

MIT SLOAN SCHOOL OF MANAGEMENT

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15.407
Fall 2003

Problem Set 1: Present Value
Solution

1. (a) The NPV of the project is $\sum_{t=1}^{10} 170,000/(1+14\%)^t - 800,000 = \$86,739.66$
 (b) At the end of 5 years, there will be 5 more years of income from the project, therefore it will be worth $\sum_{t=1}^5 170,000/(1+14\%)^t = \$583,623.76$

2. An annuity of \$1 over the next 12 years is worth

$$\frac{1}{8\%} \left[1 - \frac{1}{(1+8\%)^{12}} \right] = \$7.536$$
 Therefore, Basset can buy 2653.90 units of \$1 annuity and receive \$2563.90 every year.

3. Value of this year's production = \$1,400,000
 The growth rate of profit = $1.02 * 0.96 - 1 = -2.080\%$
 Using the annuity with growth formula, the PV of the oil well is

$$14 * 100,000 * \frac{1}{8\%+2.080\%} \left[1 - \left(\frac{1-2.080\%}{1+8\%} \right)^{18} \right]$$
 Therefore, PV of the well's production is \$11,508,076.

4. Sam wants to accumulate $\$300,000 * 1.02^{20} = \$445,784.22$ in nominal term at the end of 20 years. PV of this amount is 168,011.38.
 - (a) An annuity of \$1 over the next 20 years is worth PV of

$$\frac{1}{5\%} \left[1 - \frac{1}{(1+5\%)^{20}} \right] = \$12.462$$
 Therefore, he needs to save $168,011.38/12.462 = \$13,481.67$ every year. That is over 25% of his first year salary.
 - (b) An annuity that starts with \$1, and grow at 4% over the next 20 years is worth PV of

$$\frac{1}{5\%-5\%} \left[1 - \left(\frac{1+5\%}{1+5\%} \right)^{20} \right] = 17.419$$
 Therefore, in the first year he needs to invest \$9,645.36. After that he needs to invest 19.291% of his wealth each year.

Ignore the effect of all taxes.

5. (a) You pay the bank $200,000 * \left(\frac{1}{10\%} \left[1 - \frac{1}{(1+10\%)^{30}} \right] \right)^{-1} = 21,215.85$ each year.
 (b) You still owe the bank 21,215.85 units of a \$1 annuity (of 20 years) that is discounted at 10%
 Therefore, the principal you still owe the bank is $21,215.85 * \left(\frac{1}{10\%} \left[1 - \frac{1}{(1+10\%)^{20}} \right] \right) = 180,622.49$

- (c) To calculate PV, you need to discount your payments at the on-going rate of 6% instead of 10%:
Therefore, $PV = \$243,344.12$.
- (d) If you re-finance and pay the 10% penalty, you owe the bank PV of \$198,684.74. If you don't you owe \$243,344.12. Therefore you want to re-finance.
- 6.** You can invest \$20 into a project that pays you \$1 at the end of 1 year, \$2 at the end of two years, etc., until paying \$10 at the end of 10 years.
- (a) Calculate the IRR of the project.
- (b) The opportunity cost of the project is 15%. Is this an attractive project? Briefly explain.
- 7.** USD interest rate is 3% and British Pound interest rate is 6%. The current exchange rate is 1.5 USD per pound.
- You are offered a contract that allows you to buy 100,000 British pound with \$140,000 USD at the end of 3 years. Is this a worthwhile investment? Explain.