

MIT SLOAN SCHOOL OF MANAGEMENT

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15.407
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Problem Set 4: Forward and Futures

Due: October 21, 2003

1. Explain the similarities and differences between forwards and futures contract on the same asset. You may want to consider (but not limited to) the following points for the differences:
 - (a) How trade is conducted
 - (b) Liquidity
 - (c) Counter-party risk
 - (d) Flexibility

2. (BM) Current riskless rate is 6% per year and the term structure is flat. Consider the following commodities and assets:

Commodity/Asset	Spot price	Other Information
Magnosium	\$2,800 per ton	$\hat{y} = 4\%/year$
Oat bran	\$0.44/bushel	$\hat{y} = 5\%/year$
Biotech stock index	140.2	zero dividend yield
AW stock	\$58.00	\$2.40 dividend yield/year
5-year T-note	\$108.93	8% coupon/year
Westonian Ruble	R3.10 = \$1	Ruble interest rate 12%/year

- (a) Calculate the six-month futures price for each case.
- (b) Explain how a magnosium producer would use a futures market to lock in the selling price of a planned shipment of 1,000 tons of magnosium six months from now
- (c) Suppose the producer takes the actions recommended in your answer to (b), but after one month magnosium prices have fallen to \$2,200. What happens? Will the producer have to undertake additional futures market trades to restore its hedged position?
- (d) Does the biotech index futures provide useful information about the expected future performance of biotech stocks?
- (e) Suppose Allen Wrench stock falls suddenly by \$10 per share. Investors are confident that the cash dividend will not be reduced. What happens to the futures price?

- (f) Suppose interest rates suddenly fall. The spot rate for cash flows 6 months from now is 4% per year; it is 4.5 % for cash flows 12 months from now, 4.8% for cash flows 18 months from now, and 5% for all subsequent cashflows. What happens to the six-month futures price on the five-year Treasury note? What happens to a trader who shorted 100 notes at the future price calculated in part (a)?
- (g) An importer must make a payment of one million ruples three months from now. Explain *two* strategies the importer could use to hedge against unfavorable shifts in the ruple-dollar exchange rate.
3. You are the owner of \$100m worth of a 10yr bond that pays an annual coupon of 8%. The yield curve is currently flat at 5%. You wish to hedge your interest rate exposure, but the only instrument available to you is the 6-month future on a 2-year treasury discount bond (face value of \$100,000).
- (a) Find the two bonds' price and duration.
- (b) How will you hedge your exposure using the futures contract? Solve for the number of futures contracts you wish to buy or sell.
- (c) Suppose the yield curve does not change in 3 months. Are you still perfectly hedged now? Illustrate that by showing that you are not protected from a parallel shift in the yield curve now.
- (d) So what you need to do to protect yourself from future move of interest rates?
4. Forward price and future price is the same if the whole term structure is deterministic. This exercise asks you to show that they are the same when interest rate is known in a simple case.
- (a) Suppose the current 1-year spot rate is r_1^s , and one year later the 1-year spot rate will be r_2^s for sure. We have an asset with current price S_0 . Find the forward price of the asset (assume zero convenience yield). Denote the forward price F_0 .
- (b) We will consider a future contracts that expires at year 2. Denote the prices of the asset one year and two years from now by S_1 and S_2 . Let the current future price H_0 and the future price of the same contract one year from now be H_1 (so this is the price of a 1-year future). The special feature of a future is that it is marked-to-market. We assume this only takes place at the end of year 1. So at the end of year 1, the exchange calculate your gain/loss, and charge you with that amount immediately. So your payoff at year 1 is $H_1 - H_0$. Then when the contract expires, your gain loss is $S_2 - H_1$. The total gain/loss is still $S_2 - H_0$, but the timing is different. You need to determine H_0 . To do this, find H_1 and find a way to replicate your payoff in both time 1 and 2. More hint: Determine the amount of stock you need to buy today to replicate H_1 tomorrow. Then buy/sell some of your stock at time 1 to replicate the futures payoff.

5. The Latvian lati is pegged to the US dollar at a rate of 2 LTV per USD. This means that the Latvian central bank stands committed to buy and sell lati at that price in order to maintain the peg. However, 12-month interest rates in the US are currently at 2%, while a Latvian 12-month discount bond has a yield of 3.4%, while the overnight lati borrowing rate is at 3.3%. The spot rate is, 2 LTV/USD.
- What is the 6-month LTV/USD forward rate?
 - Can you somehow profit from the existing situation? Detail your strategy. Is it an arbitrage?
 - In your opinion, what makes the forward rate to be different than 2?
 - Now, suppose you are a speculator and wish to attack the lati. What is your strategy? What sort of risks (if any) are you running?
 - (Extra Credit) Now, suppose you are the Latvian Central Bank governor, and your charge is to defend against such an attack. What do you do? What does your success depend on and for how long can you keep it up?
6. Past weekly spot and futures prices of S&P 500 index and crude oil are given in a file `futures.csv` from the course website (<http://web.mit.edu/15.407>). Suppose today is June 10, 2002.
- Regress the changes in the spot prices on the changes in futures prices for both the S&P 500 index and crude oil and report the regression results (including point estimates and some statistics for these estimates).
 - Compute the minimum variance hedge ratio if we use the futures to hedge the risk of S&P 500 index.
 - Suppose that you have an oil reserve of 10 million barrels. You would like to use the oil futures traded on NYMEX (New York Mercantile Exchange) to hedge the price risk. How many oil futures contracts should you buy if you follow a minimum variance hedging strategy? (The contract size can be found from the Wall Street Journal, the Money & Investing section.) What is the basis risk of your hedging strategy (i.e., the standard deviation of the percentage change in the value of your total position)?
 - Suppose that you have \$200 million invested in S&P 500 index and 10 million barrels of oil reserve. You want to hedge against both risks using a combination of S&P futures and crude oil futures. What should be the number of contracts you need to buy/sell in each futures market?