MIT Sloan School of Management

J. Wang
E52-456

15.407
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Problem Set 4: Forward and Futures

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1. In terms of the contract, forwards and futures are very similar. Both of the require a transaction to be executed at a specified future date at a specified price.

However, there are a few differences:

Forward is trader over-the-counter (OTC), which means two party comes together and write a contract. Therefore, forward is very flexible and customized. However, as the buyers and sellers have different needs, the contracts are not homogenous and liquidity is low. There is also no third-party involved to protect either side from a default risk. Unless otherwise stated in the contract, there is no cash changing hand until the maturity date.

Futures is traded in an exchange. Contracts are standardized, with just a few maturities date available for each good. Because of this, liquidity is higher and it is easier to trade in and out of the market. Moreover, there is a clearing house which guarantees each side of the trade, therefore there is no default risk to anyone who trades in futures. To protect the clearing house, however, you are required to make a margin deposit, and your position is marked-to-market. Therefore you may be required to pay your loss before the maturity date.

2. (a) Magnoosium: [(2800(1.03 - 0.02)] =$2,828 per ton
Oat Bran: [0.44*(1.03 - 0.025)] =$0.4422 per bushel
Biotech: 1.03*140.2 = $144.4
Allen Wrench 1.03*[58.00 - (1.2/1.03)] = $58.54
5-year T-Note: 1.03*[108.93 - (4/1.03)] = $108.20
Ruple: $X = 3.1 * \frac{1.06}{1.03} = 3.19$

(b) The magnoosium producer would sell 1,000 tons of six-month magnoosium futures

(c) Because magnoosium prices have fallen, the magnoosium producer will receive payment from the exchange. It is not necessary for the producer to undertake additional futures market trades to restore its hedge position.

(d) No, the futures price depends on the spot price, the risk-free rate of interest, and the convenience yield.

(e) The futures price will fall to $48.24 (same calculation as above, with a spot price of $48)

(f) The 6-month rate is 3%. The new spot price of the 5-year Treasury note will be $113.18. Thus the futures price of the 5-year T-note will be 1.02*[113.18 - 4/1.02] = $111.44

The dealer who shorted 100 notes at the (previous) futures price has lost money.

(g) The importer could buy a three-month option to exchange dollars for ruples, or the importer could buy a futures contract, agreeing to exchange dollars for ruples in 3 months' time.

3. (a) Let the face value be \$100,000. The price of the bond is \$ 123,165.20 and the duration is 7.542

(b) We will need to sell some futures. The future price of the treasury bond is $\frac{100{,}000*1.05^{0.5}}{1.05^2} = \$92942.86$. The duration is 2 years. So we need to sell $\frac{\$100m*7.542}{92942.86*2} = 4,057.29$ contracts.

(c) After 3 months, the duration of the 10 year bond has decreased to 7.292, and the price has risen from 1.2317 to 1.2468 per dollar. The duration on the futures is 1.75, and the future price does no change. So the duration exposure is $\$100m * \frac{1.2468}{1.2317} * 7.292 - 4,057.29 * 92942.86 * 1.75 = 78,222,970.61$.
Since you are short on the hedge, you will suffer a loss if interest rates go up now. Suppose the rates go up by 1%. You will suffer a loss of \$6.717 million, but your hedge will only gain in value for \$5.324 million. Therefore you suffer a loss of \$1.393 million

(d) You will need to adjust your portfolio such that duration is matched again. You need to short 4538.2 contracts, so you want to sell more futures. As a matter of fact, since you have cashflow mismatch in your hedge, you want to re-balance your trade as often as possible.

4. (a) $F_0 = S_0 * (1 + r_1^s) * (1 + r_2^s)$

(b) We will show that $H_0 = F_0$. First, notice that $H_1 = F_1 = S_1 * (1 + r_2^s)$ because the futures is no longer marked-to-market. In order to do this, we need to replicate the payoff of the futures. We will do the following trades.

Time	Futures payoff	Replicating trade	Payoff
0	0	(i) Buy $(1 + r_2^s)$ of S (ii) Borrow $S_0 * (1 + r_2^s)$	0
1	$S_0 * (1 + r_2^s) - H_0$	(i) Sell off r_2^s of S (ii) Pays back your debt (iii) Borrow S_1	$S_0 * (1 + r_2^s) - H_0$
2	$S_2 - H_1$	(i) Sell off 1 stock (ii) Return your debt	$S_2 - H_1$

If this is confusing, I suggest you to draw a table and write down your holding in the asset, your liabilities, and current prices of stock for each period, before and after re-balancing. Remember that the asset sells at S_1 at time 1 and S_2 at time 2.

5. (a) It is $2 * (1.034/1.02)^{.5} = 2.0137$

(b) Yes. You can bet that the peg will hold. You sell lati forward at 2.0274 and simply wait. It is not however an arbitrage, since it is not riskless, see below.

(c) The forward is different because the market believes there might be a chance, however small, that the central bank might decide to abandon the peg. This is reflected in the higher interest rates in latvia (along with other things).

(d) First, you short as many futures contract on lati as you can. Then, before the futrues expires, borrow lati in the overnight market and sell them. If enough people do the same thing, the bank's dollar reserves could run out and the bank might decide to abandon the peg, and you gained from both spot and futures market. Since you borrowed at the overnight market (you cannot issue latvian government bonds!), you need to roll your position forward every day. While you are doing that, however, you are exposed to the Latvian (overnight) interest rates.

(e) Well, one thing you can do is increase the overnight rates, thus increasing the cost to the speculators. However, you cannot do this for a long period of time, since by increasing rates too much, you run the risk of harming the economy. Thus it becomes a game of brinkmanship between the speculators and the bank.

6. (a) A regression of changes in spot prices on changes in futures prices of S&P index

$$\Delta S_{\text{S\&P}} = \alpha_{\text{S\&P}} + \beta_{\text{S\&P}} \cdot \Delta F_{\text{S\&P}} + \epsilon_{\text{S\&P}}$$

reveals the following regression result:

Regression Output

R-Square	0.9740
Number of observations	649
Constant	0.6079 (weekly)
Standard error of constant	0.1366
Coefficient	0.9424
Standard error coefficient	0.0061
Variance of futures price change	508.17
Variance of spot price change	463.41

A regression of changes in spot prices on changes in futures prices of crude oil

$$\Delta S_{\text{oil}} = \alpha_{\text{oil}} + \beta_{\text{oil}} \cdot \Delta F_{\text{oil}} + \epsilon_{\text{oil}}$$

reveals the following regression result:

Regression Output

R-Square	0.6940
Number of observations	649
Constant	-0.0012
Standard error of constant	0.0293
Coefficient	0.8798
Standard error coefficient	0.0230
Variance of futures price change	1.6320
Variance of spot price change	1.8202

(b) The minimum hedge ratio is simply the coefficient of the regression, $\beta_{\text{S\&P}}$. If we want to use the futures to hedge the risk of S&P 500 index, the minimum hedge ratio should be 0.9424.

(c) To hedge the oil position, the firm should sell:

$$\frac{10000000}{1000} \times 0.8798 = 8798 \text{ futures contracts}$$

We know that

$$\mathrm{R}^2 = \frac{\beta_{\text{oil}}^2 \operatorname{Var}(\Delta F_{\text{oil}})}{\operatorname{Var}(\Delta S_{\text{oil}})}$$

and

$$1 - \mathrm{R}^2 = \frac{\operatorname{Var}(\epsilon)}{\operatorname{Var}(\Delta S_{\text{oil}})}$$

The basis risk of the hedging strategy is:

$$\begin{aligned} \frac{\sqrt{\operatorname{Var}(\epsilon)}}{S} &= \frac{1}{21.62} \times \sqrt{(1 - \mathrm{R}^2) \times \operatorname{Var}(\Delta S_{\text{oil}})} \\ &= 0.04625 \times \sqrt{0.3060 \times 1.8202} \\ &= 3.45\% \end{aligned}$$

(d) The point of this question is that if you simply use the individual hedge ratio, you are ignoring the correlation and you hedge will not be optimal.

First you have to normalize your holding to number of contracts. The ratio between S&P and oil is $\frac{\$200m}{10m * \$1030.74} = .0194$, i.e. you are holding 10 million units of portfolio, each unit being 1 barrel of oil and 0.0194 unit of the S&P index.

Now all you need to do is just repeat the exercise above, but run the regression of 1 unit of your portfolio against both oil and S&P futures. The regression gives you a hedge ratio of 0.01921 for S&P and 0.8783 for oil, which means you need to sell 0.1921m contracts of S&P futures and 8.783m contracts of oil futures. Correcting for the ratio in number of contract, the hedge ratio for 1 unit of S&P is 0.9901. You will notice that this is slightly different from hedging them individually. See the attached spreadsheet for the regressions and a comparison of the performance of the two hedges.