Integration of Strategic and Tactical Planning in the Aluminum Industry

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As we have indicated several times before, problem formulation usually is not straightforward; on the contrary, it requires a great deal of creativity on the part of the model builder. It also is often the case that a single model cannot provide all the support managers need in dealing with a complex set of decisions. This chapter describes a practical application to illustrate the complexities inherent in model design when both strategic and tactical decisions are involved. The emphasis is on the formulation and linking of two separate models representing distinct levels of decision making.

6.1 THE PLANNING APPROACH

Industrial logistics is concerned with the effective management of the total flow of materials, from the acquisition of raw materials to the delivery of finished goods to the final consumer. It is an important and complex field of management, which encompasses a large number of decisions and affects several organizational echelons. Included in the logistics process are decisions determining the resources of the firm (number and location of plants, number and location of warehouses, transportation facilities, communication equipment, data-processing facilities, and so forth) and the proper utilization of these resources (capacity planning, inventory control, production scheduling, dispatching, and so forth). Commonly, the resource-acquisition decisions are associated with the strategic-planning efforts of the firm, and the resource-utilization decisions are considered tactical-planning activities.

These two types of decisions, resource acquisition and resource utilization, differ in scope, level of management involvement, type of supporting information, and length of their planning horizons. If formal systems are to be designed to support these two widely different types of logistics decisions, it is logical to develop two distinct systems: one addressing the strategic-planning issues and one addressing the tactical-planning issues of the firm. However, the two systems should interact strongly with one another, since strategic decisions provide constraints that tactical planning decisions have to meet, and the execution of tactical decisions determines the resource requirements to be supplied by the higher-level strategic decisions. This suggests the development of a hierarchical planning system that is responsive to the organizational structure of the firm and defines a framework for the partitioning and linking of the planning activities.

It is the purpose of this chapter to describe an actual study dealing with the development of an integrated system to support the strategic and tactical planning of an aluminum company. The production of aluminum is a continuous process; therefore, it does not introduce the problems associated with discontinuities in lot-size production required in batch-processing operations. Linear programming becomes a very appropriate model to use in connection with the production-planning activities.

The heart of the planning system to be described is formed by two linear-programming models that interact with each other. One of these models addresses the long-range strategic issues associated with resource planning, while the other is an operational model oriented toward the tactical problems of short-range resource utilization.

We will first provide a brief background of the aluminum industry. Subsequently, we will describe the structure of both models and the way in which they are integrated to form a comprehensive planning system. Special attention should be paid to the different characteristics of the models in terms of time horizon and number of time periods, level of aggregation of the information processed, and the scope and level of management interaction.

6.2 THE ALUMINUM INDUSTRY AND SMELTER OPERATIONS

The production of aluminum from raw materials to finished goods is a fairly involved process but the smelting part of the process is straightforward (see Fig. 6.1). Calcined alumina is reduced to aluminum metal in electrolytic cells or "pots." The passing of electric current through the molten electrolyte causes an electrolytic action that reduces the alumina into two materials, molten aluminum and carbon dioxide. Periodically the aluminum is drawn off through a siphon into large crucibles, which are then used to transfer the molten aluminum to a holding furnace, where the blending or alloying of the metal takes place. Next the aluminum is cast into ingots of various sizes and shapes ranging from 5 lbs to 20,000 lbs depending on the type of equipment that will be used in processing the metal. The smelting operation is the major point of the process that is of concern to us in the present study.

The aluminum company where this project took place is one of the largest fully integrated aluminum companies in the world; the company supplies its products to approximately a hundred different countries. It has six smelters, all operating near

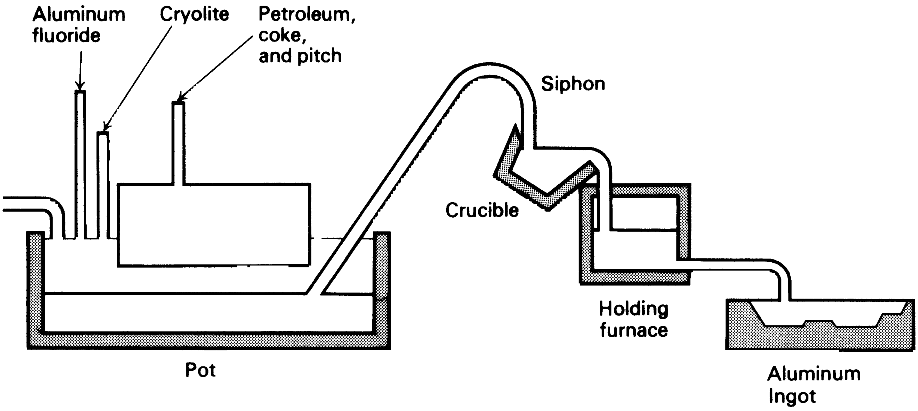

Fig. 6.1 The aluminum production process.

capacity, and the demand for ingot aluminum is expected to continue to grow. Of the six smelters, four are located in domestic locations and two are in foreign locations.

In addition to its own facilities, the company also is able to procure aluminum from external sources through "swapping" contracts. Swapping is the term used for the common practice in the aluminum industry of having a competitor produce one of *your* orders for a particular type of aluminum alloy and deliver it directly to *your* end-use customer under *your* label. The reason a company might want to do this may vary but the most common one is to save on transportation costs of the finished product by selecting a competitor's smelter that is in production closer to your customer area. These agreements generally are reciprocal in nature and are negotiated prior to the start of a year. They are actually a trade of material, since usually no money is exchanged and the major part of the negotiation concerns the amount of material that one company will produce for the other. All warrantees, complaints, and the like, will fall back on the company doing the contracting and not the company producing and delivering the aluminum. Such swapping agreements also are common in industries such as petroleum and fertilizer.

In view of the then current level of operations and the forecast growth in demand, the company was considering various alternatives for smelter-capacity expansion. One of the alternatives was the construction of a new smelter. A major concern of top management was the decision affecting the location, capacity, and the date for starting construction of the new smelter. In addition, a capacity-expansion program was required in the *existing* facilities. It was imperative to organize this effort in a well balanced form, allocating the resources of the company in those areas that offered the highest potential payoffs.

The first model to be presented is used to assess various options for capacity expansion at existing or new smelters, and for swapping contracts. It determines the sources of aluminum for meeting customer demands. A second model is used to assign incoming ingot orders to the various sources of supply, according to the capacity available at each source.

6.3 OVERVIEW OF THE STRATEGIC PLANNING MODEL

Objectives of the Strategic Logistics Model

An effective strategic model should be able to support the development of corporate logistics policies and to provide top managers with a better understanding of decisions on design of new production facilities, capacity expansion of existing facilities, acceptance of long-term contracts, and development of marketing and distribution strategies.

Specifically, our strategic-planning model was designed to assist managers in:

1. evaluating different options for increasing capacity at the existing aluminum smelters of the company;
2. measuring the economic consequences of installing a new smelter, whose size and location had yet to be determined;
3. defining the desired quantity and the price to be paid for various purchasing and swapping contracts;
4. setting general guidelines for the levels of operation at the company's smelters; and
5. assessing the attractiveness of each of the present market areas and defining a strategy for potential growth in these areas.

The Strategic Model—General Characteristics

In order to fulfill the objectives stated above, the planning model should have an overall corporate approach and should deal with aggregate information without going into details pertinent only for operating decisions. Due to the large number of interactions involved in the planning process and the continuous nature of the production activities, it was soon recognized that the most appropriate model to use in this kind of problem was a linear-programming model.

One of the first decisions that has to be made regarding the design of a model is the *total time horizon* covered and the *number of time periods* into which that time horizon is divided. In order to maintain as simple a model structure as possible, only one time period was included in the model. (This does not represent a shortcoming in an aggregated model such as this one, since multistage decisions affecting several years can be studied by changing the input to the one-time-period model properly in a *sequential* fashion.)

The model considers eleven different metal sources, six existing smelter locations, one new smelter, three swapping sources, and one other *possible* swapping source. The use of eleven sources makes it possible to (1) evaluate precisely what value to attach to current swapping agreements, (2) evaluate the marginal economic worth of the various alternative locations for a new smelter, and (3) allow for an unanticipated source of metal. At the same time, the model provides management with sufficient detailed information to answer questions regarding the operation of existing smelters (capacity planning, the level of operation desired, and so forth).

Customer areas and swapping destinations are broken into forty different market groups. These forty locations provide sufficient segmentation of the total market by allowing us to partition the market into *demand centers*, which have distinct transportation costs, customs duties, and in-transit inventory charges.

An analysis of the product line of the company revealed that, while the variety was extensive, an aggregation into only *eight major categories* provided sufficient detail for the strategic-planning model.

The primary constraints that are imposed on the planning process are the existing capacities of hot-metal and casting equipment, as well as the limits imposed on the purchasing or swapping contracts. In addition, the demand generated at each customer area should be satisfied with the present capacity; otherwise expansions of the current installations have to be made.

An important cost element in the production of aluminum ingot is the metal reduction cost. Large amounts of electricity are consumed in that process (approximately 8 KWH/lb), making the cost of electricity the single most important consideration in determining a smelter's location. The choice of location for a new smelter thus is limited to a small number of geographic areas where electrical costs are significantly low.

A second major cost factor is the cost of blending and casting the aluminum. A third relevant cost that affects decisions concerning either a new smelter location or order allocations among existing smelters is that for transportation–the total cost involved for both shipping the raw materials to a smelter and shipping the finished goods to a given customer. Because of the nature of the material (weight and volume) and the wide dispersion of the company's customers, transportation represents a large portion of the controllable variable cost of aluminum ingots. The problem is simplified in our case because the company owns its own shipping line for delivering raw materials to the smelters. The cost of transporting materials to the smelter then can be treated as a fixed cost in a first approximation. This assumption can be relaxed in a subsequent stage of the model development.

The final two major cost items to be considered when making logistic decisions are in-transit inventory and customs duties. While both of these costs are relatively small when compared with the other variable costs, they nonetheless are still large enough to warrant consideration.

Minimization of cost, instead of maximization of profit, was selected as the objective function because cost information is more readily available and prices depend on quantities purchased, type of contracts, and clients. Moreover, if demand *has* to be met, the resulting revenues are fixed and minimization of cost becomes equivalent to maximization of profit.

6.4 MATHEMATICAL FORMULATION OF THE STRATEGIC PLANNING MODEL

The Strategic Model—Notation

We now will describe the symbolic notation that is used in delineating the mathematical formulation of the strategic model.

Smelter and Purchase Source Locations

The letter “s” represents the location of smelters or purchase sources according to the following convention:

s = 1, 2, . . . , 6	existing smelter locations
s = 7	new smelter
s = 8, 9, 10	swapping
s = 11	other

Customer Areas

The letter “a” represents the forty locations of the various customer areas and swapping recipients.

Product Types

The letter “p” represents the eight different product types.

Data Requirements

r_{sp} = Reduction cost of product “p” at source “s”, in \$/ton;

c_{sp} = Casting cost of product “p” at source “s”, in \$/ton;

t_{sa} = Transportation cost from source “s” to customer area “a”, in \$/ton (the cost/ton is the same for all product types);

o_{sa} = Customs duty charged for the shipments from source “s” to customer area “a”, in \$/ton (the cost is the same for all product types);

ℓ_{sa} = Lead time and in-transit time required to ship from source “s” to customer area “a” (the time is the same for all product types);

h_p = Inventory-holding cost for product “p”, in \$/ton/day;

d_{ap} = Forecast demand for product “p” at customer location “a”, in tons;

$\underline{m}_s, \overline{m}_s$ = Lower and upper bounds respectively, for the hot-metal capacity at source “s”, in tons;

$\underline{e}_{sp}, \overline{e}_{sp}$ = Lower and upper bounds, respectively, for the casting-equipment capacity at source “s” for product “p”, in tons.

Decision Variables

Q_{sap} = Quantity of product “p” to be shipped from source “s” to customer area “a”, in tons;

M_s = Total hot-metal output at source “s”, in tons;

D = Total overseas customs duties for all products shipped from all sources to all foreign customer areas, in dollars;

I_p = Total in-transit inventory costs for product "p", in dollars;

E_{sp} = Total amount of product "p" to be cast at source "s", in tons.

The last four decision variables (M_s, D, I_p, and E_{sp}) are introduced only for convenience in interpreting the results, as will be seen in the next section.

The Strategic Model—Formulation

Using the notation described above, we can now formulate the strategic model in mathematical terms.

The Objective Function—Logistics Cost

The objective of the model is the minimization of the total logistics cost incurred, which is represented by the following expression:

$$\text{Minimize cost} = \sum_s \sum_a \sum_p (r_{sp} + c_{sp} + t_{sa})Q_{sap} + \sum_p I_p + D.$$

The first term is the sum of reduction, casting, and transportation costs; the second term is the in-transit inventory cost; and the third term, D, is the total customs duties. (In-transit inventory costs and customs duties are defined in the constraint set as a function of the variable Q_{sap}.)

Metal-Supply Constraint at Sources

$$\sum_a \sum_p Q_{sap} - M_s = 0, \qquad s = 1, 2, \ldots, 11,$$

$$\underline{m}_s \leqq M_s \leqq \overline{m}_s, \qquad s = 1, 2, \ldots, 11.$$

The first equation merely states that the total amount shipped from location s to every customer, considering all products, should be equal to M_s, which is the total metal supply at smelter (or purchasing location) s. This equation serves to define the variable M_s.

The second constraint set represents the upper and lower bounds on the total metal supply at each smelter or puchase location. Recall that constraints of this type are handled implicitly rather than explicitly whenever a bounded-variable linear-programming code is used (see Chapter 2, Section 2.6). The upper and lower bounds define the maximum hot-metal capacity and the minimum economical operational level of the smelter, respectively. When dealing with swapping or puchasing locations, they provide the range in which purchasing or swapping agreements take place.

Shadow prices associated with the metal-supply constraints indicate whether expansion (or contraction) of a smelter hot-metal capacity or purchase contract are in order.

Equipment Casting Capacity at Smelters

$$\sum_{a} Q_{sap} - E_{sp} = 0 \qquad \begin{cases} s = 1, 2, \ldots, 11, \\ p = 1, 2, \ldots, 8. \end{cases}$$

$$\underline{e}_{sp} \leqq E_{sp} \leqq \bar{e}_{sp} \qquad \begin{cases} s = 1, 2, \ldots, 11, \\ p = 1, 2, \ldots, 8. \end{cases}$$

The first equation is used as a definition of variable E_{sp}. It indicates that E_{sp}, the total amount of product p cast at smelter s, must be equal to the total amount of product p shipped from location s to all customers.

The second set of constraints imposes lower and upper bounds on the amount of product p cast in smelter s. These bounds reflect maximum casting capacity and minimum economical levels of performance, respectively, and again do not add significant computational time to the solution of the model when a linear-programming code with bounded-variable provisions is used.

Shadow prices associated with the casting-equipment capacity constraint allow the efficiencies of the various casting equipment to be ranked and suggest expansion or replacement of current equipment.

Demand Constraints

$$\sum_{s} Q_{sap} = d_{ap} \qquad \begin{cases} a = 1, 2, \ldots, 40, \\ p = 1, 2, \ldots, 8. \end{cases}$$

This set of equations specifies that the amount of each product p received at customer region a, from all sources s, has to be equal to the demand of product p at customer region a.

The shadow prices of these constraints allow the relative attractiveness of each product group at each customer area to be defined and therefore serve as basic information for marketing-penetration strategies. By ranking each market in accordance with the marginal returns to be derived by expanding its current requirements, priorities can be assigned that provide guidelines for marketing penetration. In addition, shadow prices indicate when swapping is of interest, since swapping means simply a trade-off between two customer areas. Ideally, we would like to swap an area with a very small marginal return with one that provides a very high marginal return.

Total Overseas Customs Duties

$$\sum_{s} \sum_{a} \sum_{p} o_{sa} Q_{sap} - D = 0.$$

This equation is used to define the total amount spent in customs duties, D, and permits the company to keep track of this expenditure without performing additional computations.

Total In-Transit Inventory Cost

$$\sum_{s}\sum_{a} h_p \ell_{sa} Q_{sap} - I_p = 0, \qquad p = 1, 2, \ldots, 8.$$

This equation records the total in-transit inventory cost for each product group p.

Nonnegativity of the Variables

All the variables should be nonnegative.

6.5 THE TACTICAL PLANNING MODEL

Objectives of the Tactical Model

The basic objective of the tactical model is to assist middle management in assigning ingot orders to the various possible sources of supply, in a way that is consistent with upper-level decisions, which are made with the help of the strategic planning model. The tactical model deals only with the order assignments to the four domestic smelters, although extensions of the model to incorporate the foreign smelters are straightforward. The assignments are performed on a week-by-week basis for a four-week time horizon, followed by two months of planned operations based on orders actually received and forecasts for orders to be expected in those periods.

Each order refers to a demand for a single product type. If an original order contains requirements for more than one product, the order is broken into various individual single-product orders.

The model is intended to support management decisions in the following areas:

1. Assignment of ingot orders to specific casting machines at each smelter (initially only domestic smelters are considered);
2. Effective utilization of existing production equipment;
3. Assignment of labor crews to each production center;
4. Determination of aggregated inventory levels for each product type;
5. Specification of transportation requirements, in deciding ship reservations; and
6. Identification of operational bottlenecks, which could suggest capacity expansion opportunities.

Linking the Strategic and Tactical Models

Figure 6.2 is a diagram of the total logistics system, illustrating the relationships between the strategic and tactical models. It is important to notice the hierarchical nature of this approach, in which decisions made at the strategic level define some of the constraints that have to be observed at the tactical level. Specifically, the strategic model defines the capacity expansion that should take place in the hot-metal and casting facilities to cope with the increasing aluminum demand, including the location, size, and timing of construction of a new smelter. In addition, it fixes operational

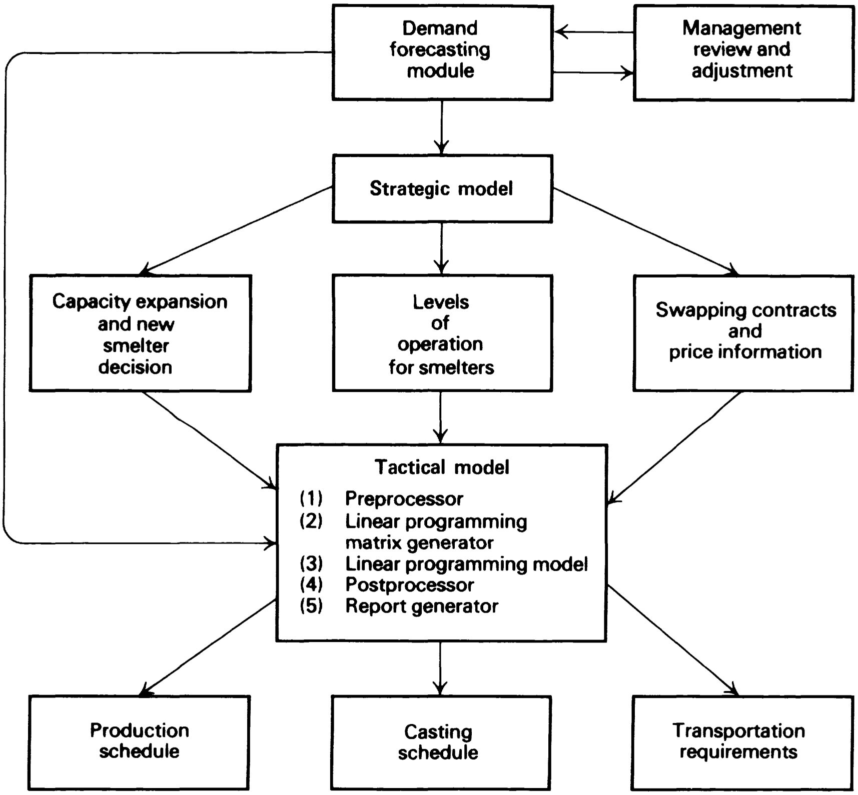

Fig. 6.2 The logistics system.

levels for hot-metal capacity, it defines swapping sources and the quantities to be swapped with these sources, and it prescribes allocation of market areas to metal sources. The strategic model essentially determines the capacity of all sources including new plants, old plants, and swapping points. The tactical model makes the most effective use of the existing production facilities by providing production, casting, and transportation schedules. Feedback from the actual implementation of the tactical-model decisions will in turn provide an important input to the strategic model to obtain a better balance between resource acquisition and resource utilization.

Segmentation of the Tactical Model

The design of the tactical model poses some serious difficulties. First, there are a number of institutional constraints resulting from the company's traditional practices in dealing with specific customers and from priority requirements of some orders, which force the schedule to allocate an order to a specific machine. These constraints must be considered in any realistic order-assignment procedure. Second, due to the large number of constraints and decision variables involved in the order-assignment process, it is mandatory to make every effort to produce a good starting solution to the model and to reduce, as much as possible, the number of constraints to be considered; otherwise the model would become computationally or economically infeasible to run and update every week. There were about one thousand outstanding customer orders to be assigned during the three-month time horizon considered in the tactical model. Thus, the dimension of the problem is such that linear programming becomes the only viable approach to consider. However, since splitting an order between two smelters was not acceptable from an operational point of view, and since the use of *integer* linear programming was out of the question due to the large model dimensions, extreme care had to be exercised to avoid order-splitting problems.

These considerations led to the design of an operating system composed of three segments: the *preprocessor*, which establishes a preliminary operation plan; the *linear-programming model*, which computes an optimal order assignment; and the *post-processor*, which consolidates the orders that might be split and produces relevant management reports. Figure 6.3 illustrates how the system has been decomposed and how the three segments interact.

The first of these segments is a preprocessor subsystem. This is a computer program that performs the following functions:

1. determines the date by which the order should be completed at each smelter (known as the ex-mill date) to satisfy the promised delivery date to the client. The ex-mill date is determined by subtracting the transportation time, from the smelter to the customer location, from the promised delivery date;
2. accepts constraints on orders that must be processed at a given smelter because of purity specifications that can be met only by using a specific machine at that smelter;
3. accepts reservations for certain blocks of casting-capacity time that are required for some special purpose; and
4. accepts the demand-forecast estimates for the two look-ahead months (orders for the current month always will be known).

The preprocessor then constructs an initial order-assignment plan, based on the minimization of freight and in-transit inventory costs. This order-assignment plan does not consider any capacity constraints and thus represents an ideal plan for distribution. The ideal plan rarely would be feasible, but it constitutes an effective solution to initiate the linear-programming model that will be formulated in terms of order reassignments. Only freight and in-transit inventory costs are selected in this initial plan assignment, in order to simplify the cost computations. Freight and in-transit inventory costs are the predominant cost elements in the operating model. Other cost elements are incorporated into the subsequent linear-programming model. The total demand determined for each product type at each smelter during each time period is accumulated, for comparison with actual casting and hot-metal capacity limitations. The final output of the preprocessor system is a list for each time period,

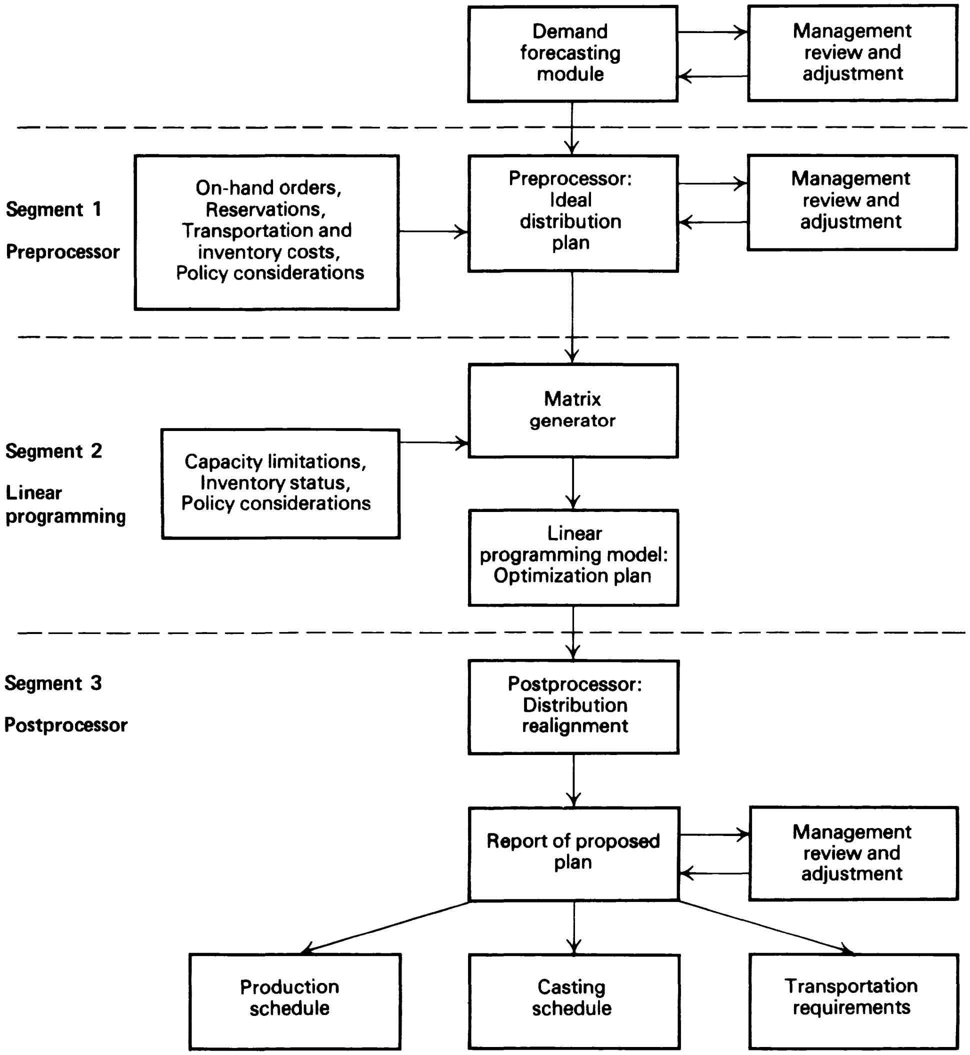

Fig. 6.3 General flow chart of the tactical system.

in order-number sequence, showing the freight-plus-in-transit-inventory cost for each order if it were shipped from each of the four smelters. Figure 6.4 is an example of this listing.

The second part of the system is the linear-programming model. This program accepts the output from the preprocessor program together with the operational constraints. The program outputs an optimum order assignment which minimizes the total logistics costs, taking into account all the constraints imposed by the production operations.

Order number	Order quantity	Source cost (Freight and in-transit inventory cost) Smelters I	II	III	IV
49001	350	10	12	27	28
49002	1000	25	20	41	44
49003	200	7	8	4	3
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
49999	290	18	16	10	8
Total assignment	6540	1554	1462	1632	1892

Note. The preprocessor produces one of these reports for each product group and for each time period. It assigns each order to minimize these costs, e.g., 49001 to smelter I, 49002 to smelter II.

Figure 6.4 Preprocessor order-assignment listing, for time period 1 and product group 1

The third segment of the system is a postprocessor program. This program is intended:

1. to consolidate those orders that may have been split between two or more smelters by the linear-programming model, since shipment from more than one source is, in general, not economical, because of the complexities created in controlling the order-processing procedures;
2. to provide the inventory policy the company should follow in order to minimize the cost of operating inventories (it should be noted that this is only for operating inventories; inventories carried for safety-stock purposes must be considered separately);
3. to provide the order-assignment output from the linear-programming model in an easily readable format;
4. to describe the marginal costs and shadow prices for the various distribution alternatives; and
5. to compute the size of the labor crews to be assigned in the coming weeks to each production center.

The main purposes of this segmented approach are to reduce the complexity and scope of the problem to a manageable size; to provide a means of seeing easily which orders could and should be changed from one time period to another; to greatly reduce the number of constraint equations in the linear-programming part of the model; to provide easy means of quickly viewing the variability of ideal demand for each of the smelters on a week-to-week basis; and to produce a report from the linear-programming output that managers can read and use for day-to-day planning operation.

There are two basic ways to reduce the number of constraint equations in the linear-programming model. One is simply to preassign some orders that must be produced on a particular machine for quality-control reasons. This preassigning is easily done in the preprocessor by reducing the capacities of the appropriate machines by the amount preassigned, and those orders will never reach the linear-programming model. The second way to reduce the size of the linear-programming model is by reducing the alternatives available for the reassignment of each order. If the choice for order assignment can be limited to only two smelters, then the corresponding constraint equation is reduced to a simple upper-bound type of constraint. The computational advantage of this approach will become clear when the linear-programming model is explained in detail.

Both the preprocessor and the postprocessor segments are relatively straightforward computer programs, which do not require much explanation. We now will review in detail the structure of the linear-programming model.

Tactical Linear-Programming Model—General Characteristics

As we indicated before, the model considers only the four smelters located in the U.S. It covers a three-month time horizon divided into six time periods of uneven duration. The first four time periods are one week each; the remaining two time periods cover the second and third months, respectively.

For scheduling purposes, each year is split into 13 periods, each of four weeks' duration. (This is an easy way to handle the uneven length of the calendar months.) A second convention adopted is the assignment of orders to time periods in terms of *when* the order must be completed at the smelter, rather than the *required delivery date* of the customer. This use of an ex-mill date (in lieu of a customer delivery date) is a common way to deal with the problem of deliveries to customer areas that are widely scattered.

An analysis of the product types revealed that while it is possible to use only eight different types for the strategic model, it is necessary to consider at least *thirteen* for the tactical model. The reason is the fact that, while some of the more subtle distinctions between product types can be ignored for long-range planning, they have to be considered for day-to-day assignments.

A number of capacity constraints must be considered in the program formulation. The first is that all the hot metal produced must be cast into some form, due to the impossibility of stocking hot metal as such. (This must be done even if there is insufficient current demand for the hot metal.) A second general constraint is that each product type requires a certain variety of production equipment. The total time required on a given machine must not exceed the capacity for that machine at the assigned smelter. This consideration is critical since, while some of the machinery is required for only one product type, other machines are jointly used for a number of different product types.

As indicated before, the preprocessor routine assigns orders based on minimizing freight and in-transit inventory costs without regard to the capacity constraints of the system. The preprocessor also lists the reassignment penalty associated with producing the order in an alternative smelter, measured by the extra freight and in-transit inventory cost that will result if a reassignment takes place. Another way to reassign an order is to produce it earlier than the time originally listed in the preprocessor program. This change results in an additional charge for inventory costs. The basic function of the linear-programming model is to decide on the best *reassignment* of orders, considering the capacity constraints at the smelters and *all* the logistics cost components (rather than only freight and in-transit inventory) as a basis for order allocation.

To account for the inherent uncertainties in each product demand, which cause unavoidable errors in the forecasts, and to prevent the model from exhausting the inventories at the end of each time period, constraints are imposed requiring the closing inventories to be at least equal to the safety stock associated with each product group.

The model is designed to minimize the total logistics cost. Major items of that cost are the order-reassignment penalties, the inventory-carrying charges, and the casting cost.

6.6 MATHEMATICAL FORMULATION OF THE TACTICAL PLANNING MODEL

The Tactical Linear-Programming Model—Notation

To facilitate the mathematical formulation of the model, a symbolic notation will be introduced to characterize its elements.

Time Periods

Each time period will be represented by the letter "t."

Smelter Locations

The four domestic smelters are represented by the subscript letter "s," where s = 1, 2, 3 or 4.

When considering transfers between smelters, instead of saying that the transfer will go from smelter "s" to smelter "s," the letter "ℓ" will be used to designate the receiving smelter. Thus the transfer will read as going from smelter "s" to smelter "ℓ."

Casting Machines

Each individual casting machine at each smelter is identified with a different number. This number is represented by the subscript "m" (m = 1, 2, . . . , 40). Thus it is possible to assign each order to a specific machine in a given smelter, since this is the way in which orders are scheduled. The model handles forty different machines, ten in each of the four smelters.

Product Groups

The 13 different product groups are represented by the letter "p" (p = 1, 2, . . . , 13). Combinations of product groups that require the use of some additional processes are represented by the letter "c" as a subscript. These product combinations introduce additional constraints imposed by secondary operations that require processing steps that use other than casting equipment (such as bundling, sawing, acid-dipping, and so forth). There are three such combinations.

Order Number

Each order is identified with a different number represented by the letter "k". This unique number is needed if we are to be able to identify where each order will be produced and whether an order has been split. It should be noted that no order contains more than one product group, and all the material in that order is to be delivered in only one time period.

Data Requirements

d_{tsp} Number of tons of product-group p assigned by the preprocessor to smelter s for production during time period t;

h_{ts} Number of tons of hot-metal capacity available at smelter s in time period t;

e_{tm} Number of hours available for the use of casting machine m during time period t;

r_{mp} Production rate of machine m for product-group p, in tons/hour;

g_{spt} Number of tons of safety stock of product group p to be carried at smelter s during time period t;

q_{k} Number of tons of aluminum contained in order k;

w_{tps} Upper bound of product-group p that can be produced at smelter s during time period t, in tons;

u_{cst} Upper bound of combination c that can be produced at smelter s in time period t, in tons;

$f_{\mathrm{s}\ell\mathrm{k}}$ Reassignment cost of transferring order k from smelter s to smelter ℓ, in dollars per ton;

a_{mp} Casting cost for the production of product-group p on machine m, in dollars per ton;

$i_{1\mathrm{p}}$ Weekly inventory-carrying cost per week for product-group p, in dollars per ton;

$i_{2\mathrm{p}}$ Monthly inventory-carrying cost for product-group p, in dollars per ton per month.

Decision Variables

P_{tmp} Number of tons of product group p to be produced on machine m during time period t;

I_{tps} Number of tons of inventory of product-group p at smelter location s at the end of time period t;

$R_{s\ell k}$ Number of tons of order k to be reassigned from smelter s to smelter ℓ.

The Tactical Linear-Programming Model—Formulation

With the notation defined above, we now can describe in exact mathematical terms the structure of the tactical linear-programming model.

The Objective Function—Logistics Cost

The objective function is to minimize the total logistics cost while fulfilling the operational constraints. The elements of the logistics cost are as follows:

$$\begin{aligned}
\text{Minimize cost} = & \sum_{s}\sum_{\ell \neq s}\sum_{k} f_{s\ell k} R_{s\ell k} && \text{Order reassignment} \\
& + \sum_{t \leqq 4}\sum_{p}\sum_{s} i_{1p} I_{tps} && \text{Weekly inventory-carrying charges} \\
& + \sum_{t=5,6}\sum_{p}\sum_{s} i_{2p} I_{tps} && \text{Monthly inventory-carrying charges} \\
& + \sum_{t}\sum_{m}\sum_{p} a_{mp} P_{tmp} && \text{Casting}
\end{aligned}$$

Demand Constraint

$$\sum_{m \text{ in } s} P_{tmp} + I_{(t-1)sp} - I_{tsp} + \sum_{k}\sum_{\ell \neq s} R_{s\ell k} - \sum_{k}\sum_{\ell \neq s} R_{\ell sk} = d_{tsp} \qquad \begin{pmatrix} t = 1, 2, \ldots, 6; \\ p = 1, 2, \ldots, 13; \\ s = 1, 2, 3, 4. \end{pmatrix}$$

This equation indicates that, for each time period, for each smelter, and for each product group, the total production in that smelter, plus the initial available inventory, minus the ending inventory, plus the reassignments out of the smelter, minus the reassignments to that smelter, should be equal to the demand assigned to the smelter by the preprocessor program. It is the material-balance equation for each product group. The quantity d_{tsp} has been determined by the preprocessor.

Hot-Metal Constraint

$$\sum_{p \text{ on } m}\sum_{m \text{ in } s} P_{tmp} = h_{ts} \qquad \begin{cases} t = 1, 2, \ldots, 6; \\ s = 1, 2, 3, 4. \end{cases}$$

This equation states that the total casting production at a smelter during time period t must equal the hot metal available for that smelter. The quantity h_{ts} has been determined by the strategic model.

Casting-Machine Capacity

$$\sum_{\text{p on m}} \frac{P_{\text{tmp}}}{r_{\text{mp}}} \leqq e_{\text{tm}} \qquad \begin{cases} \text{t} = 1, 2, \ldots, 6; \\ \text{m} = 1, 2, \ldots, 20. \end{cases}$$

This constraint merely states that the total machine time used cannot exceed the total machine time available for time period t. The conversion into hours for each machine (from tons) is based on the productivity factor for each machine. The summation is made over all product-groups p that require machine m. The quantity e_{tm} is suggested by the strategic model.

Production Bounds

$$\sum_{\text{m in s}} P_{\text{tmp}} \leqq w_{\text{tps}} \qquad \begin{cases} \text{t} = 1, 2, \ldots, 6; \\ \text{p} = 1, 2, \ldots, 13; \\ \text{s} = 1, 2, 3, 4; \\ \text{m} = 1, 2, \ldots, 20. \end{cases}$$

The production of product-group p on machine m for time period t cannot exceed the capacity of that smelter for that product during that time period.

Combination Constraints

$$\sum_{\text{p in c}} \sum_{\text{m in s}} P_{\text{tmp}} \leqq u_{\text{tsc}} \qquad \begin{cases} \text{t} = 1, 2, \ldots, 6; \\ \text{s} = 1, 2, 3, 4; \\ \text{c} = 1, 2, 3. \end{cases}$$

The number of tons of the product-group p in the combination c that are produced on machine m in smelter s during time period t cannot exceed the capacity for combination c during time period t at smelter s. This is a secondary constraint involving the capacity of a process step for certain of the product groups.

Ending-Inventory Constraints

$$I_{\text{tsp}} \geqq g_{\text{spt}} \qquad \begin{cases} \text{s} = 1, 2, 3, 4; \\ \text{t} = 1, 2, \ldots, 6; \\ \text{p} = 1, 2, \ldots, 13. \end{cases}$$

The amount of inventory of product-group p that is available at smelter s at the end of time period t should be greater than or equal to the safety stock required for product group p at smelter s.

Reassignment Balance

$$\sum_{\ell \neq \text{s}} R_{\text{s}\ell\text{k}} \leqq q_{\text{k}}, \qquad \text{for all k.}$$

The amount of order k that is reassigned from smelter s to all other smelters ℓ cannot exceed the total order quantity.

Note that neither the time period nor the product type is specified, since order k contains only one product type and it has an ex-mill date within just one time period.

This set of equations explains the role that the preprocessor plays in reducing the number of constraints of the linear-programming model. Since there are about one thousand orders to be scheduled during the three-month time horizon, in theory we should have that many reassignment constraints. However, for most orders we need only consider two alternative smelter locations; the first one corresponds to the location to which the order has been preassigned, and the second one presents a reassignment alternative. This means that only one variable is required in most reassignment-balance constraints, making that constraint a simple upper bound for that variable. This type of constraint is handled with little additional computational time by an upper-bounded linear-programming code. In the subject company, if the order assignment is to be made properly, about one hundred orders might be assigned to more than two alternative smelter locations. Therefore, using the preprocessor reduced the total set of constraints from one thousand to one hundred. Moreover, even the general reassignment-balance constraint can be handled easily by a special linear-programming code known as *generalized upper bounding*, which is available on many commercial programming systems.

Given the tendency of the linear-programming solutions to drive toward extreme points, in practice very few orders are split. The problem of split orders is thus very easy to resolve at the postprocessor level.

Nonnegativity of the Variables

All of the variables should be nonnegative.

6.7 CONCLUSION

We have described, with a fair amount of detail, a formal, integrated system to deal with some important logistics decisions in an aluminum company. We purposely have emphasized the formulational aspects of this project in order to illustrate the modeling effort required in many mathematical-programming applications.

This project also can be viewed as an example of the *hierarchical planning approach*, which was discussed in Chapter 5.

EXERCISES

I. Strategic Planning Model

1. Time Horizon of the Strategic Planning Model

Make a list of the objectives of the strategic planning model. What is the time horizon required to address each one of these specific objectives? Do these time horizons have the same length? If not, how do you think a single model could deal with the different time horizons? What is the proposed time horizon of the strategic-planning model? How many time periods does the model consider? How will the model handle the dynamic changes throughout the time horizon required to deal with each of the specified objectives?

2. *Objective Function of the Strategic Planning Model*

Review the elements of the objective function of the strategic-planning model. Why does the model minimize cost rather than maximize profit? Are reduction, casting, and transportation costs really linear? How might any nonlinearities be modeled?

How do the costs associated with the new smelter enter into the objective function? What could you do if there are trade-offs between fixed cost and variable-reduction cost in the new smelter (i.e., if there is one option involving a small fixed cost but high variable-reduction cost, which should be compared against an option consisting of a high fixed cost that generates a smaller variable-reduction cost)?

Why does the model ignore the procurement and transportation cost of raw materials to the smelters? Why are customs duties included in the objective function? (Aren't customs duties unavoidable, anyhow?)

Why aren't inventory costs other than in-transit inventory costs included in the model? Why doesn't the model impose costs for safety stocks, cycle stocks, stock-piling costs, seasonal stocks, work-in-process inventory, and so on? Why aren't the in-transit inventory costs and customs duties expressed directly in terms of the quantities Q_{sap}?

3. *Metal-Supply and Casting-Equipment Capacity Constraints*

Consider the metal-supply constraint

$$\sum_{a}\sum_{p} Q_{sap} - M_s = 0, \qquad s = 1, 2, \ldots, 11,$$

$$M_s \leqq \bar{m}_s, \qquad s = 1, 2, \ldots, 11, \tag{1}$$

$$M_s \geqq \underline{m}_s, \qquad s = 1, 2, \ldots, 11. \tag{2}$$

In total, there are 33 constraints used to express the metal-supply availability. An equivalent form for expressing these conditions will be:

$$\sum_{a}\sum_{p} Q_{sap} \leqq \bar{m}_s \qquad s = 1, 2, \ldots, 11,$$

$$\sum_{a}\sum_{p} Q_{sap} \geqq \underline{m}_s \qquad s = 1, 2, \ldots, 11.$$

which gives a total of 22 constraints. Why do you think the model uses the first formulation, which appears to unnecessarily increase the numbers of constraints required?

Consider constraints (1). What are the possible signs (positive, zero, negative) of the shadow prices associated with these constraints? How would you interpret the shadow prices? Answer the same questions with regard to constraints (2). From the values of these shadow prices, when would you consider expanding the metal capacity of a given smelter? When would you consider closing down a given smelter?

Similar questions can be posed with regard to casting-equipment capacity.

4. *Demand Constraints*

How many demand constraints are there in the strategic-planning model? What implications does the number of these constraints impose with regard to aggregation of information into product types and market regions?

What are the possible signs (positive, zero, negative) of the shadow prices associated with these constraints? How would you use this shadow-price information to decide on market-penetration strategies, and on swapping agreements? How could you deal with uncertainties in the demand requirements?

5. *Customs Duties and In-Transit Inventory Constraints*

Can these constraints be eliminated from the model? How would you accomplish this?

6. *Use of the Strategic-Planning Model to Support Managerial Objectives*

Refer to the list of objectives stated for the strategic-planning model, and discuss in detail how you think the model should be used to provide managerial support on each one of the decisions implied by the stated objectives. In particular, analyze how the model should be used in connection with the decision affecting the size and location of the new smelter.

II. Tactical Planning Model

1. *Time Horizon of the Tactical Planning Model*

Discuss each objective of the tactical model. How long is the time horizon required to address each of the specific objectives? How many time periods does the model consider? Why are multiple time periods essential in the tactical model?

2. *Segmentation of the Tactical Model*

Discuss the proposed segmentation of the tactical model. What are the functions, inputs, and outputs of the demand-forecasting module, preprocessor, matrix generator, linear-programming optimization routine, postprocessor, and report generator? How does the preprocessor help in reducing the computational requirements of the linear-programming model?

3. *Objective Function of the Tactical Linear-Programming Model*

Why is cost minimization preferred over profit maximization? What cost elements are implicit in the order-reassignment cost? Why are inventory-carrying charges divided into weekly and monthly charges? What additional cost elements might you include in a tactical model such as this one?

4. *Demand Constraints*

Discuss each term in the demand constraint. How is the righthand-side element of these constraints determined? How many constraints are there? What are the implications of these constraints with regard to aggregation of information? How would you interpret the shadow prices associated with these constraints? What feedback would these shadow prices provide to the strategic-planning model?

5. *Hot-Metal, Casting-Machine Capacity, Production Bounds, and Combination Constraints*

Analyze the nature of these constraints. Discuss their shadow-price interpretations, and the feedback implications to the strategic-planning model.

6. Ending-Inventory Constraints

Discuss the nature of these constraints. How would you specify the value of the righthand sides of these constraints? Are there alternative ways of expressing inventory targets, particularly when only safety stocks are involved and the model is going to be updated every review period? How taxing are these constraints in terms of computational requirements?

7. Reassignment-Balance Constraints

These constraints are critical for understanding the computational economies introduced by the preprocessor. Discuss what happens when only one reassignment alternative is considered for every order; what computational implications does this have? Interpret the shadow price associated with these constraints. Indicate an alternative formulation of the tactical-planning model without using the preprocessor or reassignment-decision variables.

III. Interaction between Strategic and Tactical Models

Analyze the hierarchical nature of the proposed planning system. Which outputs of the strategic model are transferred to the tactical model? What feedback from the tactical model can be useful for defining new alternatives to be tested by the strategic model? How might the problem be approached by a single model incorporating both tactical and strategic decisions? What would be the advantages and disadvantages of such an approach?

IV. Data Requirements

How much data do the strategic and planning models require? What would you do to collect this data? Which elements of the data would be most costly to collect? Which must be estimated most carefully?

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