therefore, the equilibrium policy is the most preferred tax rate for the median voter, given by

$$\frac{y^M}{\bar{y}} = v'(\tau^M \bar{y}), \quad (4-13)$$

which is different from (4-12) simply because with a continuous distribution of income we have normalized the size of the population, n , to 1.

The Median Voter Theorem is particularly useful because the simple characterization of equilibrium policy enables comparative static exercises: for example, how does the equilibrium policy change when the economic environment changes? It is clear that anything that makes the public good more valuable, for example, a shift of the $v(\cdot)$ function increasing its slope $v'(\cdot)$ everywhere, leads to greater public good provision. After all, the public good has become more valuable for everybody, including the median voter, whose preferences determine equilibrium policies.

More interesting, how does the distribution of income in this society affect the level of taxation and public good provision? Equations (4-12) and (4-13) indicate that what matters is not the shape of the whole distribution, but simply the comparison between the mean, \bar{y} , and the median, y^M . When the median becomes relatively poor compared to the mean, i.e., y^M/\bar{y} falls, $v'(\tau^M \bar{y}) = v'(G)$ needs to decrease as well. Since v is a strictly concave function, this means that G and τ^M will increase. Intuitively, when the median voter, who is at the middle point of the income distribution, becomes poorer relative to the mean, he would like to redistribute more resources towards himself. In this model, this can only be achieved by increasing taxation and investing more in the public good. Therefore, a bigger (proportional) gap between the mean and the median, i.e., a smaller y^M/\bar{y} , leads to higher taxation and higher public good provision.

This comparative static result was the main focus of Meltzer and Richard's paper. They noted that, with the extension of the franchise to poor segments of the society, the gap between the median income of those with the right to vote and the mean income in society, widened, i.e., y^M/\bar{y} was now smaller, and this increased the demand for redistribution, pushing up taxes and public expenditures.

The above comparative static is also useful for linking inequality to redistribution. The literature generally interprets the gap between the mean and the median, y^M/\bar{y} , as

a (inverse) measure of inequality. This is motivated by the fact that real world income distributions are close to log normal, like the curve shown in Figure 4.6, and most significantly, they are skewed to the right. In other words, they have the median almost always to the left of the mean. Moreover, an increase in inequality with a log normal distribution (measured for example by the variance of income or the Gini coefficient) always corresponds to the gap between the mean and the median opening up. Therefore, for log normal distributions, greater inequality is equivalent to a smaller value of y^M/\bar{y} . Nevertheless, there are many instances where greater inequality may not correspond to a bigger gap between the mean and the median, and we return to discussing some such cases below.

4.2 The Median Voter Theorem and Efficiency

Now that we have aggregated individual preferences into a particular decision rule for the society, it is natural to ask whether this social ordering has some desirable efficiency properties? Does it pick an efficient allocation, for some reasonable definition of efficiency? Or does the underlying conflict of interest emphasized above also mean that voting leads to inefficient outcomes? In this section, we will see that there is no reason to expect efficient outcomes in general, and in fact, there will be some well-defined inefficiencies arising from the fact that the median voter looks after his own interests, not those of society.

A proper discussion of efficiency needs to start with a definition of socially, or economically, efficient allocations. Two common notions, which will also arise in this book, are: *Pareto optimality* and *surplus maximization*. An allocation of resources, here just a tax rate and consequent level of spending on the public good, is Pareto optimal if there is no other allocation where everyone is better off.

It is easy to see that the level of G defined by (4-12) or (4-13) in the previous section is Pareto optimal. Given the policy instruments available, the equilibrium policy maximizes the utility of the median voter. Therefore, the median voter must be worse off at any other allocation of resources. Hence this situation is Pareto optimal.

However, Pareto optimality is not always a strong enough notion. In situations of conflict of interest, most relevant allocations make one individual better off, and some

others worse off. And we would still like to compare these situations. As an example, imagine two allocations: one gives \$10 to individual 1, and nothing to a large number of other citizens. The second gives \$9 to individual 1, and \$20 to all others who were receiving nothing under the first allocation. According to the concept of Pareto optimality, we cannot rank these two allocations in the sense that we cannot say that one is more desirable or efficient than the other. However, many people might view a move from the second to the first as “a big waste of resources”, since a large number of individuals lose \$20, and individual 1 gains \$1.

Therefore, it may be useful to look at an alternative, stronger, notion of efficiency, *surplus-maximizing*. We say that an allocation is surplus-maximizing if it maximizes the total surplus, or utility, of society. This definition implicitly builds on a utilitarian conception, going back to Jeremy Bentham, the famous 19th-century British philosopher. This utilitarian conception, which maintains that utilities across individuals can be compared, is clearly strong and problematic when applied in many contexts. Nevertheless, it is a simple way of conceptualizing a notion of aggregate efficiency weaker than Pareto optimality.

More formally, we say that an allocation is *surplus-maximizing* if it maximizes the sum of the utilities of all the individuals in the society. In the public good example above a surplus-maximizing allocation maximizes

$$U = \int_0^\infty (c^i + v(G)) dF.$$

Exploiting the fact that consumption is equal to post-tax income and all tax proceeds are used to finance the public good, we have that:

$$U = \int_0^\infty ((1 - \tau) y^i + v(\tau \bar{y})) dF.$$

The surplus-maximizing allocation can then simply be defined as τ^* such that $\tau^* = \arg \max U$, i.e., τ^* maximizes the utilitarian social welfare function, U . The first-order condition for this maximization is simply given by differentiating U (which amounts of differentiating $(1 - \tau) y^i + v(\tau \bar{y})$ for all i under the integral). Therefore, τ^* satisfies:

$$\frac{\partial U}{\partial \tau} = \int_0^\infty (-y^i + v'(\tau^* \bar{y}) \bar{y}) dF = 0,$$

or integrating the two terms separately, and making use of the fact that $\int_0^\infty y^i dF = \bar{y}$, we have

$$\begin{aligned} - \int_0^\infty y^i dF + v'(\tau^* \bar{y}) \bar{y} &= 0 \\ \Rightarrow v'(\tau^* \bar{y}) = v'(G^*) &= 1, \end{aligned} \tag{4-14}$$

where τ^* and G^* denote the surplus-maximizing policies.

Condition (4-14) says that the marginal utility gain from an incremental increase in the public good, $v'(\tau^* \bar{y})$ or $v'(G)$, has to be equal to 1. Intuitively, $v'(G)$ is the marginal benefit from greater public good provision for all agents. The marginal cost is one more dollar of tax revenue. This has different costs for all individuals, but for the utilitarian social welfare function what matters is the sum of all of these costs, which is, by definition, equal to 1.

How does this efficient (surplus-maximizing) allocation of public goods compare to the equilibrium resulting from voting? Recall from (4-13) that $v'(G^M) = y^M / \bar{y}$, where G^M denotes the median voter's choice of public good. Therefore, as long as the median income, y^M , is different from mean income, \bar{y} , the equilibrium levels of taxes and public good provision will not coincide with the surplus-maximizing ones.

Returning to the discussion above, we expect the median of a real world income distribution to be less than the mean, thus y^M / \bar{y} should be less than 1. This implies that in the political equilibrium, we will have $v'(G^M) < 1$, while for surplus-maximization, we need $v'(G^*) = 1$. The strict concavity of $v'(\cdot)$ immediately implies that $G^M > G^*$. Therefore, in economy with the median less rich than the mean income level, like in most real world income distributions, levels of public good provision and tax rates will be greater than the surplus-maximizing levels. This result, first emphasized by Bergstrom (1979), is quite intuitive. The median voter, who is poorer than average income in the society, decides the level of taxation not for utilitarian reasons, but to redistribute resources to himself. This entails setting higher taxes than would be socially efficient from a surplus maximizing perspective, since he bears less of the burden of taxation than the average individual in the society, but benefits equally from public goods.

Equally important, an increase in inequality corresponding to a larger gap between

the mean and the median increases the equilibrium rate of taxation, and therefore the gap between the equilibrium and the surplus-maximizing allocations. We will return to discussing cases where this may not be true below.

It is also noteworthy that when $y^M > \bar{y}$ instead there is inefficient underprovision. In this case, the median voter has an income which is greater than the mean, and therefore dislikes taxes, because taxation imposes a greater burden on him than on the rest of the society. Therefore, realizing that to provide the public good, which is for everybody, he is taxing himself more than the rest of the society, the median voter provides too little public good relative to the surplus-maximizing amount. Nevertheless, recall that the case where the median is richer than the mean is empirically much less relevant.

5 Our Workhorse Models

In this section, we will introduce two models that will be used throughout the book. The discussion of the public good provision model in the previous section illustrated how the level of taxation, even when the proceeds are used to finance public goods, is determined by distributional conflict. As already explained, our theory of democracy and democratization will be based on political and distributional conflict, and in an effort to isolate the major interactions, we will often use models of pure redistribution, where the proceeds of proportional taxation are redistributed lump sum to the citizens. In addition, the major conflict will be between those who lose from redistribution versus those who benefit from redistribution—two groups will refer to loosely as the rich and the poor. Hence, a two-class model consisting of only the rich and the poor is a natural starting point. This model will be discussed in the next two subsections. Another advantage of a two-class model is that a form of the Median Voter Theorem will even if the policy space is multi-dimensional. This is because the poor are the majority and we restrict the policy space so that no intra-poor conflict can ever emerge. In this case the policies preferred by the poor will win over policies preferred by the rich. Later in this section, we will extend this model by introducing another group, the middle class, and show how this changes a range of the predictions of the model, including the relationship between inequality and redistribution.

5.1 The Two-Class Model of Redistributive Politics

Consider a society consisting of two sorts of individuals, the rich with fixed income y^r and the poor with income $y^p < y^r$. Total population is normalized to 1, a fraction $\lambda > 1/2$ of the agents are poor, with income y^p , and the remaining fraction $1 - \lambda$ are rich with income y^r . Mean income is denoted by \bar{y} . Our focus is on distributional conflict, so it is important to parameterize inequality. To do so, we introduce the notation θ as the share of total income accruing to the poor, hence, we have that:

$$y^p = \frac{\theta \bar{y}}{\lambda} \text{ and } y^r = \frac{(1 - \theta) \bar{y}}{1 - \lambda}. \quad (4-15)$$

Notice that an increase in θ represents a fall in inequality. Of course we need $y^p < \bar{y} < y^r$ which requires that

$$\frac{\theta y}{\lambda} < \frac{(1 - \theta)y}{1 - \lambda} \text{ or } \theta < \lambda.$$

The political system determines a linear nonnegative tax rate $\tau \geq 0$, the proceeds of which are redistributed lump sum to all citizens. Moreover, note that this tax rate has to be bounded above by 100 percent, i.e., $\tau \leq 1$. Let the resulting lump-sum transfer be T . Then, the government budget constraint is:

$$T = \tau (\lambda y^p + (1 - \lambda) y^r) = \tau \bar{y}. \quad (4-16)$$

This equation emphasizes that there are proportional income taxes, and equal redistribution of the proceeds, so higher taxes are more redistributive as in the public good example of the previous section. For example, a higher τ increases the lump sum transfer, and since rich and poor agents receive the same transfer but pay taxes proportional to their incomes, rich agents bear a greater tax burden.

All individuals in this society maximize their consumption, which is equal to their post-tax income, which denoted by $\hat{y}^i(\tau)$ for individual i at tax rate τ . Moreover, we use the superscript i to denote social classes as well as individuals, so for most of the discussion we have $i = p$ or r . Using the government budget constraint, (4-16), we have that, when the tax rate is τ , the indirect utility of individual i and his post-tax income are

$$V(y^i | \tau) = \hat{y}^i(\tau) = (1 - \tau) y^i + \tau \bar{y}. \quad (4-17)$$

Since there are more poor agents than rich agents, the “median voter” is a poor agent (think of ordering all the individuals from low to high income). Democratic politics (for example, political competition or open agenda) will then lead to the tax rate most preferred by the median voter, here a poor agent.

Let this equilibrium tax rate be τ^p (to emphasize that this will be the most preferred tax rate by the poor). We can find it by maximizing the post-tax income of the poor agent, i.e., by

$$\tau^p = \arg \max_{\tau} V(y^p | \tau) = (1 - \tau)y^p + \tau\bar{y}. \quad (4-18)$$

This is a simple unconstrained maximization problem, and to find a solution we simply have to look at the first order condition. Let us write this first-order condition somewhat more generally, taking into account that corner solutions are possible at $\tau = 0$ and $\tau = 1$, in the form of a complementary slackness condition:

$$\begin{cases} \bar{y} - y^p = 0 & \text{and } \tau \in (0, 1) \\ \bar{y} - y^p > 0 & \text{and } \tau = 1 \\ \bar{y} - y^p < 0 & \text{and } \tau = 0 \end{cases} \quad (4-19)$$

These conditions take into account that the solution to the maximization problem (4-18) may not be interior (i.e. may not be $0 < \tau < 1$), and we may end up with that corner solution, where the tax rate is the maximum or the minimum that it can be. Inspection of (4-19), in fact, reveals that this will be the case. By definition, the rich are richer than the poor, so average income is greater than the income of the poor, that is, $\bar{y} - y^p > 0$. As a result, democratic politics in this society will lead to the 100 percent tax rate, $\tau = 1$. As a result, post-tax incomes will be fully equalized in this society.

Such an extreme result is clearly not realistic. Moreover, with a corner solution, there are no interesting comparative static results. For example, the extent of inequality in this society, as parameterized by θ , may change, but there will be no effect on the equilibrium tax rate.

This corner solution did not arise in the public good model, since there was a natural source of concavity: greater tax revenues were redistributive in the form of a public good, but the marginal utility from the public good was declining, or equivalently utility function $v(G)$ was strictly concave, i.e., $v'' < 0$. In the original, Meltzer-Richard model, there was also no corner solution, because individuals had an economic decision, that of

labor supply, and greater taxes reduced labor supply, encouraging people to work less. The poor realize that higher taxes create a disincentive effect, and a 100 percent tax rate would basically shut down all economic activity. Hence, the equilibrium tax rate was interior.

Similarly, to get away from this corner solution here, we need to introduce some costs of raising taxes. We will do so by introducing a general deadweight cost of taxation related to the tax rate. The greater are taxes, the greater are the costs because of disincentive effects, other distortions, or because of costs related to tax collection and enforcement.

More specifically, we assume that there is an aggregate cost, coming out of the government budget constraint of $C(\tau)\bar{y}$ when the tax rate is τ . Average income, \bar{y} , is included simply as a normalization. We adopt this normalization throughout the book because we do not want the equilibrium tax rate to depend in an arbitrary way on the scale of the economy. For example, if we vary \bar{y} we do not want equilibrium tax rates to rise simply because the costs of taxation are fixed while the benefits of taxation to the median voter increase. It seems likely that as \bar{y} increases the costs of taxation also increase (for example the wages of tax inspectors increase) and this normalization takes this into account. We assume that $C(0) = 0$, so that there are no costs when there is no taxation; $C'(\cdot) > 0$ so that costs are increasing in the level of taxation; $C''(\cdot) > 0$, so that these costs are strictly convex, that is they increase faster and faster as tax rates increase (thus ensuring the second-order conditions of the maximization problems to be satisfied); and finally, $C'(0) = 0$ and $C'(1) = 1$, so that an interior solution is ensured: the first says that marginal costs are very small when the tax rate is low, and the second implies that costs increase rapidly at very high levels of taxation. Together with the convexity assumption, both of these are plausible: they emphasize that the disincentive effects of taxation become substantial as tax rates become very high. Think, for example, of the incentives to work and to produce when there is a 100 percent tax rate on your earnings.

Introducing this cost of taxation into the government budget constraint, we obtain an updated version of the constraint (4-16):

$$T = \tau\bar{y} - C(\tau)\bar{y}, \tag{4-20}$$

where the right hand-side is the amount of tax revenues raised by the government after

netting out the costs of taxation, and the left hand-side is what the government is spending on lump sum redistribution.

Incorporating the costs of taxation, we have the indirect utility of a poor agent as

$$V(y^p | \tau) = (1 - \tau)y^p + \tau\bar{y} - C(\tau)\bar{y}.$$

The first order condition of maximizing this indirect utility now gives

$$\bar{y} - y^p - C'(\tau^p)\bar{y} = 0. \tag{4-21}$$

The assumption that $C'(1) = 1$ ensures that there is an interior most preferred tax rate by poor agents, τ^p —at $\tau = 1$ we would have $\bar{y} - y^p - \bar{y} < 0$, and we could increase indirect utility by reducing taxes. Moreover, the assumption that $C''(\cdot) > 0$ ensures that the second-order condition for maximization is satisfied, and that (4-21) gives a maximum not a minimum. More explicitly, the second-order condition is $-C''(\tau^p)\bar{y} < 0$, which is always true given $C''(\cdot) > 0$. Equation (4-21) therefore implicitly defines the most preferred tax rate of a poor agent, and the political equilibrium tax rate.

Notice that we do not need preferences to be single peaked in this case for this tax rate to emerge as the political equilibrium outcome, since the poor form an absolute majority. This will be discussed in greater detail below. But for the record, it is also true that preferences are single peaked in this case. This follows immediately from the fact that indirect utility is concave, or that its second derivative is negative, which we just established for the second-order condition of the maximization problem.

Now using the definitions in (4-15), we can write the equation for τ^p in a more convenient form:

$$\left(\frac{\lambda - \theta}{\lambda}\right) = C'(\tau^p) \tag{4-22}$$

where both sides of (4-22) are positive since $\theta < \lambda$ by the fact that the poor have less income than the rich.

Equation (4-22) is useful for comparative statics. Most importantly, consider an increase in θ , so that a greater share of income accrues to the poor, or the gap between the rich and the poor narrows. Since there is a minus sign in front of θ , the left side of (4-22) falls. Therefore, for (4-22) to hold, τ^p must change so that the value of the right side falls

as well. Since $C'' > 0$, when τ^p increases the derivative *increases*, therefore for the right side to fall τ^p must fall. This establishes that lower inequality (higher θ) induces a lower tax rate. Or written mathematically,

$$\frac{\partial \tau^p}{\partial \theta} < 0.$$

It is also the case that total (net) tax revenues as a proportion of national income increase when inequality rises. Total net tax revenues as a proportion of national income are

$$\frac{\tau^p \bar{y} - C(\tau^p) \bar{y}}{y} = \tau^p - C(\tau^p).$$

Notice that $\partial (\tau^p - C(\tau^p)) / \partial \theta = (1 - C'(\tau^p)) \cdot \partial \tau^p / \partial \theta$. We know that higher inequality leads to higher taxes, i.e., $\partial \tau^p / \partial \theta < 0$. Moreover, (4-22) implies that $C'(\tau^p) = (\lambda - \theta) / \lambda < 1$, so $1 - C'(\tau^p) > 0$, which implies that $\partial (\tau^p - C(\tau^p)) / \partial \theta < 0$, in other words, greater inequality leads to a higher proportion of net tax revenues in national income, as argued by Meltzer and Richard (1981) in the context of a slightly different model as we discussed above.

Finally, it is useful to conclude this subsection with a brief discussion of efficiency. This model, taxes are purely redistributive and create distortionary costs as captured by to function $C(\tau^p)$. Therefore, in terms of the efficiency concepts introduced above, the political equilibrium here is inefficient compared to the utilitarian social optimum, which would involve no taxation (in turn, the political equilibrium allocation is Pareto optimal, since it is impossible to avoid the distortionary costs without reducing the utility of the poor). That taxation creates distortionary costs will be a feature of most of the models we will see throughout this book. In some sense, this is plausible, since that taxation creates disincentive effects, distorting the allocation of resources. This might suggest that democracy will be inefficient (at least from a surplus maximizing perspective) since in general there will income redistribution with consequent costs in a democracy. Nevertheless, there are also plausible reasons in general for why greater redistribution might improve the allocation of resources. For example, we can imagine a situation where agents undertake investments in human capital, and the poor are credit constrained and underinvest relative to the optimal amount. Then redistributive taxation, by increasing

the post-tax incomes of the poor, may contribute to aggregate human capital investments and improve the allocation of resources (e.g., Galor and Zeira, 1993, Benabou, 1999, Acemoglu and Robinson, 2000). Moreover, we show later, democracy may in fact be more efficient than dictatorship even when there are taxes raised in democracy. This is because dictatorships may allocate resources to socially wasteful activities, such as repression, to stay in power and the costs of taxation may well be less than the costs of repression.

5.2 The Two-Class Model with Multidimensional Policies

In the previous subsection, we discussed our basic two-class model with a unidimensional policy, the tax rate (the proceeds of which were redistributed lump sum to all the citizens). Recall that indirect utility in that model is given by

$$V(y^i | \tau) = \hat{y}^i(\tau) = (1 - \tau)y^i + (\tau - C(\tau))\bar{y}, \quad (4-23)$$

and that, in addition, we assumed there were only two income levels, that for the rich, y^r , and that for the poor, y^p . Imagine a generalization of that model for a second where income in the population has a more general distribution given by $F(y^i)$. How does this affect the analysis?

The answer is that much of the analysis remains unchanged because, as already noted before, preferences are single peaked. In particular, the indirect utility function $V(y^i | \tau)$ is strictly concave in the unique policy instrument, the tax rate τ , or in other words, $V''(y^i | \tau) = -C''(\tau)\bar{y} < 0$. Given single-peaked preferences, the MVT, Proposition 4.1, applies, and democratic politics elects the most preferred policy of the median voter, the individual M such that $F(y^M) = 1/2$. This tax rate is simply given by τ^M such that:

$$\frac{\bar{y} - y^M}{\bar{y}} = C'(\tau^M),$$

and now taxes and redistribution increase when y^M falls relative to mean income, \bar{y} .

Now imagine a richer policy space in this more general model. For example, suppose that taxes are of the following potentially progressive form:

$$\text{Taxes}(y^i) = \tau_1 y^i + \tau_2 (y^i)^2$$

where $\tau_2 > 0$ implies that taxes increase more than proportionately with income. Suppose also that costs of taxation are $C(\tau_1, \tau_2)$ with the function C strictly increasing in both of its arguments. Then, the government budget constraint becomes

$$T = \tau_1 \bar{y} + \tau_2 (\sigma^2 + \bar{y}^2) - C(\tau_1, \tau_2) \bar{y},$$

where σ^2 is the variance of income distribution, and as before the first two terms on the right hand side are what is raised from taxation, and the last term is what is lost due to distortions, and the left-hand side is the per-person lump-sum redistribution. In writing this expression, we used the fact that $\sigma^2 = \int (y^i - \bar{y})^2 dF = \int (y^i)^2 dF - \bar{y}^2$.

The indirect utility of individual i is then obtained as

$$V(y^i | \tau) = \hat{y}^i(\tau) = (1 - \tau_1) y^i - \tau_2 (y^i)^2 + \tau_1 \bar{y} + \tau_2 (\sigma^2 + \bar{y}^2) - C(\tau_1, \tau_2) \bar{y}. \quad (4-24)$$

If we return to the two-class model, even though this model has a two-dimensional policy space, the political equilibrium is still well behaved, because the poor are the absolute majority. In particular, in this case with progressive taxes the indirect utility of the poor can be written as

$$V(y^p | \tau) = \hat{y}^p(\tau) = (1 - \tau_1) y^p - \tau_2 (y^p)^2 + \tau_1 \bar{y} + \tau_2 (\sigma^2 + \bar{y}^2) - C(\tau_1, \tau_2) \bar{y}. \quad (4-25)$$

The most preferred policies for a poor agent can be obtained by simply maximizing this expression with respect to τ_1 and τ_2 , which gives these most preferred policies, τ_1^p and τ_2^p as

$$\begin{aligned} \left(\frac{\lambda - \theta}{\lambda} \right) &= \frac{\partial C(\tau_1^p, \tau_2^p)}{\partial \tau_1} \\ \left(\frac{\sigma^2}{\bar{y}} + \bar{y} \right) &= \frac{\partial C(\tau_1^p, \tau_2^p)}{\partial \tau_2}. \end{aligned}$$

Here the optimal levels of τ_1^p and τ_2^p are determined by setting the marginal increase in net redistribution equal to the marginal distortion costs due to that form of taxation.

This discussion illustrates that an additional reason for us to focus on the two-class model is that it highlights the fundamental conflict between the relatively rich and the relatively poor without forcing us to limit the policy space to a single dimension. The key observation is that the set of feasible policies is limited enough to avoid any intra-poor

conflict. This stops a coalition between some sub-set of the poor and the rich ever forming a potential majority. These restrictions imply that we can always talk about what the ‘poor’ want and this is what will arise as a political equilibrium under majority voting. Even though in much of the book, we will work with the pure redistribution model of the previous subsection, it is useful to note that we can extend the two-class model to consider more general forms of redistributive politics.

5.3 The Three-Class Model with the Middle Class

In this section, we will introduce a three-class model, which features three distinct social groups, the rich, the middle class and the poor. We will see how in this model, greater inequality can actually reduce redistribution. Moreover, this model will play an important role when we discuss the role of the middle class in the transition to democracy and the consolidation of democracy.

We assume that there are three groups of agents, the rich of size λ^r , the middle class of size λ^m and the poor of size λ^p . We normalize total population to 1 as before, thus $\sum_i \lambda^i = 1$, and assume that $\lambda^p > \lambda^m > \lambda^r$, that is, the poor are the most populous, and then the middle class, and the rich are the smallest group in the population. Also, we denote average income by \bar{y} as before, and introduce the notation that

$$y^r = \frac{\theta^r}{\lambda^r} \bar{y}, y^m = \frac{\theta^m}{\lambda^m} \bar{y}, \text{ and } y^p = \frac{\theta^p}{\lambda^p} \bar{y}. \quad (4-26)$$

This implies that group i has a share θ^i of the economy’s total income, and naturally $\sum_i \theta^i = 1$. Moreover, we assume that

$$\frac{\theta^r}{\lambda^r} > \frac{\theta^m}{\lambda^m} > \frac{\theta^p}{\lambda^p},$$

so that the rich are richer than the middle class, who are in turn richer than the poor.

As before, we assume that the political system determines a linear nonnegative tax rate $\tau \geq 0$, the proceeds of which are redistributed lump sum, and there is an aggregate cost of taxation $C(\tau) \bar{y}$, with $C(\cdot)$ strictly increasing, differentiable and convex, and $C(0) = C'(0) = 0$, and $C'(1) = 1$. Therefore, total tax revenues, after the cost of taxation are subtracted, are $T = (\tau - C(\tau)) \bar{y}$, and since this amount will be redistributed

lump-sum to all individuals, the post-tax income level of an agent with income y^i when the tax rate is τ is given by

$$\hat{y}^i = (1 - \tau) y^i + (\tau - C(\tau)) \bar{y}. \quad (4-27)$$

Utility is defined over consumption, and the consumption level of an individual is simply equal to this post-tax income level, i.e., $c^i = \hat{y}^i$.

Given this setup, we can define the most preferred tax rates of rich, middle-class and poor agents. For any group that is the tax rate that maximizes \hat{y}^i . Given the condition that $C'(1) = 1$, we know that no group would like to set a 100 percent tax rate, but some groups, in particular the rich, and perhaps the middle class, may want to set a zero tax rate. Therefore, the most preferred tax rate of group i satisfies the following condition with complementary slackness:

$$\bar{y} - y^i \leq C'(\tau^i) \bar{y} \text{ and } \tau^i \geq 0$$

or

$$\left(\frac{\lambda^i - \theta^i}{\lambda^i} \right) \leq C'(\tau^i) \text{ and } \tau^i \geq 0 \quad (4-28)$$

Since $y^r > \bar{y}$ by definition, we have that for the rich (4-28) holds as an inequality, and $\tau^r = 0$ as before. Moreover, since $\bar{y} > y^p$, the most preferred tax rate of the poor is positive, i.e., $\tau^p > 0$, given by:

$$\left(\frac{\lambda^p - \theta^p}{\lambda^p} \right) = C'(\tau^p). \quad (4-29)$$

The reader might recognize this equation as the condition determining the most preferred tax rate of poor agents in the two-class model, (4-22), with $\lambda^i = \lambda$ and $\theta^i = \theta$.

The most preferred to tax rate of the middle class could be zero or positive depending on whether y^m is greater or less than mean income \bar{y} . In most real world income distributions, the rich are sufficiently rich that the median is less than the mean. If this is the case the median person is a member of the middle class and we have that

$$\frac{\theta^m}{\lambda^m} < 1 \text{ or } \bar{y} > y^m,$$

then the complementary slackness condition (4-28) for the middle class would hold as equality and τ^m would be given by

$$\left(\frac{\lambda^m - \theta^m}{\lambda^m}\right) = C'(\tau^m), \quad (4-30)$$

and $\tau^m > 0$. However, by virtue of the fact that the middle class are richer than the poor, i.e., $\theta^m/\lambda^m > \theta^p/\lambda^p$, we also have that

$$\tau^p > \tau^m$$

The nature of political equilibrium in democracy will depend crucially on the relative sizes of the three groups. In particular, the assumption above that $\lambda^p > \lambda^m > \lambda^r$ immediately implies $\lambda^r < 1/2$, so the rich are not the majority. This leaves us with two interesting cases:

1. $\lambda^p \geq 1/2$, so the poor are the majority, and unrestricted majority voting will generate their most preferred policy, τ^p .
2. $\lambda^p < 1/2$, so the poor are not the majority, and the median voter will be a middle-class agent. In that case, unrestricted majority voting will lead to the most-preferred policy of the middle class, in this case τ^m .

We will now separately analyze these two cases.

First suppose that $\lambda^p \geq 1/2$. Then the poor are the majority, and democratic politics will lead to their most preferred tax rate, τ^p , as given by (4-29). The comparative statics of this equilibrium are very similar to those of the two-class model, but what matters now is θ^p , which is now a measure of the gap between the poor and average income, not necessarily the gap between the poor in the rich. For example, when θ^p declines, so that poor become relatively poorer, their most preferred tax rate τ^p increases. In other words,

$$\frac{\partial \tau^p}{\partial \theta^p} < 0.$$

However, note that this can happen while the gap between the rich and the poor remains constant. For example, we could have a simultaneous decline in θ^p and θ^r , compensated by an increase in θ^m . In this case, the poor would still vote for, and obtain, higher taxes,

but they are not poorer relative to the rich. They are simply poorer relative to average income.

This observation already shows that the relationship between inequality and redistribution will now depend on exactly what measure of inequality we use. For example, a common measure in the literature is the Gini coefficient or the standard deviation of the logarithm of individual income. Now consider a change in income distribution such that the middle class becomes poor, i.e., θ^m falls, and the rich become richer, that is, θ^r increases, without any change in θ^p . In this model with $\lambda^p \geq 1/2$ this has no effect on redistribution and taxation, whereas according to both measures, income inequality has increased. In fact, if $y^m < \bar{y}$, according to the more rigorous and demanding definition of a mean-preserving spread (see Rothschild and Stiglitz, 1976), we have a more unequal distribution—one that is a mean-preserving spread of the original one, meaning that the distribution now has more weight at the tails than the original distribution. Similarly, if we used the measure of inequality that's the gap between the rich and the poor (for example, the often used measure of the ratio of 90th and 10th percentiles of the income distribution), again inequality has increased, but there is no effect on redistribution. Instead, this model makes a very specific prediction: the extent of redistribution should depend on the gap between the poor and average incomes. But this prediction doesn't necessarily map into a relationship between redistribution and a standard measure of inequality.

Next consider the case where $\lambda^p < 1/2$, so that the poor are not the absolute majority, and the median voter will be from the middle class. In this case, the political equilibrium is given by the tax rate that maximizes the indirect utility of the middle-class agent, and this has a first-order condition given by

$$\left(\frac{\lambda^m - \theta^m}{\lambda^m} \right) \leq C'(\tau^m) \text{ and } \tau^m \geq 0. \quad (4-31)$$

This implies that if $\lambda^m - \theta^m < 0$, or $y^m > \bar{y}$, so that the middle class are richer than average (which happens if the poor are especially poor, pulling down the average), they would have an ideal tax rate identical to those of the rich, equal to 0, i.e., $\tau^m = 0$.

A more realistic case, in line with the skewed nature of real world income distributions is one where the median of the income distribution is to be left of the mean, or the middle

class are poorer than average because the rich are especially rich, pulling up the average. In this case, the political equilibrium tax rate is given by (4-31) holding as equality, or by (4-30) above. The comparative statics of this equilibrium tax rate are similar to those of the most preferred tax rate of the poor. In particular, we have

$$\frac{\partial \tau^m}{\partial \theta^m} < 0,$$

so that when the middle class become poorer relative to the average, they desire higher taxes. Now the relationship between measures of inequality and redistribution is even more tenuous. For example, consider a change in the distribution that reduces θ^p , so that the poor become poorer, simultaneously also increasing θ^m and θ^r . Most measures would show this as an increase in inequality, but the equilibrium tax rate will actually decline.

What do we know in practice about the relationship between inequality and redistribution? In Chapter 2 we saw that there has been some controversy in the economics literature about this relationship both in time-series and in cross-section. Nevertheless, the historical evidence (Lindert, 2002, for an overview) is certainly consistent with the notion that democratizations in more unequal societies lead to much more radical redistribution. We discussed there how many of the existing studies suffer from flawed research design, but that once we control properly for factors which influence the levels relationship between inequality and redistribution (such as political institutions, parties and the power of the rich) greater inequality does seem to lead to greater redistribution.

Nevertheless, these concerns about the relationship between inequality and redistributive taxation notwithstanding, it is straightforward to see that the burden of taxation on the rich is heavier when inequality is greater. To see this, let us return to the two-class model, and suppose that we hold the tax rate constant as inequality increases. What is the effect of this on the burden of taxation on the rich? To answer this question, let us first define the burden of taxation as the net redistribution away from the rich at some tax rate τ . This is clearly defined as

$$\begin{aligned} \text{Burden}(\tau) &= \tau y^r - (\tau - C(\tau)) \bar{y} \\ &= \left[\tau \frac{1 - \theta}{1 - \lambda} - (\tau - C(\tau)) \right] \bar{y} \end{aligned}$$

It is immediately clear that as inequality increases, i.e. θ falls, this burden increases. This simply reflects the fact that with constant average incomes, transfers are constant, and as inequality increases, a greater fraction of tax revenues are collected from the rich. This is a very important observation for our analysis in Part II of the book, because a crucial feature will be how the attitudes of the rich towards democracy and taxation change when inequality changes. This observation implies that, even with unchanged tax rates, this burden increases, and therefore with great inequality, the rich will be typically more opposed to democracy.

6 Democracy and Political Equality

6.1 Probabilistic Voting and Swing Voters

Let us return to the nonexistence of voting equilibria in models without single-peaked preferences. The source of the problem can be seen in equation (4-8) above, which links the probability of winning an election for a party to the preferences of the median voter, when preferences are single peaked. Generally, we can think of an equation like (4-8) as applying at the level of each agent and capturing his voting decision. More specifically, let $p^i(q_A, q_B)$ be the probability that individual i votes for party A offering policy q_A rather than party B , which is committed to policy q_B . This is given by:

$$p^i(q_A, q_B) = \begin{cases} 1 & \text{if } V^i(q_A) > V^i(q_B) \\ \frac{1}{2} & \text{if } V^i(q_A) = V^i(q_B) \\ 0 & \text{if } V^i(q_A) < V^i(q_B) \end{cases} \quad (4-32)$$

The important feature of the voting behavior summarized by (4-32) is that it is *discontinuous*. For example, suppose that the policy vector in question, q , is unidimensional, and that individual i 's preferences are single peaked, with his most preferred policy denoted by q^i . Then, when the two parties offer the following policies $q_A = q_B + \varepsilon < q^i$, where ε is a small number, in the limit infinitesimally small and close to 0. Here, this individual will prefer party A , which is offering a policy closer to his most preferred point. But now imagine party B changes its policy by a very small amount, increasing it by 2ε . This causes a discontinuous change in $p^i(q_A, q_B)$ from 1 to 0.

A very promising approach in the literature to deal with the problems arising from

this discontinuity has been to introduce considerations that would make the above voting behavior function continuous. The most common way of doing so is to presume that there are some non-policy related reasons for uncertainty in individual's preferences, so that a small change in policy only gets a small response in terms of voting behavior. This has been the approach taken by Lindbeck and Weibull (1987), and then later extended and applied extensively by Dixit and Londregan (1995) and Persson and Tabellini (2000). This model is essentially an interpretation of models of probabilistic voting which were developed by political scientists in the 1970's and 1980's (see Coughlin, 1992). An important advantage of this approach is that it introduces concepts of *swing voters*, that is, voters whose behavior is more crucial for the political equilibrium than others, and via this channel, it gives us a richer way of parameterizing *political power* in democratic politics—that is, a situation where some groups have more say (or political power) in democratic politics than others.

More specifically, let the society consist of I distinct groups of voters (all voters within a group having the same economic characteristics). Examples would be the rich and the poor in the two-class model or the rich, the middle class and the poor in the three-class model. There is electoral competition between two parties, A and B , and let π_P^i be the fraction of voters in group i voting for party P where $P = A, B$, and let λ^i be the share of voters in group i and naturally $\sum_{i=1}^I \lambda^i = 1$. Then the expected vote share of party P is

$$\pi_P = \sum_{i=1}^I \lambda^i \pi_P^i.$$

Under Downsian electoral competition, since all voters in i have the same economic preferences, π_P^i is given by (4-32) above, and jumps discontinuously from 0 to 1 as voters in group i always vote with certainty for the party that promises the policy that they prefer more. As summarized in Proposition 4.2, this type of Downsian electoral competition leads to the policy most preferred by the median voter. We will see now how different outcomes emerge when ideological differences are incorporated into voting behavior.

Instead, imagine that an individual j in group i has the following preferences:

$$\tilde{V}^{ji}(q, P) = V^i(q) + \tilde{\sigma}^{ij}(P) \tag{4-33}$$

when party P comes to power, where q is a vector of economic policies chosen by the party in power. Here, $V^i(q)$ is the indirect utility of agents in group i as before, and captures their economic interests. In addition, the term $\sigma^{ij}(P)$ can be interpreted as non-policy related benefits that the individual receives from party P . The most obvious source of these preferences would be ideological. So this model allows individuals within the same economic group to have different ideological preferences.

We can now use a simple normalization, so that we have

$$\tilde{V}^{ji}(q, A) = V^i(q), \text{ and } \tilde{V}^{ji}(q, B) = V^i(q) + \tilde{\sigma}^{ij} \quad (4-34)$$

In that case, the voting behavior of individual j can be represented by an equation similar to (4-32):

$$p^{ji}(q_A, q_B) = \begin{cases} 1 & \text{if } V^i(q_A) - V^i(q_B) > \tilde{\sigma}^{ij} \\ \frac{1}{2} & \text{if } V^i(q_A) - V^i(q_B) = \tilde{\sigma}^{ij} \\ 0 & \text{if } V^i(q_A) - V^i(q_B) < \tilde{\sigma}^{ij} \end{cases} \quad (4-35)$$

In addition, let the distribution of $\tilde{\sigma}^{ij}$ be given by the smooth cumulative distribution function F^i defined over $(-\infty, +\infty)$, with the associated probability density function f^i . Then, we have that

$$\pi_A^i = F^i(V^i(q_A) - V^i(q_B)).$$

Furthermore, and somewhat differently from before, suppose that parties maximize their expected vote share.⁴ In this case, party A sets this policy platform q_A to maximize:

$$\pi_A = \sum_{i=1}^I \lambda^i F^i(V^i(q_A) - V^i(q_B)). \quad (4-36)$$

Party B faces a symmetric problem, which can be simply thought of as minimizing π_A . Equilibrium policies will then be determined as the Nash equilibrium over a game where both parties make simultaneous policy announcements to maximize their vote share. Let us first look at the first-order condition of party A with respect to its own policy choice, q_A , taking the policy choices of the other party, q_B , as given. This is:

$$\sum_{i=1}^I \lambda^i f^i(V^i(q_A) - V^i(q_B)) \frac{\partial V^i(q_A)}{\partial q_A} = 0.$$

⁴Before they simply wanted their vote share to be greater than 1/2, and here, we could obtain similar results by assuming the same preferences, but also some aggregate ideological shocks. The assumption that they maximize their vote share simplifies the discussion.

Since the problem of party B is symmetric, it will also promise the same policy, hence in equilibrium we will have policy convergence with $q_A = q_B$. Therefore, $V^i(q_A) = V^i(q_B)$, and equilibrium policies, announced by both parties, are given by

$$\sum_{i=1}^I \lambda^i f^i(0) \frac{\partial V^i(q_A)}{\partial q_A} = 0. \quad (4-37)$$

Equation (4-37), which gives equilibrium policies, also corresponds to the solution to the maximization of the following weighted utilitarian social welfare function:

$$\sum_{i=1}^I \chi^i V^i(q), \quad (4-38)$$

where

$$\chi^i = \lambda^i f^i(0)$$

are the weights that different groups receive in the social welfare function. We state this result as a proposition for future reference:

Proposition 4.3: (Probabilistic Voting Theorem) Consider a set of policy choices Q , let $q \in Q$ be a policy vector, and let preferences be given by (4-34) as a function of policy and which party is in power, with the distribution function of $\tilde{\sigma}^{ij}$ as F^i . Then equilibrium policy is given by q^* that maximizes (4-38).

There are two additional noteworthy features here. First, an equilibrium always exists as long as the second-order conditions associated with the first-order conditions in (4-37) are satisfied. Therefore, the probabilistic voting model partially avoids the non-existence problems associated either with the failure of single-peakedness or with the multidimensionality of policy spaces.

Second, and potentially more important, this model gives us a way of thinking about and parameterizing the different political power of various groups. Note that if the $f^i(0)$'s, the density of ideological biases between parties at the point where both parties give the same utility, i.e., at $V^i(q_A) = V^i(q_B)$, are identical across groups, we have that $\chi^i = \lambda^i$, and (4-38) becomes exactly the utilitarian social welfare function. The actual equilibrium in this political economy game differs from the maximization of this utilitarian social welfare function because different groups have different sensitivities to policy. For

example, imagine two groups i and i' such that i is much more “ideological,” so that there will be individuals in this group with strong preferences towards party A or party B . This corresponds to the distribution function F^i having relatively large weights at the tails. In contrast, imagine group i' is not very ideological, and the majority of the group will vote for the party that gives them slightly better economic policies. This will correspond to little weights at the tails of $F^{i'}$, and therefore a large value of $f^{i'}(0)$. In this case, voters from group i' become the “swing voters” receiving more weight in the political competition game because they are more responsive to changes in policies.

Now let us briefly return to our basic two-class model with a unique policy instrument, the linear tax rate, τ . Given that the poor are the majority, i.e., $\lambda > 1/2$, Downsian political competition simply maximized the indirect utility of the poor, $V^p(\tau)$. With probabilistic voting, instead, the outcome would depend on the sensitivities of the two different groups to transfers. Let those be χ and $1-\chi$ for the poor and the rich respectively. Then, the equilibrium tax rate would be that which maximizes:

$$\max_{\tau} \chi\lambda((1-\tau)y^p + \tau\bar{y} - C(\tau)\bar{y}) + (1-\chi)(1-\lambda)((1-\tau)y^r + \tau\bar{y} - C(\tau)\bar{y}),$$

which has a first-order condition that can be written as

$$\frac{\chi\theta + (1-\chi)(1-\theta)}{\chi\lambda + (1-\chi)(1-\lambda)} = 1 - C'(\tau). \quad (4-39)$$

It is instructive to compare (4-39) with (4-22) which determined equilibrium policy in the two-class model with Downsian political competition. It is clear that the Downsian outcome is a special case of the current model for $\chi = 1$, in which case (4-39) becomes identical to (4-22). However, for all values of $\chi < 1$, the preferences of the rich also matter for equilibrium policies. Moreover, the smaller is χ , the more political power the rich have despite the fact that they are the minority. In this model, as emphasized before, the political power of the rich will be related to how responsive they are to transfers. For example, if the poor are concerned with other issues or vote on non-policy related ideological grounds, then χ can be very low, and democracy can yield policies quite similar to those most preferred by the rich.

This is an important point to highlight, since throughout we have emphasized that democracies generate more pro-poor policies than nondemocracies. If in fact we have that

as $\chi \rightarrow 0$ and the tax rate generated by democratic politics tends to that most preferred by the rich, there will be little difference between democracies and nondemocracies. Also, as $\chi \rightarrow 1/2$, the tax generated by democratic politics tends to that most preferred by a utilitarian social planner. Our perspective is that there are often reasons for the rich to be powerful in democracies even when they are a minority, i.e., $\chi < 1$ may be a good approximation to reality. In the next subsection, we will see another reason for why the rich may have substantial power in democracy, that money matters in politics and the rich can form effective interest groups lobbying politicians. Nevertheless, both the evidence discussed so far and introspection suggest that we certainly live far from the case where $\chi = 0$, even from that of $\chi = 1/2$. As a result, democracies do not simply cater to the preferences of the rich the same way as a typical nondemocracy would do, nor did they simply maximize utilitarian social welfare function.

6.2 Lobbying

Consider next a very different model of policy determination, and lobbying model. In a lobbying model, different groups make campaign contributions or pay money to politicians in order to induce them to adopt a policy that they prefer. Does introducing lobbying affect our results so far? The answer is clearly yes, since with lobbying power comes not only from voting, but also from a variety of other sources, including whether various groups are organized, how much resources they have available, and their marginal willingness to pay for changes in different policies. Nevertheless, the most important result for us will be that even with lobbying, equilibrium policies will look like the solution to a weighted utilitarian social welfare maximization problem.

To see this, we will quickly review the lobbying model due to Grossman and Helpman (1994, 2000). Imagine again that there are I groups of agents, with the same economic preferences. The utility of an agent in group i , when the policy that implemented is given by the vector q , is equal to

$$V^i(q) - \gamma^i(q)$$

where $V^i(q)$ is the usual indirect utility function, and $\gamma^i(q)$ is the per-person lobbying contribution from group i . We will allow these contributions to be a function of the policy

implemented by the politician, and to emphasize this, it is written with q as an explicit argument.

We assume, following Grossman and Helpman that there is a politician in power, and he has a utility function of the form

$$G(q) \equiv \sum_{i=1}^I \lambda^i \gamma^i(q) + a \sum_{i=1}^I \lambda^i V^i(q), \quad (4-40)$$

where as before λ^i is the share of group i in the population. The first term in (4-40) is the monetary receipts of the politician, and the second term is utilitarian aggregate welfare. Therefore, the parameter a determines how much the politician cares about aggregate welfare. When $a = 0$, he only cares about money, and when $a \rightarrow \infty$, he acts as a utilitarian social planner. One reason why politicians might care about aggregate welfare is because of electoral politics, for example as in the last subsection, since the vote share that they receive might depend on the welfare of each group.

Now consider the problem of an individual j in group i . By contributing some money, he might be able to sway the politician to adopt a policy more favorable to his group. But he is one of many members in his group, and there is a natural free-rider problem. He might let others make the contribution, and simply enjoy the benefits. This will typically be an outcome if groups are unorganized (for example, there is no effective organization coordinating their lobbying activity and excluding non-contributing members from some of the benefits etc.). On the other hand, organized groups might be able to collect contributions from their members in order to maximize group welfare.

We will think that out of the I groups of agents, $J < I$ of those are organized as lobbies, and can collect money among their members in order to further the interests of the group. The remaining $I - J$ are unorganized, and will make no contributions. Without loss of any generality, let us rank the groups such that groups $i = 1, \dots, J$ to be the organized ones.

The lobbying game takes the following form: every organized lobby i simultaneously offers a schedule $\gamma^i(q) \geq 0$ which denotes the payments they would make to the politician when policy q is adopted. After observing the schedules, the politician chooses q . Notice the important assumption here that contributions to politicians (campaign contributions

or bribes) can be conditioned on the actual policy that's implemented by the politicians. This assumption may be a good approximation to reality in some situations, but in others, lobbies might simply have to make upfront contributions and hope that these help the parties that are expected to implement policies favorable to them get elected.

This is a potentially very complex game, since various different agents (here lobbies) are choosing functions (rather than scalars or factors). Nevertheless, noticing the fact that this looks like an auction model along the lines of the work by Bernheim and Whinston (1992), it can be shown that the equilibrium has a simple form. In particular, Grossman and Helpman establish the following proposition:

Proposition 4.4: (Grossman-Helpman Lobbying Equilibrium) In the lobbying game described above, contribution functions for groups $i = 1, 2, \dots, J$, $\{\hat{\gamma}^i(\cdot)\}_{i=1,2,\dots,J}$ and policy q^* constitute a subgame perfect Nash equilibrium if:

1. $\hat{\gamma}^i(\cdot)$ is feasible in the sense that $0 \leq \hat{\gamma}^i(q) \leq V^i(q)$.
2. The politician chooses the policy that maximizes its welfare, i.e.,

$$q^* \in \arg \max_q \left(\sum_{i=1}^J \lambda^i \hat{\gamma}^i(q) + a \sum_{j=1}^I \lambda^j V^j(q) \right)$$

3. There are no profitable deviations for any lobby, $i = 1, 2, \dots, J$, i.e.,

$$q^* \in \arg \max_q \left(V^i(q) - \hat{\gamma}^i(q) + \sum_{i'=1}^J \lambda^{i'} \hat{\gamma}^{i'}(q) + a \sum_{i'=1}^I \lambda^{i'} V^{i'}(q) \right) \text{ for all } i = 1, 2, \dots, J \quad (4.41)$$

4. There exists a policy q^i for every lobby $i = 1, 2, \dots, J$ such that

$$q^i \in \arg \max_q \left(\sum_{i'=1}^J \lambda^{i'} \hat{\gamma}^{i'}(q) + a \sum_{i'=1}^I \lambda^{i'} V^{i'}(q) \right)$$

and satisfies $\hat{\gamma}^i(q^i) = 0$. That is, the contribution function of each lobby is such that there exists a policy that makes no contributions to the politician, and gives her the same utility.

Though this proposition at first looks complicated, it is really quite intuitive. Conditions 1, 2 and 3 are easy to understand. No group would ever offer the contribution that does not satisfy Condition 1.

Condition 2 has to hold, since the politician chooses the policy. If Condition 3 did not hold, then the lobby could change its contribution schedule slightly and improve its welfare.⁵

Next, consider condition 4. Basically, this condition is ensuring that the lobby is not making a payment to the politician above the minimum that is required. If this condition were not true, the lobby could reduce all its contributions, and still induce the same behavior.

Next suppose that these contribution functions are differentiable. Then it has to be the case that for every policy choice, q_k , within the vector q , we must have from the first-order condition of the politician that

$$\sum_{i=1}^J \lambda^i \frac{\partial \hat{\gamma}^i(q)}{\partial q_k} + a \sum_{i=1}^I \lambda^i \frac{\partial V^i(q)}{\partial q_k} = 0 \text{ for all } k = 1, 2, \dots, K$$

and from the first-order condition of each lobby that

⁵In particular suppose that this condition does not hold for lobby $i = 1$, and instead of q^* , some \hat{q} maximizes (4-41). More formally, denote the difference in the values of (4-41) evaluated at these two vectors by $\Delta > 0$.

Then consider the following contribution schedule for this lobby

$$\tilde{\gamma}^1(q) = \sum_{i=1}^J \lambda^i \hat{\gamma}^i(q^*) + a \sum_{i=1}^I \lambda^i V^i(q^*) - \sum_{i=2}^J \lambda^i \hat{\gamma}^i(q) - a \sum_{i=1}^n \lambda^i V^i(q) + \varepsilon c^1(q)$$

where $c^1(q)$ reaches its maximum at $q = \hat{q}$.

Following this contribution offer by lobby 1, the politician would choose $q = \hat{q}$ for any $\varepsilon > 0$. To see this note that by construction

$$\begin{aligned} & V^i(\hat{q}) - \hat{\gamma}^i(\hat{q}) + \sum_{i'=1}^J \lambda^{i'} \hat{\gamma}^{i'}(\hat{q}) + a \sum_{i'=1}^I \lambda^{i'} V^{i'}(\hat{q}) \\ & > V^i(q^*) - \hat{\gamma}^i(q^*) + \sum_{i'=1}^J \lambda^{i'} \hat{\gamma}^{i'}(q^*) + a \sum_{i'=1}^I \lambda^{i'} V^{i'}(q^*) \end{aligned}$$

Therefore, with this new contribution function, the politician is comparing the utility of sticking with the old policy of q^* , which is $\sum_{i'=1}^J \lambda^{i'} \hat{\gamma}^{i'}(q^*) + a \sum_{i'=1}^I \lambda^{i'} V^{i'}(q^*)$ to deviating to \hat{q} , which is $\sum_{i'=1}^J \lambda^{i'} \hat{\gamma}^{i'}(\hat{q}) + a \sum_{i'=1}^I \lambda^{i'} V^{i'}(\hat{q}) + \varepsilon c^1(\hat{q})$. The latter utility is greater for any $\varepsilon > 0$.

The change in the welfare of lobby 1 as a result of changing its strategy is then

$$\Delta - \varepsilon c^1(q)$$

Since $\Delta > 0$, for small enough ε , the lobby gains from this change, showing that the original allocation could not have been an equilibrium.

$$\frac{\partial \hat{\gamma}^i(q)}{\partial q_k} + a \frac{\partial V^i(q)}{\partial q_k} + \sum_{i'=1}^I \lambda^{i'} \frac{\partial \hat{\gamma}^{i'}(q)}{\partial q_k} + a \sum_{i'=1}^I \lambda^{i'} \frac{\partial V^{i'}(q)}{\partial q_k} = 0$$

for all $k = 1, 2, \dots, K$ and $i = 1, 2, \dots, J$

Combining these two first-order conditions, we obtain

$$\frac{\partial \hat{\gamma}^i(q)}{\partial q_k} = \frac{\partial V^i(q)}{\partial q_k} \quad (4-42)$$

for all $k = 1, 2, \dots, K$ and $i = 1, 2, \dots, J$. Intuitively, at the margin each lobby is willing to pay for a change in policy exactly as much as this policy will bring them in terms of marginal return.

But then this implies that the equilibrium can be characterized as a solution to maximizing the following function

$$q^* = \arg \max_q \left(\sum_{j=1}^J \lambda^j V^j(q) + a \sum_{j=1}^I \lambda^j V^j(q) \right),$$

or in other words by maximizing a weighted social welfare function, with individuals in unorganized groups getting a weight of a and those in organized group receiving a weight of $1 + a$. Intuitively, $1/a$ measures how much money matters in politics, and the more money matters, the more weight groups that can lobby receive. As $a \rightarrow \infty$, we converge to the utilitarian social welfare function.

A key implication of this lobbying model for our purposes is that we can think of the rich having more political power in democracy if they can manage to organize as an effective lobby. Moreover, we can also link their exact power to the parameter a . As an application, consider our two-class model of redistribution, and assume that the poor are unorganized, but the rich are an organized lobby. The results in this subsection imply that the lobbying equilibria will be given by maximizing

$$\max_{\tau} a\lambda((1-\tau)y^p + \tau\bar{y} - C(\tau)\bar{y}) + (1+a)(1-\lambda)((1-\tau)y^r + \tau\bar{y} - C(\tau)\bar{y}),$$

which has a first-order condition that can be written, again in the complementary slackness fashion, as

$$\frac{a\theta + (1+a)(1-\theta)}{a\lambda + (1+a)(1-\lambda)} \leq 1 - C'(\tau) \text{ and } \tau \geq 0 \quad (4-43)$$

As $a \rightarrow \infty$, we obtain the case of maximizing utilitarian social welfare function. As $a \rightarrow 0$, equilibrium policy simply maximizes the utility of the rich agents, who become much more influential in democratic politics because of their organized lobby. Interestingly, in this case, irrespective of the value of a , we have that $\tau = 0$, since even with utilitarian social welfare function, there should be no distortionary taxation, as we saw above.

More interestingly, it is possible to combine elements from the probabilistic voting model, where different groups have different amounts of political power, and the lobbying model. For example, we could have that equilibrium policy is given by:

$$q^* = \arg \max_q \left(\sum_{j=1}^J \lambda^j V^j(q) + a \sum_{j=1}^I \lambda^j \chi^j V^j(q) \right)$$

where χ^i 's are political power parameters coming from electoral politics. Applying this model combining features from electoral competition and lobbying to our two-class model of redistribution, we see that there will be redistributive taxation, i.e., $\tau > 0$, if the poor are sufficiently powerful in electoral politics, e.g., $\chi^p > \chi^r$, so as to offset the effects of the power of the rich which derive from their lobbying activities.

7 Partisan Politics and Political Capture

The model of Downsian political competition where the equilibrium policy is the ideal point of the median voter is our basic model of democracy in this book. In the last section, we enriched this model by introducing notions of political power. There are two issues that we have so far ignored or only touched upon slightly. First, political parties might also have ideologies, and these ideological biases may also have an effect on equilibrium policies. Second, an important question is whether certain groups will be able to capture the political agenda (for example via lobbying as in the last section), and how this could be influential in democratic politics. In this section, we will introduce ideological parties (partisan politics), and show how this affects the implications of the Downsian political competition model, and also use this model to discuss issues of political capture. We will see that as long as there are no issues of probabilistic voting (ideological considerations on the side of voters), the predictions of the model of Downsian political competition apply as before, and there are very strong forces towards convergence of policies to the

preferences of the median voter. However, either when there are ideological considerations on the side of voters as well, or when there are problems of commitment on the side of parties, the ideological preferences of parties will also affect equilibrium policy. This will give us another channel through which certain groups can influence equilibrium policy: by capturing the agendas of political parties.

7.1 Electoral Competition with Partisan Parties

In the basic Downsian model of political competition, the objective functions of the parties were given by (4-9), which only valued the rent from coming to power. By ideological parties, we mean parties that have preferences over policies as well as over whether they come to power.

To formalize these notions, imagine that there is a single dimension of policy, again denoted q from a convex and compact set Q , and let there be two parties A and B . We now replace the equation (4-9) by

$$\begin{aligned} \text{Party } A & : \max_{q_A} P(q_A, q_B) (R + W_A(q_A)) + (1 - P(q_A, q_B))W_A(q_B) & (4-44) \\ \text{Party } B & : \max_{q_B} (1 - P(q_A, q_B)) (R + W_B(q_B)) + P(q_A, q_B)W_B(q_A), \end{aligned}$$

where $W_A(q)$ and $W_B(q)$ denote the ‘utility functions’ of parties A and B , and R is a rent from being in office, which is assumed to be non-negative. Parties now maximize their ‘expected utility’ taking into account the voting behavior of the citizens as summarized by the function $P(q_A, q_B)$. This expected utility consists of their ideological preferences over policies that are implemented, and the rent from coming to office.

To start with, we consider the case where $P(q_A, q_B)$ is given by (4-8), for example, because preferences are single peaked and there are no ideological considerations on the side of the voters (we shall later come to probabilistic voting and thus to more smooth versions of equation (4-8)).

Moreover, suppose that the utility functions of the parties are smooth and strictly quasi-concave (i.e., single peaked), with ideal policies q_A^* and q_B^* , i.e.,

$$q_A^* = \arg \max_{q_A} W_A(q_A) \quad \text{and} \quad q_B^* = \arg \max_{q_B} W_B(q_B).$$

In other words, $W'_A(q_A^*) = 0$ and $W'_B(q_B^*) = 0$.

A model of partisan politics along these lines was first formalized by Wittman (1983), who used it to argue that there may not be policy convergence when parties have ideological biases. We will also use this model to talk about issues of capture of the political agenda by one of the groups.

Finally, we assume that both parties choose their policies (policy platforms) simultaneously. Therefore, the predictions of this model can be summarized by the corresponding *Nash Equilibrium*, where each party chooses the policy which maximizes its utility given the policy of the other party. Nash Equilibrium policy platforms, (q_A^N, q_B^N) , will satisfy the following conditions:

$$q_A^N = \arg \max_{q_A} P(q_A, q_B^N) [R + W_A(q_A)] + [1 - P(q_A, q_B^N)] W_A(q_B^N),$$

and, simultaneously,

$$q_B^N = \arg \max_{q_B} [1 - P(q_A^N, q_B)] [R + W_B(q_B)] + P(q_A^N, q_B) W_B(q_A^N).$$

Intuitively these conditions state that in a Nash Equilibrium, taking q_B^N as given, q_A^N should maximize party A 's expected utility. At the same time it must be true that, taking q_A^N as given, q_B^N should maximize B 's expected utility.

The problem in characterizing this Nash equilibrium is that the function $P(q_A, q_B)$, as shown by (4-8), is not differentiable. Nevertheless, it is possible to establish the following proposition, which was first proved by Calvert (1985), and shows that even with partisan politics, there will be policy convergence, and this convergence will typically be to the most preferred point of the median voter.

Proposition 4.5: (Policy Convergence with Partisan Politics) Consider the partisan politics model described above, with ideal points of the two parties q_A^* and q_B^* , and the ideal point of the median voter corresponding to q^M . Suppose also that the probability of party A winning the election is given by $P(q_A, q_B)$, as in (4-8). We have that:

- If $q_A^* \geq q^M \geq q_B^*$ or if $q_B^* \geq q^M \geq q_A^*$, then the unique equilibrium involves $q_A = q_B = q^M$ and each party wins the election with probability one half.

- Otherwise, the unique equilibrium involves $q_A = q_B = q^M$ if $R > 0$, and if $R = 0$, then $q_A = q_B = q_A^*$ when $V^M(q_A^*) > V^M(q_B^*)$ and $q_A = q_B = q_B^*$ when $V^M(q_A^*) < V^M(q_B^*)$.

Therefore, the basic result is that although there can be exceptions when there are no rents from coming to office and both parties have the same type of ideological bias, there are very strong forces towards policy convergence. As the discussion will illustrate, the source of these powerful forces is equation (4-8), which implies that the policy that comes closer to the median voter's preferences will win relative to another policy.

Proposition 4.5 is relatively straightforward to prove, and here we simply give an outline and the basic intuition. Start with the first case where the preferences of the median voter are intermediate with respect to the ideal points of the two parties. Consider first the situation in which $q_A = q^M \neq q_B$. Then, we have that $P(q_A, q_B) = 1$, and party A is winning for sure. The utility of party B is given by: $W_B(q^M)$. Now imagine a deviation by party B to $q_B = q^M$. Now we will have that $P(q_A, q_B) = 1/2$, so the utility of party B changes to $R/2 + W_B(q^M) > W_B(q^M)$, hence the deviation is profitable, and $q_A = q^M \neq q_B$ cannot be an equilibrium (in the case where $R = 0$, the argument is a little different, and now party A can change its policy to something slightly away from q^M towards its ideal point q_A^* and still win the election and now implement a policy closer to its preferences.)

Similarly, consider now a situation where $q_A \neq q^M \neq q_B$, and suppose without loss of any generality that $q_A^* > q^M > q_B^*$ and $V^M(q_A) > V^M(q_B)$, so that we again have $P(q_A, q_B) = 1$. It is clear that we must have $q_A \geq q^M$, otherwise, party A could find a policy q'_A such that $V^M(q'_A) > V^M(q_B)$ and $q'_A \geq q^M$ preferable to any $q_A \in (q^M, q_B)$. But then party B is obtaining utility $W_B(q_A)$, and by changing its policy to $q_B = q^M$ it will obtain utility $R + W_B(q^M)$ if $q_A > q^M$ and $R/2 + W_B(q^M)$ if $q_A = q^M$. By the fact that $q_A \geq q^M$ both of these are greater than its initial utility, $W_B(q_A)$, hence, no policy announcements with $q_A \neq q^M \neq q_B$ can be an equilibrium. Therefore, the equilibrium must have $q_A = q_B = q^M$, i.e., convergence to the median. Intuitively, the median voter's ideal point is preferable to each party relative to the other party's ideal point, and moreover increases their likelihood of coming to power. Therefore, no policy other than

the median voter's ideal point can ever be implemented in equilibrium.

Next let us consider the case where $q_B^* > q_A^* > q^M$ (other configurations give analogous results) Now, suppose that we have $q_A = q_A^*$. What should party B do? Clearly, any policy $q_B > q_A^*$ will lose the election. $q_B = q_A^*$ will win the election with probability 1/2 and is preferable. But in fact party B can do better. It can set $q_B = q_A^* - \varepsilon$ which is closer to the median voter's preferences, and by the fact that voters' preferences are single peaked, this is preferable to q_A^* , and therefore will win the election for party B . Although this policy is worse for party B than q_A^* (since $q_B^* > q_A^*$), for ε small enough, the difference is minuscule, whereas the gain in terms of the rent from coming to power is first-order. This argument only breaks down when $R = 0$, and in this case, the best thing that party B can do is to offer $q_B = q_A^*$ (or any other policy $q_B > q_A^*$ for that matter, since it does not care about coming to power, and in either case, q_A^* will be the equilibrium policy).

We therefore see that policy convergence to the median is a rather strong force, but they can be exceptions especially when rents from coming to power are nonexistent.

Nevertheless, the above results depend crucially on the form of the $P(q_A, q_B)$ function, which created very strong returns to being closer to the most preferred point of the median voter. We saw in the last section how in the presence of ideological considerations on the side of the voters, $P(q_A, q_B)$ can become a continuous function. If that's the case, then policy convergence will break down. To see this, suppose that $P(q_A, q_B)$ is a continuous and differentiable function, and suppose that it reaches its maximum for each party at q^M (i.e., being closer to the median voter's preferences is still beneficial in terms of the probability of being elected—that we make this point which maximizes winning probabilities the median voter's ideal point is simply a normalization without any consequences). In that case, the Nash equilibrium of the policy competition game between the two parties will be a pair of policies q_A^N, q_B^N such that the following first-order conditions hold:

$$\begin{aligned} \frac{\partial P(q_A^N, q_B^N)}{\partial q_A} (W_A(q_A^N) + R - W_A(q_B^N)) + P(q_A^N, q_B^N) W'_A(q_A^N) &= 0, \\ -\frac{\partial P(q_A^N, q_B^N)}{\partial q_B} (W_B(q_B^N) + R - W_B(q_A^N)) - P(q_A^N, q_B^N) W'_B(q_B^N) &= 0. \end{aligned} \quad (4-45)$$

The first term on both lines is the gain in terms of the utility of winning times the

change in the probability of winning in response to a policy change, and the second term is the product of the current probability of winning times the gain in terms of improvements in the party's utility because of the policy change. When these two marginal effects are equal to each other, each party is playing its best response. When both parties are playing their best responses, we have the Nash equilibrium.

Inspection of these first-order conditions will reveal that both parties offering the ideal point of the median voter, i.e., $q_A = q_B = q^M$, is typically not an equilibrium, despite the fact that the probability of winning is higher for both parties at this point. The reason is that now parties are trading off ideological benefits coming from their partisan views against the probability of winning. To see this, suppose that $q_A = q_B = q^M$, and party A deviates and offers a policy slightly away from q^M and closer to its ideal point, q_A^* . The first effect of this on party A 's utility is a loss $\frac{\partial P(q^M, q^M)}{\partial q_A} R < 0$, where the negative sign follows from the fact that any move away from q^M necessarily reduces the probability of winning by definition. But nevertheless, as long as $P(q^M, q^M)W'_A(q^M)$ is large, i.e., party A is sufficiently ideological, the deviation can be profitable, i.e., we can have $P(q^M, q^M)W'_A(q^M) > -\frac{\partial P(q^M, q^M)}{\partial q_A} R$. In this case, we will get non-convergence in the two parties' platforms, and equilibrium policy will be different from the preferences of the median voter, and will depend on the preferences of the parties' ideologies.

Moreover, suppose that both parties are ideologically biased in one direction relative to the populace. In particular, suppose that $q_B^* > q_A^* > q^M$. Now it is easy to see that $q_A = q_B = q_A^*$ can be an equilibrium because by shifting its platform to $q_B = q_A^* - \varepsilon$, party will only increase its chance of winning the election continuously, since $P(q_A, q_B)$ is a continuous and smooth function. We summarize this discussion with the following proposition:

Proposition 4.6: (Policy Non-Convergence with Partisan Politics and Probabilistic Voting) Suppose that $P(q_A, q_B)$ is a continuous and smooth function because of probabilistic voting (or other reasons). Then there can exist in equilibrium where $q_A \neq q_B \neq q^M$ even if $R > 0$.

The reason why this proposition is important for us is that it suggests that certain groups can be quite powerful in democratic politics if they can manage to control the

ideological leanings of the parties. For example, suppose that the policy instrument in question is a tax rate. But somehow suppose that the richer segments of the society can control the political agenda and, by campaign contributions or other methods, they can influence the two parties's ideal points, such that they achieve $\tau_A < \tau_B < \tau^M$. Our discussion suggest that this will typically result in a tax rate less than that preferred by the median, and therefore, the rich can control equilibrium policies, at least to some degree, by their control of the party system.

7.2 Commitment and Convergence

An important assumption so far is that parties announce policy platforms and then they can commit to the policies that they have announced in those platforms. This way, parties could basically compete by varying the policies that they will implement when in office. However, as emphasized by Alesina (1988), the assumption of commitment is not necessarily plausible. In these one-shot models, what is there to stop the politicians from changing policies to their ideal point once they come the power? Nothing. There is no potential punishment (there would have been some punishment if we were in the world with repeated elections, but this is beyond the scope of our treatment here).

Therefore, it is important to see what happens when we remove this commitment assumption. So consider the model of the last section, but assume that parties can choose whichever policy they like when they come to office. Suppose also that $P(q_A, q_B)$ is given by (4-8). This means that announcements before the election are nothing other than cheap talk, and in a subgame perfect equilibrium, voters will realize that once they come to power, parties will implement their ideal points. Therefore, they will simply compare $V^i(q_A^*)$ and $V^i(q_B^*)$, and vote for whichever party has an ideal point closer to their ideal point. The result will be that the party with an ideal point closer to that of the median voter will win. We therefore have that

Proposition 4.7: (Policy Non-Convergence With Partisan Politics and No Commitment) Suppose that there is no commitment to policy platforms in the above model of partisan politics. Then in the unique equilibrium, we have that: if $V^M(q_A^*) > V^M(q_B^*)$ party A comes to power with probability 1 and the equilib-

rium policy is q_A^* ; if $V^M(q_B^*) > V^M(q_A^*)$ party B comes to power with probability 1 and the equilibrium policy is q_B^* ; and if $V^M(q_A^*) = V^M(q_B^*)$ each party comes the power with probability 1/2 and the equilibrium policy is $q_A^* = q_B^*$.

In this model of partisan politics without commitment, we see that parties' policy preferences matter more. This implies that control of the political agenda and parties internal structures may now be much more important in determining equilibrium policies.

Even though the analysis in this section shows that the way that some groups, in particular, richer segments of the society, may influence equilibrium policies through political capture is quite different from their effects in the probabilistic voting model and lobbying model, the overall result is the same: certain groups can be more powerful in democratic politics than suggested by the basic model of Downsian political competition. This result is important for our analysis of the emergence and consolidation of democracy below, but what matters is this general qualitative tendency. Therefore, in the rest of the book, we will use the formalization of the probabilistic voting and lobbying models, which can be summarized by saying that equilibrium policies will be such that they maximize a weighted utilitarian social welfare function

$$\sum_{i=1}^I \chi^i V^i(q),$$

and in this formulation, the parameters χ^i 's denotes the political power of the groups. Given the analysis in this section, however, we will think of these parameters more generally as resulting from political capture as well as lobbying and probabilistic voting.

8 Conclusion

In this chapter we have developed some basic models of democratic politics. We also discussed in detail the workhorse models and some of their properties that we will use to characterize democracy in the rest of the book. Our analysis will focus on the two-class model where the MVT applies, even when the policy space is multi-dimensional. In this model the median voter will be a poor agent and his preferences will determine what happens in a democracy. We will also consider extensively two substantive extensions

of this model. First, the three class model where the middle class enter as a separate group from the rich and the poor. Second, the reduced form model of democracy where different groups 'power' can vary depending on whether or not they are swing voters, whether they are an organized lobby etc. In this chapter we discussed in detail different microfoundations for the power parameter χ^i , but for the rest of the book we simply work with this reduced form rather than present detailed models where lobbying, party capture, or probabilistic voting is explicitly introduced.

Figure 4.1: Non-Single Peaked Preferences

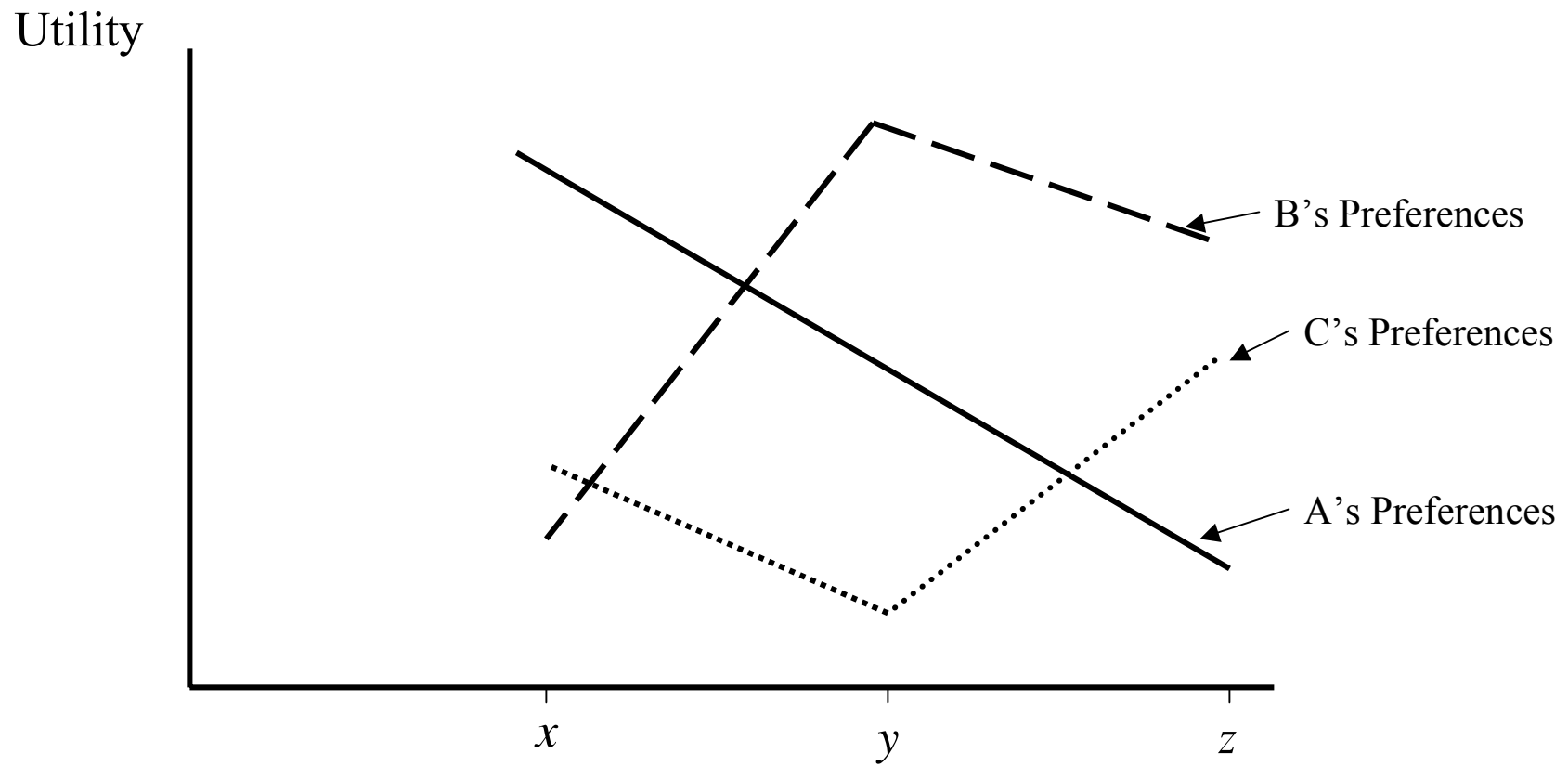


Figure 4.2: Single Peaked Preferences

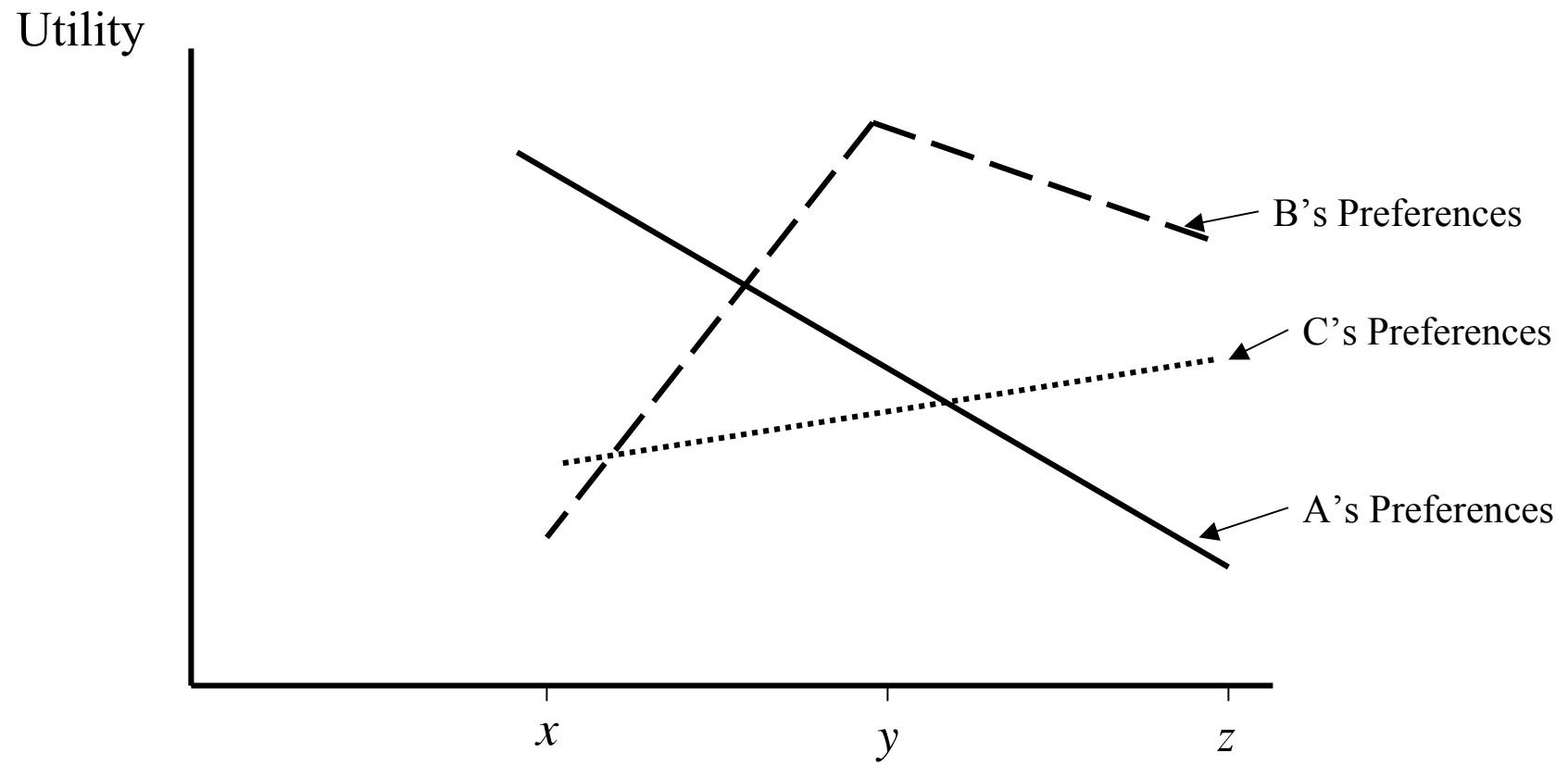


Figure 4.3: A (Strictly) Concave Function

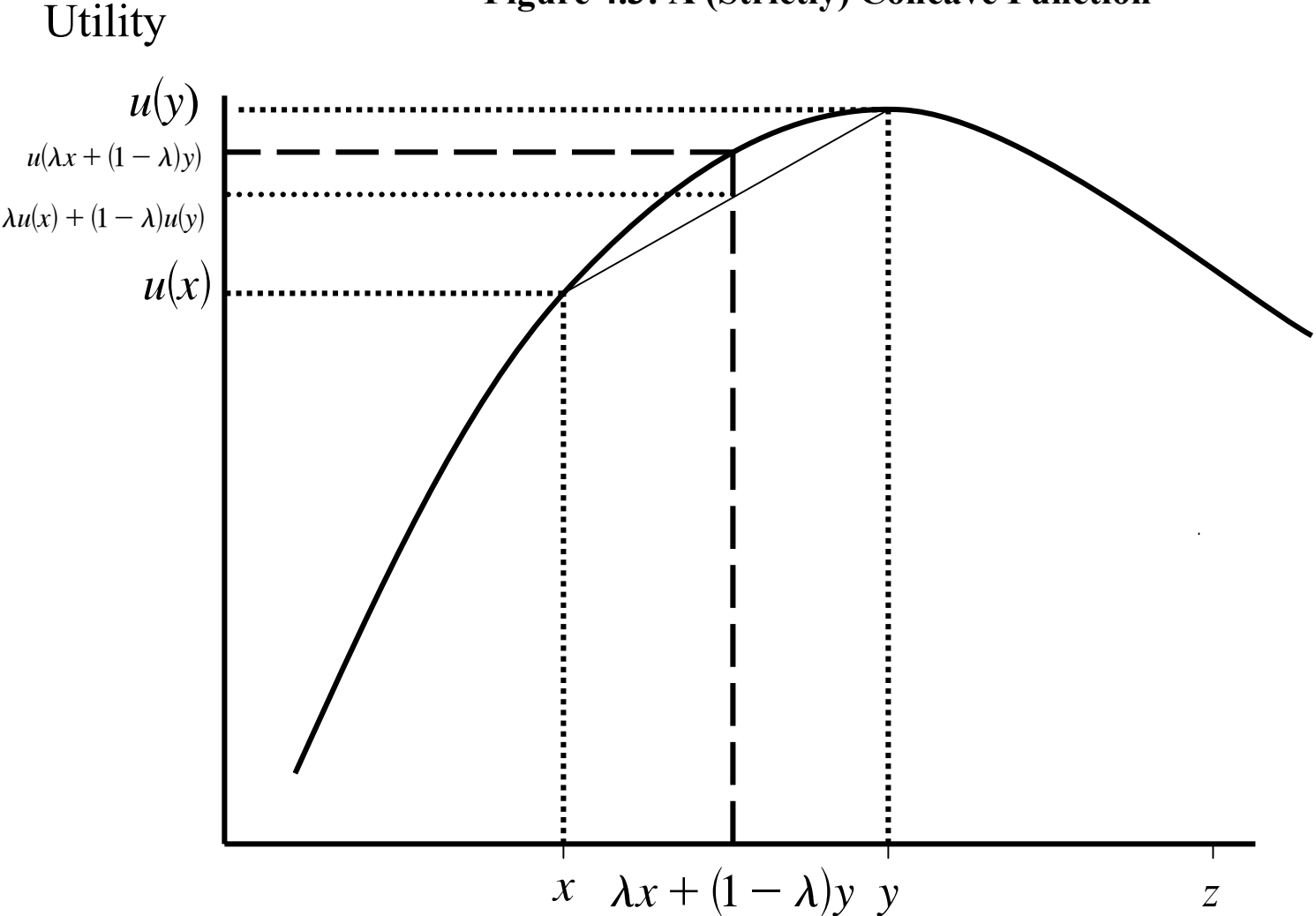


Figure 4.4: A (Strictly) Quasi-Concave Function

Utility

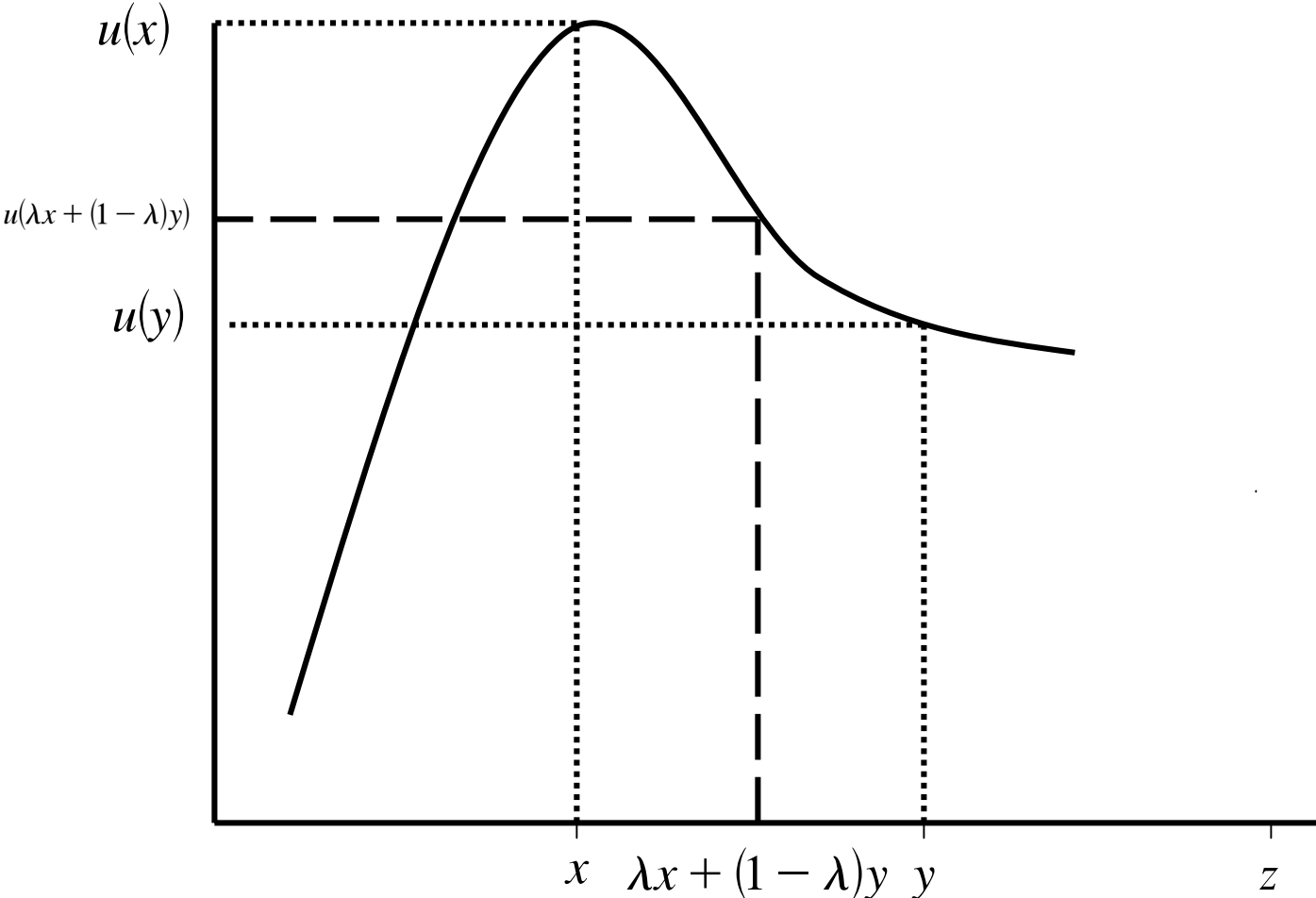


Figure 4.5: A Function which is not Quasi-Concave

Utility

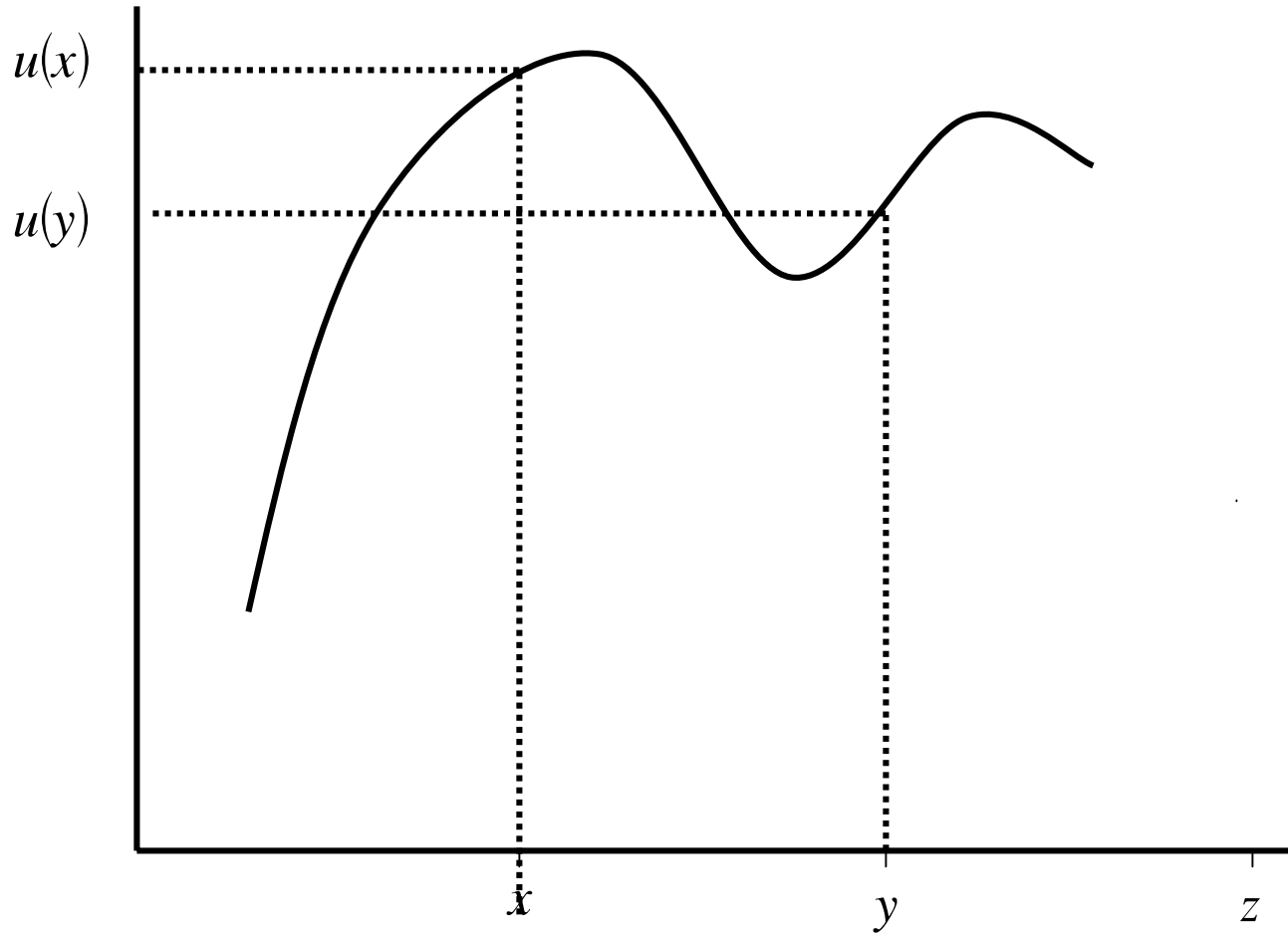


Figure 4.6: The Log-Normal Distribution

